Name .... Teacher ....

MORIAH COLLEGE

Year 12 2007 Pre-Trial

Extension 1 MATHEMATICS

Time Allowed: 2 hours plus 5 minutes reading time

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Instructions:

- Answer every question. All questions are of equal value.
- Show all necessary working. Draw clear, well labelled diagrams.

Question 1. (12 Marks) Use a SEPARATE Booklet **Marks**

(a) Find the exact value of $\cos\left(\frac{5\pi}{4}\right)$. **1**

(b) Find:

(i) $\int e^{3x-5}\,dx$ **1**

(ii) $\frac{d}{dx}(\tan^{-1} 4x)$. **2**

(c) Evaluate $\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx$. **2**

(d) The point P divides the line AB externally in the ratio $1:4$. Find P if A is (2, 1) and B is (-4, 5). **2**

(e) $\lim_{x\to 0} \frac{\sin x}{3x}$ **1**

(f) Find the values of x which satisfy $\frac{3}{x-1} \le 3$. **3**

Question 2. (12 Marks) Use a SEPARATE Booklet **Marks**

(a) (i) Show that the curves $y = 4x$ and $y = x^3$ intersect at the point where $x = 2$. **2**

(ii) Find the acute angle between the two curves at $x = 2$, correct to the nearest degree. **2**

(b) Find the derivative of $f(x) = 5x^2 - x$ from first principles using the definition : **2**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

(c) Find the exact value of $\int_0^{\frac{\pi}{3}} 3\cos x \sin^2 x \, dx$. **2**

(d) Find the exact values of

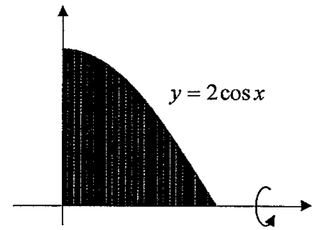
(i) $\sin^{-1}\left(\cos\frac{\pi}{6}\right)$. **1**

(ii) $\cos\left(2\sin^{-1}\frac{3}{7}\right)$. **3**

Question 3. (12 Marks) Use a SEPARATE Booklet **Marks**

(a) (i) Show that $\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$. **2**

(ii) The shaded region bounded by the curve $y = 2\cos x$ and the coordinate axes is rotated around the x-axis to form a solid.

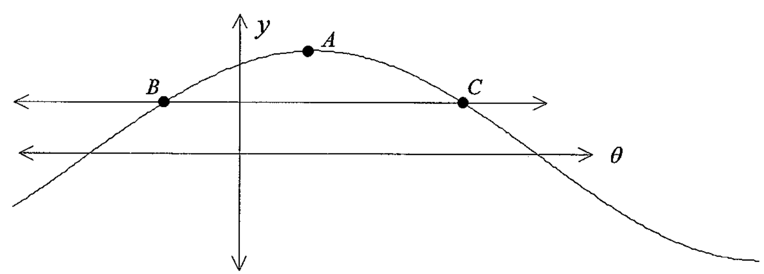


Using part (i), find the volume of the solid in terms of π. **3**

(b) (i) Express $\sin 2\theta + \sqrt{3}\cos 2\theta$ in the form $R\sin(2\theta + \alpha)$, where $0 \le \alpha \le \frac{\pi}{2}$ and $R > 0$. **2**

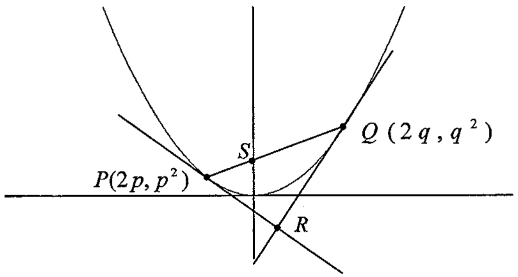
(ii) The graphs of $y = \sin 2\theta + \sqrt{3}\cos 2\theta$ and $y = 1$ are shown on the real number plane below. The point A is a relative maximum and the points B and C are points of intersection.

Find the coordinates of the points A, B and C. **5**



Question 4. (12 Marks) Use a SEPARATE Booklet **Marks**

(a)



The diagram shows the graph of the parabola $x^2 = 4y$. The line PQ is a focal chord which intersects the y-axis at S.

(i) Prove that the equation of the tangent to the parabola at P is $y = px - p^2$. **2**

(ii) The tangents at P and Q meet at point R. Find the coordinates of R. **2**

(iii) Find the equation of the line PQ and hence, show that $pq = -1$. **2**

(iv) Find the Cartesian equation of the locus of R, as P moves on the parabola. **2**

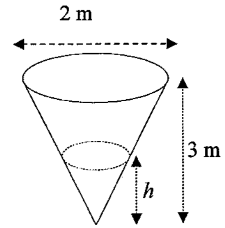
(b) Given that $\frac{d}{dx}(x\log x) = \log x + 1$, find the exact value of **2**

$$\int_e^{e^2} \frac{1+\log x}{x\log x}\,dx.$$

(c) Use the substitution $t = \tan\theta$ to show that $\sqrt{\frac{1-\sin 2\theta}{1+\sin 2\theta}} = \frac{1-\tan\theta}{1+\tan\theta}$, where $-1 < \tan\theta \le 1$. **2**

Question 5. (12 Marks) Use a SEPARATE Booklet **Marks**

(a)



A conical container, with diameter 2 metres and height 3 metres, is being filled with water at a rate of $0\cdot5\,\text{m}^3$/min. The volume of water V in the container at any time t, is given by:

$$V = \frac{\pi h^3}{27} \quad \text{(cubic metres)}$$

(i) Find $\frac{dV}{dh}$ and hence, show that the rate of increase in the height h, of the water at any time t is given by: **2**

$$\frac{dh}{dt} = \frac{9}{2\pi h^2} \quad \text{(metres/min)}$$

(ii) Find the exact rate of increase in the height h, of the liquid when the container is $\frac{1}{4}$ full. Give your answer in simplest form. **2**

(b) The line $y = mx$ is a tangent to the curve $y = \log x$.

(i) Find the value of m. **3**

(ii) Hence, find the range of values of k such that the equation $kx = \log x$ has two distinct roots. **2**

(c) Prove that $\frac{\sin 5\theta}{\sin\theta} - \frac{\cos 5\theta}{\cos\theta} = 4\cos 2\theta$ **3**

Question 6. (12 Marks) Use a SEPARATE Booklet **Marks**

(a) Use the principle of mathematical induction to prove that for all integers $n \geq 1$, **3**

$$2+10+24+...+n(3n-1)=n^2(n+1).$$

(b) Consider the function $f(x)=\dfrac{x}{\sqrt{1-x^2}}$

(i) Write down the domain of $f(x)$. **1**

(ii) Show that $f'(x)=\dfrac{1}{(1-x^2)^{\frac{3}{2}}}$. **2**

(iii) Find the equation of the tangent to $f(x)=\dfrac{x}{\sqrt{1-x^2}}$ at $x=0$. **1**

(iv) Sketch a graph of $y=f(x)$ clearly showing the tangent at $x=0$, and any asymptotes. **2**

(v) Sketch an accurate graph of $y=f^{-1}(x)$ on the same set of axes. **1**

(vi) Find the inverse function $y=f^{-1}(x)$. **2**

Question 7. (12 Marks) Use a SEPARATE Booklet **Marks**

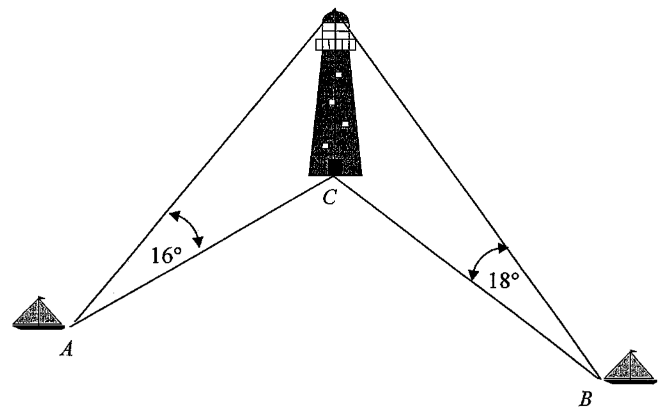
(a) A boat sails from a point A to a point B.

At point A the captain of the ship measures the angle of elevation of the top of a lighthouse as $16°$ and the bearing of the lighthouse as $040°$.

At point B the captain of the ship measures the angle of elevation of the top of the lighthouse as $18°$ and the bearing of the lighthouse as $340°$.

The top of the lighthouse is known to be 80 m above sea level.

The diagram below shows the angles of elevation of the top of the lighthouse from A and B.



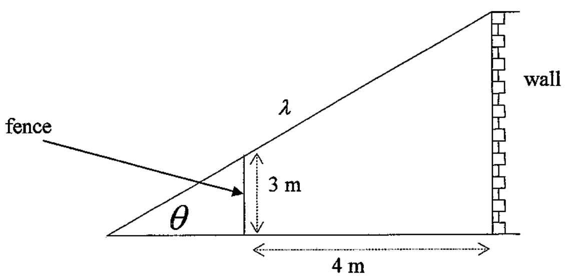
(i) Draw a bearing diagram showing the relative positions of A, B and C and use your diagram to explain why $\angle ACB = 60°$. **1**

(ii) Hence, find the distance between A and B, correct to the nearest metre. **3**

(iii) Hence, find the bearing of B from A, to the nearest degree. **2**

Marks

(b) A ladder λ m long is leaning against a vertical wall so that it just touches the top of a fence that is 3 m high and 4 m from the wall.
The ladder is inclined at an angle of θ radians to the ground.



(i) Prove that the length of the ladder $\lambda = \dfrac{3}{\sin\theta} + \dfrac{4}{\cos\theta}$. **2**

(ii) Show that if $\lambda = 10$ then the angle θ satisfies the equation **2**

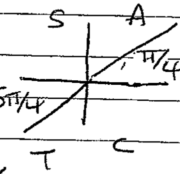
$\sin(2\theta) = \sin(\theta + \varphi)$, where $\tan\varphi = \dfrac{3}{4}$.

(iii) Hence, find the value(s) of θ. **2**

End of Paper

Question 1:

(a) $\cos\left(\frac{5\pi}{4}\right) = -\cos\frac{\pi}{4}$



$= -\frac{1}{\sqrt{2}}.$ ✓

(b) (i) $\int e^{3x-5}\,dx.$

$= \frac{1}{3}e^{3x-5} + c$ ✓

(ii) $\frac{d}{dx}\left(\tan^{-1}4x\right)$

$= \frac{1}{1+(4x)^2}\cdot 4$

$= \frac{4}{1+16x^2}$ ✓

(c) $\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx.$

$= \int_0^1 \frac{1}{\sqrt{(\sqrt{2})^2-x^2}}\,dx$

$= \left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^1$ ✓

$= \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}0$

$= \frac{\pi}{4} - 0$

$= \pi/4.$ ✓

(d) A $(2,1)$ B $(-4,5)$

$-1 : 4$

$P\left(\frac{2\times4 + -1\times-4}{3}, \frac{4\times1 + -1\times5}{3}\right)$ ✓

$= \left(4, -\frac{1}{3}\right)$ ✓

(e) $\lim_{x\to0} \frac{\sin x}{3x}$

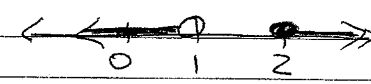
$= \frac{1}{3}\lim_{x\to0}\frac{\sin x}{x}$

$= \frac{1}{3}\times 1$

$= \frac{1}{3}.$ ✓

(f) $\frac{3}{x-1} \leq 3$

(i) $x \neq 1$



(ii) Assume true

$\frac{3}{x-1} = 3$ ✓

$3x-3=3$

$3x=6$

$x=2$

(iii) test $x=0$

$-3 \leq 3$ (✓ True

$\therefore x<1,\ x\geq 2.$ ✓ ✓

2) a) i)

$$x^3 = 4x$$
$$x^3 - 4x = 0$$
$$x(x^2-4) = 0$$
$$x = 0, \quad x^2 = 4$$
$$x = \pm 2$$

$\therefore$ the curves $y=4x$ and $y=x^3$ intersect at the point where $x=2$.

ii) $\frac{dy}{dx} = 4 \qquad \frac{dy}{dx} = 3x^2$

at $x=2$

$$\frac{dy}{dx} = 3 \times 2^2$$
$$= 3 \times 4$$
$$= 12$$

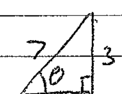
$$\tan\theta = \left| \frac{4-12}{1+(4)(12)} \right|$$
$$= \left| \frac{-8}{1+48} \right|$$
$$= \frac{8}{49}$$
$$\theta = \text{INV} \tan\left(\frac{8}{49}\right)$$
$$= 9^\circ$$

b) $$f'(x) = \lim_{h \to 0} \frac{5(x+h)^2 - (x+h) - (5x^2 - x)}{h}$$
$$= \lim_{h \to 0} \frac{5(x^2+2xh+h^2) - x - h - 5x^2 + x}{h}$$
$$= \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{x} - h - \cancel{5x^2} + \cancel{x}}{h}$$
$$= \lim_{h \to 0} 10x + 5h - 1$$
$$= 10x - 1$$

c) $$\int_0^{\frac{\pi}{3}} 3\cos x \sin^2 x \, dx = \left[\sin^3 x\right]_0^{\frac{\pi}{3}} \qquad \frac{d}{dx}\sin^3 x = 3\cos x \sin^2 x$$
$$= \sin^3\frac{\pi}{3} - \sin^3 0$$
$$= \left(\frac{\sqrt{3}}{2}\right)^3 - 0$$
$$= \frac{\sqrt{27}}{8}$$
$$= \frac{3\sqrt{3}}{8}$$

d) i) $$\sin^{-1}\left(\cos\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right)\right)$$
$$= \sin^{-1}\left(\sin\left(\frac{2\pi}{6}\right)\right)$$
$$= \frac{\pi}{3}$$

ii) $$\cos\left(\sin^{-1}\frac{3}{7} + \sin^{-1}\frac{3}{7}\right) = \cos^2\left(\sin^{-1}\frac{3}{7}\right) - \sin^2\left(\sin^{-1}\frac{3}{7}\right)$$



$\sqrt{7^2-3^2} = \sqrt{40}$
$= 2\sqrt{10}$

$$= \left(\cos\left(\cos^{-1}\frac{2\sqrt{10}}{7}\right)\right)^2 - \left(\frac{3}{7}\right)^2$$
$$= \left(\frac{2\sqrt{10}}{7}\right)^2 - \frac{9}{49}$$
$$= \frac{40}{49} - \frac{9}{49} = \frac{31}{49}$$

Question 3:

(a) (i)

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = \cos^2 x - (1 - \cos^2 x)$$
$$\cos 2x = \cos^2 x - 1 + \cos^2 x.$$
$$\cos 2x = 2\cos^2 x - 1$$
$$2\cos^2 x = \cos 2x + 1$$
$$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$$
$$\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$$

($\frac{1}{2}$ for setting out)

(ii) $V = \pi \int_0^{\pi/2} y^2 \, dx$ $\quad y = 2\cos x$

$y = 0$:
$2\cos x = 0$
$\cos x = 0$
$x = \pi/2$.

$$= \pi \int_0^{\pi/2} (2\cos x)^2 \, dx.$$
$$= \pi \int_0^{\pi/2} (4\cos^2 x) \, dx$$
$$= 4\pi \int_0^{\pi/2} \frac{1}{2}(\cos 2x + 1) \, dx.$$
$$= 2\pi \int_0^{\frac{\pi}{2}} (\cos 2x + 1) \, dx$$
$$= 2\pi \left[\frac{\sin 2x}{2} + x \right]_0^{\pi/2}.$$
$$= 2\pi \left[\pi/2 - 0 \right]$$
$$= \pi^2 \text{ units}^3.$$

(b) (i) $\sin 2\theta + \sqrt{3}\cos 2\theta = R\sin(2\theta + \alpha)$

$$R = \sqrt{1+3} = 2.$$

$$\sin(2\theta + \alpha) = \sin 2\theta \cos\alpha + \sin\alpha \cos 2\theta$$

$\cos\alpha = 1$ $\quad \tan\alpha = \sqrt{3}$
$\sin\alpha = \sqrt{3}$ $\quad \alpha = \frac{\pi}{3}.$

$$R\sin(2\theta + \alpha) = 2\sin\left(2\theta + \frac{\pi}{3}\right)$$

(ii) $\sin 2\theta + \sqrt{3}\cos 2\theta = 2\sin\left(2\theta + \frac{\pi}{3}\right).$

For A, $y = 2$.

$$\Rightarrow 2\sin\left(2\theta + \frac{\pi}{3}\right) = 2$$
$$\sin\left(2\theta + \frac{\pi}{3}\right) = 1$$
$$\sin^{-1}(1) = 2\theta + \frac{\pi}{3}$$
$$\frac{\pi}{2} = 2\theta + \frac{\pi}{3}$$
$$2\theta = \frac{\pi}{2} - \frac{\pi}{3}$$
$$2\theta = \frac{\pi}{6}$$
$$\theta = \frac{\pi}{12}$$

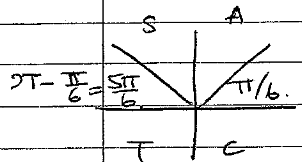
$\therefore A\left(\frac{\pi}{12}, 2\right)$

For B and C: $y = 1$.

$$2\sin\left(2\theta + \frac{\pi}{3}\right) = 1$$

$$\sin\left(2\theta + \frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sin^{-1}\left(\frac{1}{2}\right) = 2\theta + \frac{\pi}{3}$$



$$\frac{\pi}{6} = 2\theta + \frac{\pi}{3}, \quad \frac{5\pi}{6} = 2\theta + \frac{\pi}{3}$$

$$2\theta = \frac{\pi}{6} - \frac{\pi}{3}, \quad 2\theta = \frac{5\pi}{6} - \frac{\pi}{3}$$

$$2\theta = -\frac{\pi}{6} \qquad 2\theta = \frac{\pi}{2}$$

$$\theta = -\frac{\pi}{12} \qquad \theta = \frac{\pi}{4}$$

$\therefore B = \left(-\frac{\pi}{12}, 1\right) \qquad C = \left(\frac{\pi}{4}, 1\right)$

4) a) i) $x^2 = 4y$

$$y = \frac{x^2}{4} \qquad \frac{dy}{dx} = \frac{2x}{4}$$

$$m_{tangent} = \frac{2x}{4} = \frac{x}{2}$$

at P, $x = 2p$

$\Rightarrow m_{tangent} = \frac{2p}{2} = p$ (1)

$\therefore$ eq^n of tangent:

$$y - p^2 = p(x - 2p)$$

$$y = px - 2p^2 + p^2$$

$$y = px - p^2 \quad (1)$$

ii) tangent at Q must have eq^n: $y = qx - q^2$

$\therefore$ at intersection of tangent from P and tangent from Q:

$$px - p^2 = qx - q^2$$

$$(p-q)x = p^2 - q^2$$

$$x = \frac{p^2 - q^2}{(p-q)}$$

$$x = \frac{(p+q)(p-q)}{(p-q)}$$

$$x = p + q \quad (1)$$

Sub x into $y = px - p^2$

$$y = p(p+q) - p^2$$

$$= p^2 + pq - p^2$$

$$= pq$$

$\therefore R = (p+q, pq)$ (1)

iii) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{p^2 - q^2}{2p - 2q}$

$= \frac{(p+q)(p-q)}{2(p-q)}$

$= \frac{p+q}{2}$

PQ passes through $P(2p, p^2)$

$\Rightarrow \quad y - p^2 = \frac{p+q}{2}(x - 2p)$

$y - p^2 = \frac{(p+q)x}{2} - p^2 - pq$ ✓ 1

$y = \frac{(p+q)}{2}x - pq$

PQ passes through $S(0,1)$:

$\Rightarrow \quad 1 = -pq$

$pq = -1$ ✓ 1

iv) $\Rightarrow \quad R = (p+q, -1)$

$\therefore$ R moves on the line $y = -1$

b) $\frac{d}{dx}(x \log x) = \log x + 1$

$\int_e^{e^2} \frac{1 + \log x}{x \log x}\,dx = \left[\log|x \log x|\right]_e^{e^2}$ ✓ 1

$= \log|e^2 \log e^2| - \log|e \log e|$

$= \log|2e^2 \log e| - \log|e|$

$= \log\left|\frac{2e^2}{e}\right|$

$= \log|2e|$

$= \log 2 + 1$ ✓ 1

c) $LHS = \sqrt{\frac{1 - \frac{2t}{1+t^2}}{1 + \frac{2t}{1+t^2}}}$ $\qquad RHS = \frac{1 - \tan\theta}{1 + \tan\theta}$

$= \sqrt{\frac{1 + t^2 - 2t}{1 + t^2 + 2t}}$

$= \sqrt{\frac{(t-1)^2}{(t+1)^2}}$

$= \frac{t-1}{t+1}$

$= \frac{1 - \tan\theta}{\tan\theta + 1}$

$\therefore LHS = RHS$

5) a) i) $\frac{dV}{dh} = \frac{3\pi h^2}{27}$

$= \frac{\pi h^2}{9}$ (1)

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{9}{\pi h^2} \times \frac{1}{2}$

$= \frac{9}{2\pi h^2}$ (1)

ii) $\frac{1}{4} V_{TOT} = \frac{1}{4} \times \frac{\pi (3)^3}{27}$

$= \frac{\pi}{4}$ (1)

when $V = \frac{\pi}{4}$

$\frac{\pi}{4} = \frac{\pi h^3}{27}$

$\frac{27}{4} = h^3$

$h = \frac{3}{4^{\frac{1}{3}}}$ (1)

$= \frac{3}{2^{\frac{2}{3}}}$

Rate of increase in h is:

$\frac{dh}{dt} = \frac{9}{2\pi\left(\frac{3}{2^{\frac{2}{3}}}\right)^2}$

$= \frac{2^{\frac{4}{3}}}{2\pi}$

$= \frac{2^{\frac{1}{3}}}{\pi}$

$= \frac{\sqrt[3]{2}}{\pi}$ (1)

b) i) $y = \log x$ $\qquad y = mx$

$\frac{dy}{dx} = \frac{1}{x}$ $\qquad \frac{dy}{dx} = m$

$\Rightarrow m = \frac{1}{x}$

at pt of intersection

$mx = \log x$

$\frac{x}{x} = \log x$

$1 = \log x$

$e = x$

$\therefore m = \frac{1}{e}$ when $y = mx$ is a tangent to $y = \log x$

ii) The range of values for which $kx = \log x$ has two distinct roots is

$0 < k < \frac{1}{e}$

c) $\frac{\sin 5\theta}{\sin\theta} - \frac{\cos 5\theta}{\cos\theta} = \frac{\sin 5\theta\cos\theta - \cos 5\theta \sin\theta}{\sin\theta\cos\theta}$

$= \frac{\sin(5\theta - \theta)}{\sin\theta\cos\theta}$ ✓

$= \frac{\sin 4\theta}{\frac{1}{2}\sin 2\theta}$

$= \frac{4\sin 2\theta\cos 2\theta}{\sin 2\theta} = 4\cos 2\theta$

6) a) Prove that $2+10+24+\dots+n(3n+1)=n^2(n+1)$ for all integers $n \geq 1$,

Let $n=1$

$LHS = 2$ $\quad$ $RHS = 1^2(1+1)$
$= 1 \times 2$
$= 2$

$\therefore$ proposition is true for $n=1$ $\quad$ 1

Assume ✓ true for $n=k$; where $k \geq 1$

$$2+10+24+\dots+k(3k+1)=k^2(k+1)$$ 1

Prove true for $n=k+1$

$$2+10+24+\dots+k(3k+1)+(k+1)(3(k+1)+1)=(k+1)^2((k+1)+1)$$

$LHS = k^2(k+1)+(k+1)(3(k+1)+1)$ $\quad$ $RHS = (k+1)^2(k+2)$
$= (k+1)(k^2+3k+3+1)$
$= (k+1)(k^2+3k+2)$
$= (k+1)(k+1)(k+2)$ $\quad$ 1
$= (k+1)^2(k+2)$

$\therefore$ LHS=RHS
$\therefore$ proposition is true for $n=k+1$ if it is true for $n=k$.

$\therefore$ As the proposition is true for $n=1$ it is true for $n \geq 1$.

6) b) i) D: $x \in \mathbb{R}$, $-1 < x < 1$ $\quad$ 1

ii) let $u = x$ and $v = (1-x^2)^{\frac{1}{2}}$

$u' = 1$ $\quad$ $v' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \times -2x$
$= -x(1-x^2)^{-\frac{1}{2}}$

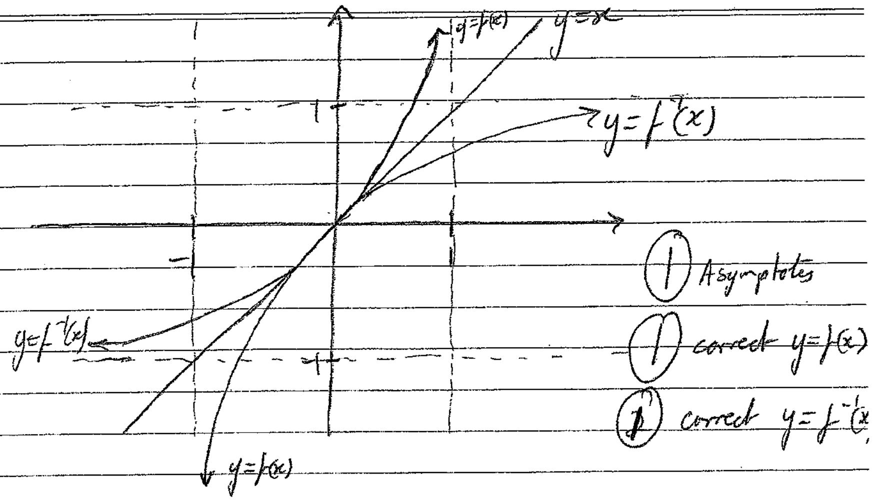
$$y' = \frac{vu' - uv'}{v^2}$$
$$= \frac{(1-x^2)^{\frac{1}{2}} + x^2(1-x^2)^{-\frac{1}{2}}}{(1-x^2)}$$ 1
$$= \frac{(1-x^2)^{\frac{1}{2}} + x^2(1-x)^{-\frac{1}{2}}}{1-x^2} \times \frac{(1-x^2)^{\frac{1}{2}}}{(1-x^2)^{\frac{1}{2}}}$$
$$= \frac{1-x^2+x^2}{(1-x^2)(1-x^2)^{\frac{1}{2}}}$$ 1
$$= \frac{1}{(1-x^2)^{\frac{3}{2}}}$$

iii) $m_{tangent} = \frac{1}{(1-0^2)^{\frac{3}{2}}}$ $\quad$ $f(0) = \frac{0}{\sqrt{1-0^2}}$
$= 1$ $\quad$ $= 0$

$$y - 0 = 1(x-0)$$

eqⁿ of tangent: $y = x$ $\quad$ 1

iv)



① Asymptotes

① correct $y=f(x)$

① correct $y=f^{-1}(x)$

$f(-x)$ is $-ve$ $\quad$ $f'(x)=1$ when $x=0$

$f(x)$ is $+ve$

v) $y=f(x)$

$$y=\frac{x}{\sqrt{1-x^2}}$$

$\therefore$ inverse is:

$$x=\frac{y}{\sqrt{1-y^2}} \quad 1$$

$$x\sqrt{1-y^2}=y$$

$$x^2(1-y^2)=y^2$$

$$x^2-x^2y^2=y^2$$

$$-y^2-x^2y^2=-x^2$$

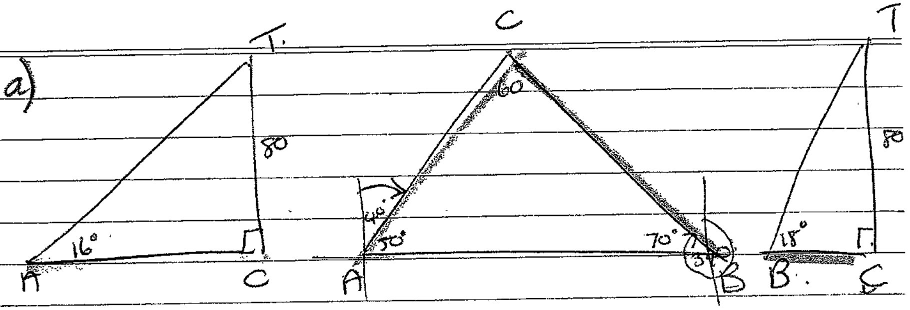
$$y^2+x^2y^2=x^2$$

$$y^2(1+x^2)=x^2 \quad \therefore f^{-1}(x)=\frac{x}{\sqrt{1+x^2}}$$

$$y^2=\frac{x^2}{1+x^2}$$

$$y=\frac{x}{\sqrt{1+x^2}} \quad 1$$

7) a)



$\angle ACB = 60°$. $\quad$ $\angle CAB = 50°$

$\angle CBA = 70°$ ✓ 1

$\therefore$ 3∠ of $\Delta = 180°$.

In Δ ACT:

$$\frac{AC}{80}=\cot 16°$$

$$AC=\frac{80}{\tan 16°} ✓$$

In Δ BCT:

$$\frac{BC}{80}=\cot 18°$$

$$BC=\frac{80}{\tan 18°} ✓$$

In Δ ABC

By cosine rule:

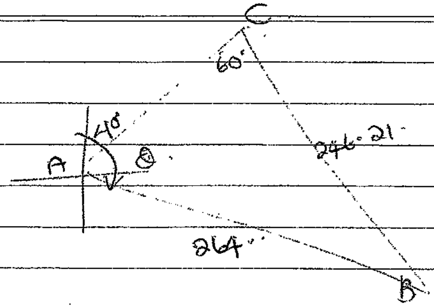
$$AB^2 = AC^2+BC^2-2(AC)(BC)\cos 60° ✓$$

$$AB^2=\left(\frac{80}{\tan 16}\right)^2+\left(\frac{80}{\tan 18}\right)^2-2\left(\frac{80}{\tan 16}\right)\left(\frac{80}{\tan 18}\right)\cos 60°$$

$AB^2 = 69766$ ✓ $\quad$ 3

$AB = 264.13$

$AB = 264.13$ (to nearest m).



$$\frac{\sin\theta}{246.21} = \frac{\sin 60}{264}$$

$$\sin\theta = \frac{\sin 60}{264} \times 246.21$$

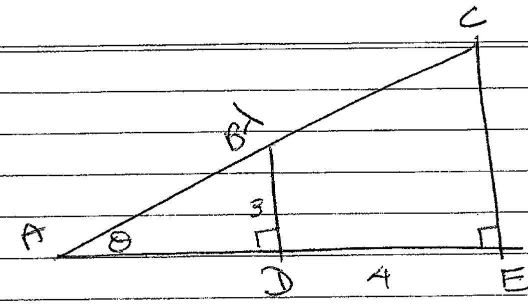
$$\sin\theta = 0.807$$ ✓ 2

$$\theta = 53°50'$$

$\therefore$ bearing $= 40° + 53°50'$ ✓

$= 93°50'$

7)b)i)



In ΔACE:

$$\tan\theta = \frac{3}{AD}$$

$$AD = \frac{3}{\tan\theta}$$ ✓

$$\cos\theta = \frac{AE}{\lambda}$$

$$\lambda = \frac{AE}{\cos\theta}$$

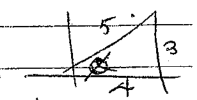
$$\lambda = \frac{AD + 4}{\cos\theta}$$ ✓

$$\lambda = \frac{\frac{3}{\tan\theta} + 4}{\cos\theta}$$ ✓

$$\lambda = \frac{3\cancel{\cos\theta}}{\sin\theta} \times \frac{1}{\cancel{\cos\theta}} + \frac{4}{\cos\theta}$$ 2

$$\lambda = \frac{3}{\sin\theta} + \frac{4}{\cos\theta}$$ ✓

if $\lambda = 10$: $$10 = \frac{3\cos\theta + 4\sin\theta}{\sin\theta\cos\theta}$$ ✓



$$10\sin\theta\cos\theta = 3\cos\theta + 4\sin\theta$$ ✓

$$5(2\sin\theta\cos\theta) = 3\cos\theta + 4\sin\theta$$ ✓ 2

$$2\sin\theta\cos\theta = \frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta$$ ✓

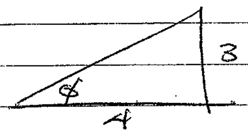
$$\sin 2\theta = \sin\phi\cos\theta + \cos\phi\sin\theta$$

$$= \sin(\theta + \phi)$$ ✓

iii) $\sin 2\theta = \sin(\theta + \phi)$

$\therefore 2\theta = \theta + \phi$ ✓ or $2\theta = 180° - (\theta + \phi)$ ✓

$\theta = \phi$ $\qquad$ $3\theta = 180 - \phi$.



2.

$\phi = \tan^{-1} 3/4$

$\phi = 36°52'$

$\therefore \theta = 36°52'$ ✓ or $3\theta = 180 - 36°52'$

$3\theta = 143°8'$

$\theta = 47°43'$ ✓