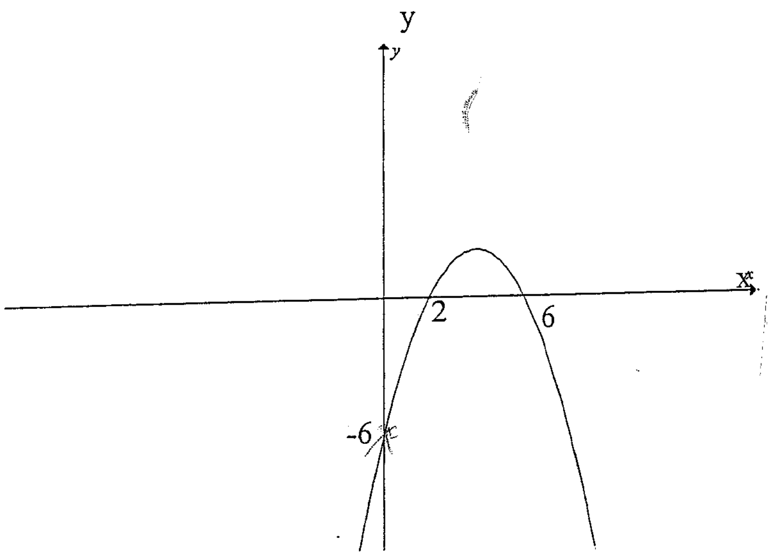
MORIAH COLLEGE
MATHEMATICS DEPARTMENT

3U Test on Quadratic Theory

1 Solve $x^2 \geq 2x$

2 Find the equation of the parabola below.



3 If α, β are the roots of $2x^2 - 13x + 9 = 0$, show that $(\alpha + 1)(\beta + 1) = 12$

4 If $2n^2 + 3n - 1 \equiv an(n-1) + bn + c$ find a, b, c.

5 Solve $(2x^2 + x)^2 + (2x^2 + x) - 2 = 0$ for x.

6 If α, β are the roots of $2x^2 + 7x + 4 = 0$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

7 Show that $x^2 + (2+k)x + k = 0$ has two distinct roots for all values of k.

11 For what values of m will $x^2 + 3x + 6 = m(x+2)$, have

i) 2 distinct roots

ii) equal roots

iii) no roots.

-Illustrate on a graph the situation when the roots are equal (from part ii) showing $y = x^2 + 3x + 6$ and $y = m(x+2)$ for various values of m.

8 Write $y = 2x^2 + 3x - 4$ in the form $y = a(x - p)^2 + q$ and hence find the vertex.

9 Show that $x^2 + 2ax + a^2 + b^2$ is positive definite.

10 For $ax^2 + bx + c = 0$, one root is 3 times the other. Show that $16ac = 3b^2$

MORIAH COLLEGE
MATHEMATICS DEPARTMENT

3U Test on Quadratic Theory

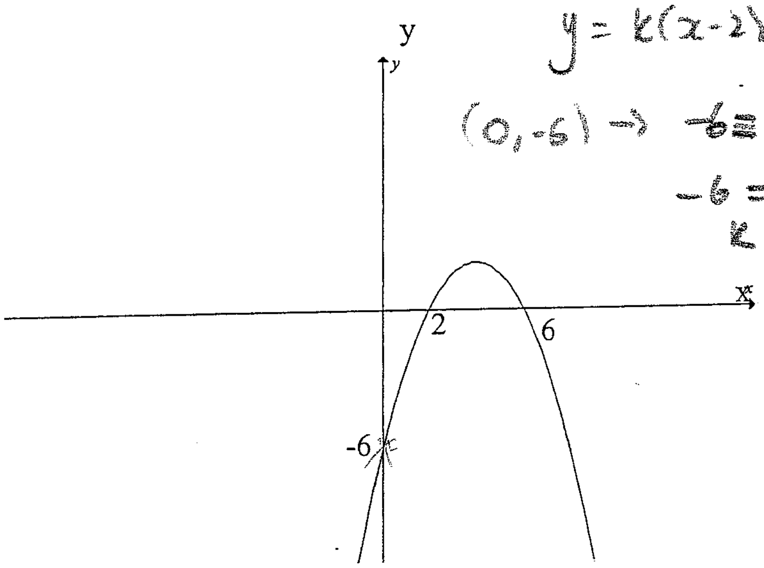
1 Solve $x^2 \geq 2x$

$x^2 - 2x \geq 0$

$x(x-2) \geq 0$

$x \geq 2$, or , $x \leq 0$

2 Find the equation of the parabola below.



$y = k(x-2)(x-6)$

$(0,-6) \to -6 = k(-2)(-6)$

$-6 = 12k$

$k = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}(x-2)(x-6)$

3 If α, β are the roots of $2x^2 - 13x + 9 = 0$, show that $(\alpha+1)(\beta+1) = 12$

$\frac{-13}{2} = -(\alpha+\beta)$

① $\frac{13}{2} = (\alpha+\beta)$

② $\frac{9}{2} = \alpha\beta$

1

$(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$\frac{9}{2} + \frac{13}{2} + 1$

$= 12$

$\therefore (\alpha+1)(\beta+1) = 12$

4 If $2n^2+3n-1 \equiv an(n-1)+bn+c$ find a, b, c.

$2n^2+3n-1 \equiv an^2 - an + bn + c$

$\therefore \boxed{a=2}$

$\boxed{c=-1}$

$3 = b-a$

$3 = b-2$

$\boxed{b=5}$

5 Solve $(2x^2+x)^2+(2x^2+x)-2=0$ for x.

let $2x^2+x = m$

$\therefore m^2+m-2=0$

$(m+2)(m-1)=0$

$\therefore m=-2$ or $m=1$

$\therefore 2x^2+x=-2$ or $2x^2+x=1$.

$2x^2+x+2=0$ has no solution.

$2x^2+x-1=0$

$x = \frac{-1\pm\sqrt{1-4\times2\times-1}}{2\times2}$

$= \frac{-1\pm\sqrt{9}}{4}$

$= \frac{-1\pm3}{4}$

$x=1$ or $x=-\frac{1}{2}$

6 If α, β are the roots of $2x^2+7x+4=0$, find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$.

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$\alpha\beta = \frac{4}{2}$.

$\boxed{\alpha\beta = \frac{4}{2}}$.

$\boxed{\alpha+\beta = -\frac{7}{2}}$.

$\therefore \alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

$= \frac{49}{4} - 4$

$= \frac{33}{4} = 8\frac{1}{4}$

$\therefore \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{33}{4} \times \frac{1}{2}$

$= \frac{33}{8}$

$= 4\frac{1}{8}$

7 Show that $x^2+(2+k)x+k=0$ has two distinct roots for all values of k.

$\Delta = (2+k)^2 - 4k$

$= (2+k)^2 - 4k$

$= 4+4k+k^2-4k$

$= k^2+4$

$k^2+4 > 0$ for all k.

8 Write $y = 2x^2 + 3x - 4$ in the form $y = a(x - p)^2 + q$ and hence find the vertex.

$$y = 2(x^2 + \tfrac{3}{2}x + \tfrac{9}{16}) - 4 - 2(\tfrac{9}{16})$$
$$= 2(x + \tfrac{3}{4})^2 - 5\tfrac{1}{8}$$

9 Show that $x^2 + 2ax + a^2 + b^2$ is positive definite.

$a > 0$. ✓

$$\Delta = (2a)^2 - 4 \times (a^2 + b^2)$$
$$= 4a^2 - 4a^2 - 4b^2$$
$$= 4(a^2 - a^2 - b^2)$$
$$= -4b^2$$
$$< 0$$

$\therefore$ pos. definite as $a > 0$, and $\Delta < 0$.

10 For $ax^2 + bx + c = 0$, one root is 3 times the other. Show that $16ac = 3b^2$

let roots be $\alpha, 3\alpha$.

$$3\alpha + \alpha = -\frac{b}{a}$$
$$4\alpha = -\frac{b}{a}$$
$$\alpha = \frac{-b}{4a}$$

$$\therefore a \times \left(\frac{-b}{4a}\right)^2 + b\left(\frac{-b}{4a}\right) + c = 0$$
$$a\left(\frac{b^2}{16a^2}\right) + b\left(\frac{-b}{4a}\right) + c = 0$$
$$\frac{b^2}{16a} - \frac{b^2}{4a} + c = 0$$
$$\frac{b^2}{16a} - \frac{4b^2}{16a} + c = 0.$$
$$\frac{-3b^2}{16a} + c = 0$$
$$\therefore 16ac = 3b^2$$

11 For what values of m will $x^2+3x+6=m(x+2)$, have

$x^2+3x+6=mx+2m$

$x^2+3x-mx+6-2m=0$

$x^2+x(3-m)+6-2m=0$

i) 2 distinct roots

$\Delta>0$

$(3-m)^2-4\times(6-2m)>0.$

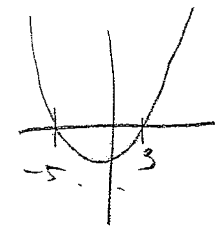
$9-6m+m^2-24+8m>0$

$m^2+2m-15>0$

$m=\frac{-2\pm\sqrt{4+60}}{2}$

$m=\frac{-2\pm 8}{2}$

$m>3$ or $m<-5$



ii) equal roots

$\Delta=0$

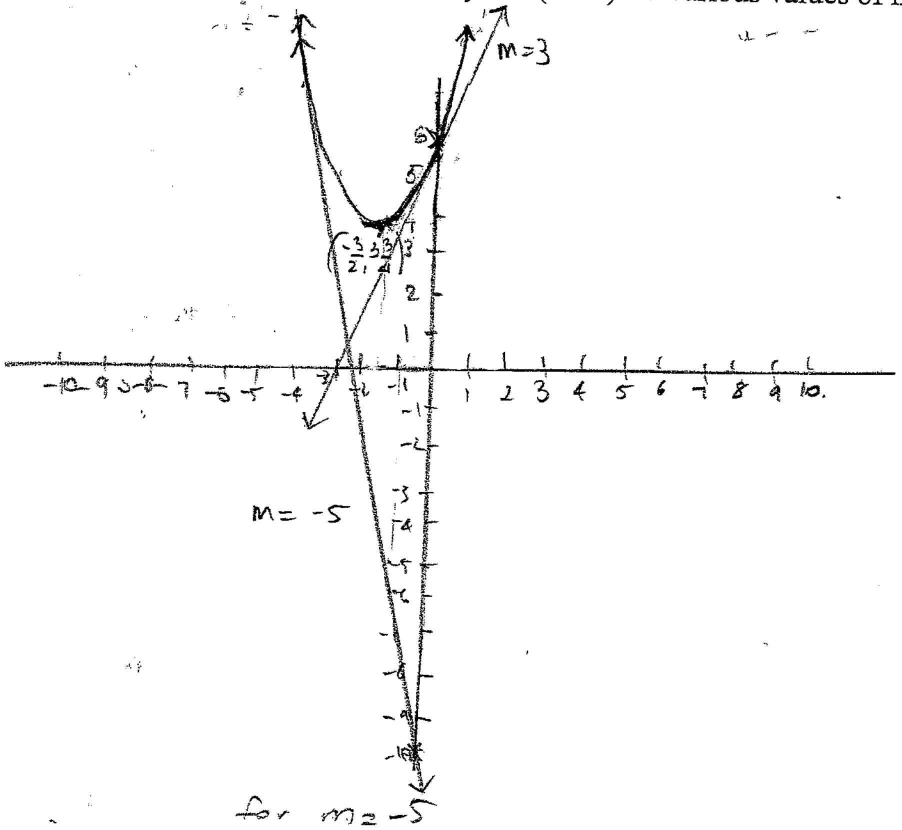
$m=3$, or, $m=-5$

iii) no roots.

$\Delta<0.$

$-5<m<3$

Illustrate on a graph the situation when the roots are equal (from part ii) showing $y=x^2+3x+6$ and $y=m(x+2)$ for various values of m.



for m=3

$x^2+3x+6=3x+6$

$x^2=0.$

$x=0$

for m=-5

$x^2+3x+6=-5x-10$

$x^2+8x+16=0.$

$(x+4)^2=0$

$x=-4$