MORIAH COLLEGE

MATHEMATICS DEPARTMENT

Coordinate Geometry

Name .... Class: ....

Question 1

Show the points $A(-2, -3)$, $B\ (1, 1)$ and $C(7, 9)$ are collinear.

Question 2

In this question, you must use the "k" method.

Find the equation of the line through the point of intersection of the two lines $2x-3y-2=0$ and $x=2y$ which is parallel to the line $2x-y+5=0$.

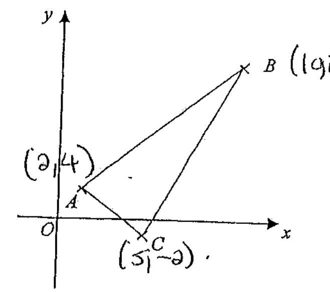
Name .... Class: ....

Question 3

Prove that $3x+4y-15=0$ is a tangent to the circle $x^2+y^2=9$

Question 4

The triangle ABC has vertices $A(2,4)$, $B(10, 10)$ and $C(5, -2)$.



i) Find the equation of the line AB.

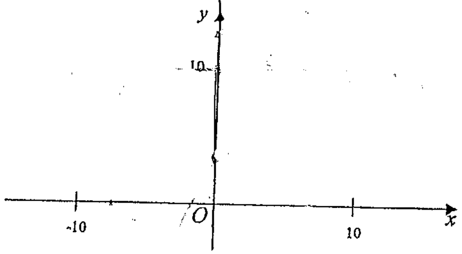
ii) Find the length of AB

iii) Find the area of the triangle ABC

Name .... Class: ....

Question 5

i) Plot the points $A(6, 10)$, $B(-6, 10)$, $C(0, 12)$ on the number plane.



ii) *Write down* the equation of the perpendicular bisector of AB

iii) The circumcentre of a triangle is the point where the perpendicular bisectors of the sides meet. Find the coordinates of the circumcentre

iv) Find the equation of the circumcircle, which is the circle passing through A, B and C.

Name .... Class: ....

Question 6

i) Divide the interval PQ externally in the ratio 5:2, where P is $(-2, 3)$ and Q is $(4, 15)$

ii) A centroid is the point which divides any median of a triangle in the ratio 2:1. Find the centroid of the triangle with vertices $A(-2,4)$, $B(-8, 10)$ and $C(2, 4)$.

iii) A is $(-5, 2)$ B is $(3, 10)$ and P is $(1, 8)$. In what ratio does P divide AB?

Name .... Class: ....

Question 7

The points $OPQR$ form a parallelogram. Take the points as

$$O(0,\ 0),\ P(a,\ 0),\ Q(b + a,\ c) \text{ and } R(b,\ c).$$

Prove, using coordinate geometry, that if the diagonals are perpendicular then the figure is a rhombus.

20 mins

MORIAH COLLEGE
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Name SOLUTIONS Class: 1 hour

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Question 1
Show the points $A(-2,-3)$, $B(1,1)$ and $C(7,9)$ are collinear.

$m(AB) = \frac{1--3}{1--2} = \frac{4}{3}$

$m(BC) = \frac{9-1}{7-1} = \frac{8}{6} = \frac{4}{3}$

$\therefore$ same gradient + common point B

$\therefore$ collinear.

Question 2
In this question, you must use the "k" method.
Find the equation of the line through the point of intersection of the two lines $2x-3y-2=0$ and $x=2y$ which is parallel to the line $2x-y+5=0$.

$(2x-3y-2)+k(x-2y)=0$

$\parallel$ $2x+5=y$. $\boxed{\therefore m = 2.}$

$2x-3y-2+kx-2ky=0$

$-3y-2ky=-2x-kx+2$

$y(-3-2k)=x(-2-k)+2$

$y = \frac{x(-2-k)}{-3-2k} + \frac{2}{-3-2k}$

same gradient

$\therefore \frac{-2-k}{-3-2k} = 2$

$-2-k=-6-4k$

$3k=-4$

$k=\frac{-4}{3}$

$(2x-3y-2)-\frac{4}{3}(x-2y)=0$

$2x-y-6=0$

Name .... Class: ....

Question 3
Prove that $3x+4y-15=0$ is a tangent to the circle $x^2+y^2=9$ $\quad (x_1,y_1)\ (0,0)$. $\quad r=3$.

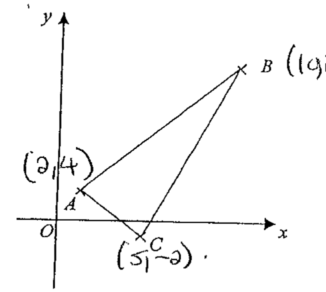
tangent if $\perp d = 3$.

$\perp d = \frac{|(3)(0)+(4)(0)-15|}{\sqrt{3^2+4^2}}$

$= \frac{|-15|}{5}$

$= 3$.

Question 4
The triangle ABC has vertices $A(2,4)$, $B(10,10)$ and $C(5,-2)$.



i) Find the equation of the line AB.

$m(AB) = \frac{10-4}{10-2} = \frac{6}{8} = \frac{3}{4}$. $\quad (2,4)$

eq: $y-4=\frac{3}{4}(x-2)$

$4y-16=3x-6$.

$4y=3x+10$.

ii) Find the length of AB

$d(AB) = \sqrt{(10-2)^2+(10-4)^2}$

$=\sqrt{64+36}$

$=10$.

iii) Find the area of the triangle ABC

$\perp d$ from C to AB: $\quad (x_1,y_1)\ (5,-2) \quad 3x-4y+10=0$.

$\perp d = \frac{|(3)(5)+(-4)(-2)+10|}{\sqrt{3^2+(-4)^2}}$

$= \frac{|15+8+10|}{5}$

$= \frac{33}{5}$.

$\therefore$ Area of $\Delta = \frac{1}{2}bh$

$= \frac{1}{2}\cdot 10 \cdot \frac{33}{5}$

$= 33u^2$

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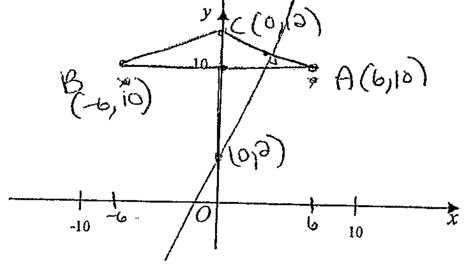
Name .... Class: ....

Question 5

i) Plot the points $A(6, 10)$, $B(-6, 10)$, $C(0, 12)$ on the number plane.



✓ 1.

1

ii) *Write down* the equation of the perpendicular bisector of AB

$x = 0$. ✓ 1.

1

iii) The circumcentre of a triangle is the point where the perpendicular bisectors of the sides meet. Find the coordinates of the circumcentre

midpt $(AC) = \left(\frac{0+6}{2}, \frac{12+10}{2}\right) = (3, 11)$ ✓

$m(AC) = \frac{10-12}{6-0} = \frac{-2}{6} = -\frac{1}{3}$ ✓ 3.

$\therefore \perp m = +3$.

eq of $\perp$ bisector: $y - 11 = 3(x-3)$

$y - 11 = 3x - 9$

$y = 3x + 2$. ✓

4

$\therefore$ at $x = 0$, $y = 2$. circumcentre $(0, 2)$. ✓ 1.

iv) Find the equation of the circumcircle, which is the circle passing through A, B and C.

$r = 10$. ✓ centre $(0, 2)$

$(x-0)^2 + (y-2)^2 = 10^2$ 2.

$x^2 + (y-2)^2 = 100$. ✓

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8.

Name .... Class: ....

Question 6

i) Divide the interval PQ externally in the ratio 5:2, where P is $(-2, 3)$ and Q is $(4, 15)$

$k \; l$ $x_1 y_1$ $x_2 y_2$

$D = (x, y)$

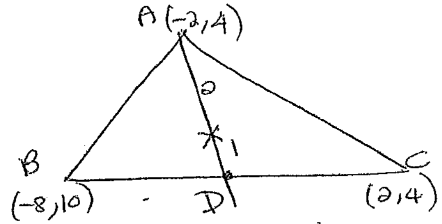
$x = \frac{(-2)(-2) + (5)(4)}{5-2}$ | $y = \frac{(-2)(3) + (5)(15)}{5-2}$

$x = \frac{4+20}{3}$ | $y = \frac{-6+75}{3}$

$x = 8$ 1 | $y = \frac{69}{3}$ 1

$(8, 23)$

ii) A centroid is the point which divides any median of a triangle in the ratio 2:1. Find the centroid of the triangle with vertices $A(-2,4)$, $B(-8, 10)$ and $C(2, 4)$.



$A(-2, 4)$ $D(-3, 7)$ $k : l$ $2 : 1$

$x = \frac{(1)(-2) + (2)(-3)}{3}$ $y = \frac{(1)(4) + }{3}$

$x = \frac{-2-6}{3}$, $y = \frac{4+14}{3}$ 1

$x = -\frac{8}{3}$, $y = \frac{18}{3}$. 1

midpt $(BC) = \left(\frac{-8+2}{2}, \frac{10+4}{2}\right)$

$D = (-3, 7)$. 1

centroid $\left(\frac{-8}{3}, \frac{18}{3}\right)$

$= \left(-\frac{8}{3}, 6\right)$. 1

iii) A is $(-5, 2)$ B is $(3, 10)$ and P is $(1, 8)$. In what ratio does P divide AB?

ratio : $k : l$.

$1 = \frac{(l)(-5) + (k)(3)}{k+l}$ 1

$1 = \frac{-5l + 3k}{k+l}$

$k + l = -5l + 3k$ 1

$-2k = -6l$

$k = 3l$.

$\frac{k}{l} = \frac{3}{1}$. 1

$\therefore$ ratio $3 : 1$

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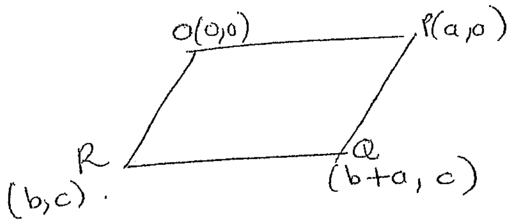
Name .... Class: ....

Question 7

The points *OPQR* form a parallelogram. Take the points as

$O(0, 0)$, $P(a, 0)$, $Q(b + a, c)$ and $R(b, c)$.

Prove, using coordinate geometry, that if the diagonals are perpendicular then the figure is a rhombus.



$m(OQ) = \frac{c-0}{b+a-0} = \frac{c}{b+a}.$

$m(PR) = \frac{0-c}{a-b} = \frac{-c}{a-b}.$

if $OQ \perp PR$

$\frac{c}{b+a} = \frac{a-b}{c}$

$c^2 = (a-b)(a+b)$

$c^2 = a^2 - b^2.$

if rhombus $d(OR) = d(RQ).$

$\sqrt{(0-b)^2+(0-c)^2} = \sqrt{(b+a-b)^2+(c-c)^2}.$

$\sqrt{b^2+c^2} = \sqrt{a^2}$

$b^2+c^2 = a^2$

$c^2 = a^2 - b^2.$

$\therefore$ rhombus adj. sides $=$.

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