Moriah College Mathematics Department
Year 11- 3 unit
Introduction to calculus

Name: ______________________ Class: ....

Question 1.

a. If $y = 3t^3 + 4t^2 - 5t + 1$, find $\frac{dy}{dt}$

b. find $D_x\left[(\sqrt{x-4})(x^2-3)\right]$

c. If $g(x) = \frac{2x+4}{x^2-3}$, find $g'(x)$

d. Find the gradient of the tangent to the curve $f(x)=2x^2-3x+5$ at $x=2$ by first principles using $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

e. Differentiate the following:

i) $\left(\sqrt[3]{x^5}\right)$

ii) $\frac{3}{x\sqrt{x}}$

iii) $\frac{5x^3+3x^2-7}{2x}$

iv) $\left(x+\frac{1}{x}\right)^2$

f. Find p if $h'(p)=-2$ and $h(p)=(2p-3)^2$

Question 2.

a) If $g(x)=\frac{3}{x}+\sqrt{x}$, find $g'(4)$

b) Show that the gradient of the normal to the curve $y=\frac{1}{x}$ is always positive and hence, show graphically what this means.

c) Find the equation of the tangent to the curve $y=\frac{x+1}{x-3}$ at $x=1$

(leave answer in general form)

d) If $y = 2\sqrt{x} + \sqrt{2x}$, show that $\frac{dy}{dx} = \frac{1+\sqrt{2}}{\sqrt{2x}}$, for $x > 0$

e) Find the coordinates of the point on the curve $y = -2x^2 + x$ at which the tangent is parallel to the line $y = -x + 5$

Question 3

a. Find any x value for which the normal to the curve $y = 5x - x^2$ has a gradient of 3

b. The tangent to $y = ax^2 + bx - 4$ at the point (-1 , 1) on the curve has a gradient of -7. Find a and b.

c. If $f(x) = 2x^2 - 2x - 1$, show that $f(x) + f'(x) + f''(x) = 2x^2 + 2x + 1$

Moriah College Mathematics Department

Year 11– 3 unit

Introduction to calculus

Name: michael lazan .... Class: ....

Question 1.

a. If $y = 3t^3 + 4t^2 - 5t + 1$, find $\frac{dy}{dt}$

$$\frac{dy}{dt} = 9t^2 + 8t - 5$$

b. find $D_x\left[\left(\sqrt{x-4}\right)\left(x^2-3\right)\right]$ $= (\sqrt{x-4})(2x) + (x^2-3)(\tfrac{1}{2}(x-4)^{-\frac{1}{2}} \times 1)$.

$$= 2x\sqrt{x-4} + \frac{1}{2}(x^2-3)\sqrt{\frac{1}{x-4}}$$

$$= 2x\sqrt{x-4} + \frac{x^2-3}{2\sqrt{x-4}}$$

c. If $g(x) = \frac{2x+4}{x^2-3}$, find $g'(x)$

$$g'(x) = \frac{(x^2-3)(2) - (2x+4)(2x)}{(x^2-3)^2}$$

$$= \frac{2x^2-6 - 4x^2-8x}{(x^2-3)^2}$$

$$= \frac{-(2x^2+8x+6)}{(x^2-3)^2}$$

$$= \frac{-2(x^2+4x+3)}{(x^2-3)^2}$$

$$= \frac{-2\cancel{(x+3)}(x+1)}{(x+3)(x-3)^2}$$

$$= \frac{-2(x+1)}{(x+3)(x-3)^2}$$

d. Find the gradient of the tangent to the curve $f(x)=2x^2-3x+5$ at $x=2$ by first principles using $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) + 5 - 2x^2 + 3x - 5}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - \cancel{3x} - 3h + \cancel{5} - 2x^2 + \cancel{3x} - \cancel{5}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - 3h - \cancel{2x^2}}{h}$$

$$= \lim_{h\to 0} \frac{h(4x+2h-3)}{h}$$

$$= \lim_{h\to 0} 4x + 2h - 3$$

$= 4x - 3$

at $x = 2$

$f'(x) = 5$

$\therefore$ grad of tgt $= 5$

e. Differentiate the following:

i) $\left(\sqrt[3]{x^5}\right) = (x)^{\frac{5}{3}}$

$\frac{dy}{dx} = \frac{5}{3}(x)^{\frac{2}{3}}$

$= \frac{5\sqrt[3]{x^2}}{3}$

ii) $\frac{3}{x\sqrt{x}} = \frac{3}{x^{\frac{3}{2}}}$

$= 3x^{-\frac{3}{2}}$

$\frac{dy}{dx} = 3 \cdot -\frac{3}{2} x^{-\frac{5}{2}}$

$= -\frac{9}{2} x^{-\frac{5}{2}}$

$= \frac{-9}{2\sqrt{x^5}}$

iii) $\frac{5x^3+3x^2-7}{2x} = \frac{1}{2}\left[\frac{5x^3}{x} + \frac{3x^2}{x} - \frac{7}{x}\right]$

$= \frac{1}{2}\left[5x^2 + 3x - \frac{7}{x}\right]$

$= \frac{5}{2}x^2 + \frac{3}{2}x - \frac{7}{2x}$

$= \frac{5}{2}x^2 + \frac{3}{2}x - 7(2x)^{-1}$

$\frac{dy}{dx} = 5x + \frac{3}{2} + \frac{7}{2}x^{-2}$

$\frac{dy}{dx} = 5x + \frac{3}{2} + \frac{14}{4x^2}$

iv) $\left(x+\frac{1}{x}\right)^2$ $\quad \frac{dy}{dx} = 2\left(x+\frac{1}{x}\right) \times \left(1 - x^{-2}\right)$

$= 2\left(x+\frac{1}{x}\right) \times \left(1-\frac{1}{x^2}\right)$

f. Find p if $h'(p) = -2$ and $h(p) = (2p-3)^2$

$h'(p) = 2(2p-3) \times 2.$

$= 4(2p-3).$

$\therefore \; 4(2p-3) = -2$

$8p - 12 = -2$

$8p = 10$

$p = \frac{10}{8}$

$= \frac{5}{4}$

Question 2.

a) If $g(x) = \frac{3}{x} + \sqrt{x}$, find $g'(4)$

$g(x) = 3x^{-1} + x^{\frac{1}{2}}$

$g'(x) = -3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{-3}{x^2} + \frac{1}{2} \times \frac{1}{\sqrt{x}}$

$= \frac{-3}{x^2} + \frac{1}{2\sqrt{x}}$

$g'(4) = \frac{-3}{16} + \frac{1}{2\times 2}$

$= \frac{-3}{16} + \frac{1}{4}$

$= \frac{-3}{16} + \frac{4}{16}$

$g'(4) = \frac{1}{16}$

b) Show that the gradient of the normal to the curve $y = \frac{1}{x}$ is always positive and hence, show graphically what this means.

$y = \frac{1}{x}$

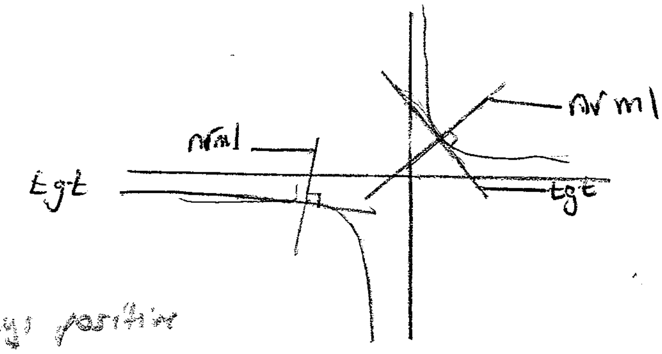
$= x^{-1}$

$y' = -1x^{-2}$

$= \frac{-1}{x^2}$

$\therefore$ grad of nrml

$= x^2$ which is always positive



c) Find the equation of the tangent to the curve $y = \frac{x+1}{x-3}$ at $x = 1$

(leave answer in general form)

when $x = 1$

$y = -1$ $\quad P(1, -1)$

$y = \frac{x+1}{x-3}$

$y' = \frac{(x-3)(1) - (x+1)(1)}{(x-3)^2}$

$= \frac{(x-3) - (x+1)}{(x-3)^2}$

$= x - 3 - x - 1$

$= \frac{-4}{(x-3)^2}$

at $x = 1$

$y'(1) = \frac{-4}{(-2)^2}$

$= -1$

$y + 1 = -1(x-1)$

$y = -x + 1 - 1$

$y = -x$

$\therefore y + x = 0$

d) If $y = 2\sqrt{x} + \sqrt{2x}$, show that $\frac{dy}{dx} = \frac{1+\sqrt{2}}{\sqrt{2x}}$, for $x > 0$

$y = 2x^{\frac{1}{2}} + (2x)^{\frac{1}{2}}$

$y' = x^{-\frac{1}{2}} + \frac{1}{2}(2x)^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{x}} + \frac{-1}{4}(2x)^{-\frac{3}{2}}$

$= \frac{1}{\sqrt{x}} + \frac{-1}{4\sqrt{(2x)^3}}$

$= \frac{1}{\sqrt{x}} - \frac{1}{4\sqrt{8x^3}}$

e) Find the coordinates of the point P on the curve $y = -2x^2 + x$ at which the tangent is parallel to the line $y = -x + 5$

$m = -1$

$y' = -4x + 1$

$-4x + 1 = -1$

$4x = 2$

$x = \frac{1}{2}$

$y = 0$

$P\left(\frac{1}{2}, 0\right)$

Question 3

a. Find any x value for which the normal to the curve $y = 5x - x^2$ has a gradient of 3

$y' = 5 - 2x$

$\therefore \; 5 - 2x = 3$

$2x = 2$

$x = 1$

b. The tangent to $y = ax^2 + bx - 4$ at the point (-1 , 1) on the curve has a gradient of -7. Find a and b.

$y' = 2ax + b$

$2ax + b = -7$ at $(-1, 1)$

① $-2a + b = -7$

$\boxed{\text{ALSO}}$

$1 = a - b - 4$

$5 = a - b$

② $a - b = 5$

$\therefore \; -a = -2$

$a = 2$

$b = -3$

c. If $f(x) = 2x^2 - 2x - 1$, show that $f(x) + f'(x) + f''(x) = 2x^2 + 2x + 1$

$f(x) = 2x^2 - 2x - 1$

$f'(x) = 4x - 2$

$f''(x) = 4$

$\therefore \; f(x) + f'(x) + f''(x) = 2x^2 - 2x - 1 + 4x - 2 + 4$

$= 2x^2 + 2x + 1$