Michael Hazan ~~[illegible]~~ Ms D.

MORIAH COLLEGE MATHEMATICS DEPARTMENT

Year 11 Extension 1 Common Test **March, 2006**

- **Answer these questions on separate sheets with your name on top of each sheet.**
- **Start each question on a new sheet.**

Question 1 – *Use a separate answer sheet*

a) Let $f(x) = x^2 - x + 3$ and $g(x) = 7x - 5$:

i) Evaluate $f(-2)$.

ii) Find $f(g(2))$

iii) Find $f(x) + g(x-2)$.

b) State the natural **domain** and **range** for each of the following functions:

i) $y = 3^x$

ii) $y = \sqrt{9 - x^2}$

iii) $y = \dfrac{1}{x-1}$

c) i) Show that $f(x) = x^3 - 16x$ is an odd function.

ii) Describe the symmetry property associated with this type of function.

Question 2 – *Use a separate answer sheet*

Simplify:

i) $\sqrt{(p^2-1)^2+4p^2}$

ii) $\dfrac{1}{\sqrt{1-a}+\sqrt{1+a}}+\dfrac{1}{\sqrt{1-a}-\sqrt{1+a}}$

b) Find the values of x and y if

$$x+y\sqrt{5}=\frac{\sqrt{5}}{3+\sqrt{5}}$$

c) Solve for x:

i) $x^2 \leq 2x$

ii) $\dfrac{4}{|x|}<3$

iii) $\dfrac{2x}{x-3}>-1$

Question 3 – *Use a separate answer sheet*

a) Sketch the following including all essential features.

i) $y = (x^2 - 4)(x + 2)$

ii) $(x + 3)^2 + (y - 1)^2 = 9$

iii) $y = 2^{-(x+2)}$

iv) $y = \dfrac{x - 3}{x + 2}$

b) A function $f(x)$ is defined by

$$f(x) = \begin{cases} x^2 - 1 & for \quad x \le -1 \\ 2x - 4 & for \quad x > -1 \end{cases}$$

i) Sketch the function.

ii) Find the range of the function.

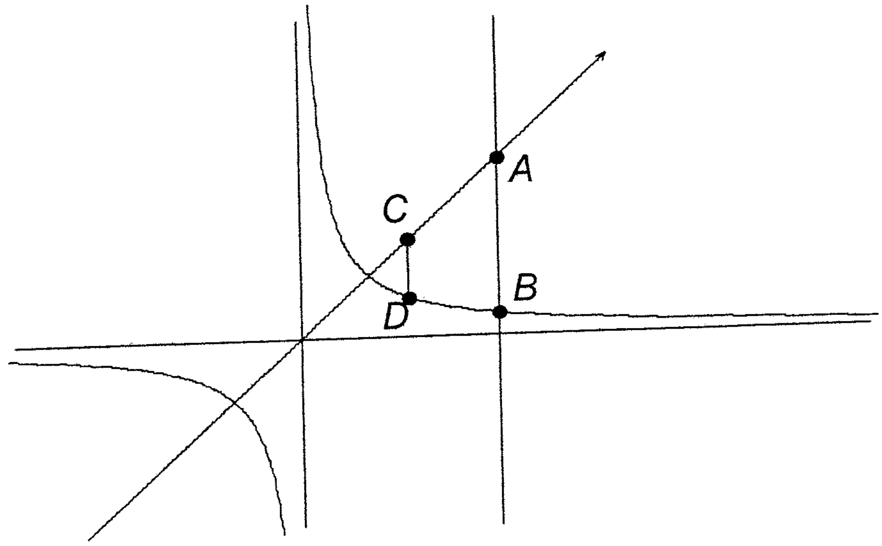
c) i) Sketch the graphs of $y = |x - 1|$ and $y = (x + 1)^2$ on the same number plane.

ii) Use your sketch to help you solve the equation $|x - 1| = (x + 1)^2$.

iii) On your graph, shade the region where $y \le |x - 1|$ and $y \ge (x + 1)^2$.

Question 4 – *Use a separate answer sheet*

a) The diagram below shows the hyperbola $y = \frac{1}{x}$, the line $y = x$ and the line $x = 3$ all on the same number plane.



i) Find the points of intersection of the hyperbola $y = \frac{1}{x}$ and the line $y = x$.

ii) Hence, or otherwise, solve the inequality $\frac{1}{x} \geq x$.

iii) By finding the coordinates of A and B, find the length of the vertical line segment AB.

iv) The vertical line segment CD, shown in the diagram, has a length of 1 unit. Find the x-coordinate of C.

v) Hence, find the y-coordinate of D, giving your answer in exact form.

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Q1.

$f(x) = x^2 - x + 3$

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a)

i) $f(-2) = (-2)^2 - (-2) + 3$
$= 4 + 2 + 3$
$= 9$

ii) $g(2) = 14 - 5$
$= 9$
$\therefore f(g(2)) = f(9)$
$= (9)^2 - 9 + 3$
$= 75$

iii) $f(x) + g(x-2) = x^2 - x + 3 + 7(x-2) - 5$
$= x^2 - x + 3 + 7x - 14 - 5$
$= x^2 + 6x - 16$

b) i) $y = 3^x$

R: $y : y > 0$ D: $x : x \in \mathbb{R}$

ii) $y = \sqrt{9 - x^2}$

D: $-3 \leq x \leq 3$ R: $3 \leq y \leq 0$

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iii) $y = \frac{1}{x-1}$

D: $x : x \in R$ $x \neq 1$
R: $y : y \in \mathbb{R}, y \neq 0$

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c) i) $f(x) = x^3 - 16x$

$f(-x) = -x^3 + 16x$

$-f(x) = -x^3 + 16x$

$\therefore$ since $f(-x) = -f(x)$ above is an odd function.

ii) This function has point symmetry. ie. it is symmetrical, rotated 180°'s around origin.

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Q2

i) $\sqrt{(p^2-1)^2+4p^2} = \sqrt{p^4-2p^2+1+4p^2}$

$= \sqrt{p^4+2p^2+1}$

$= \sqrt{(p^2+1)^2}$

$= p^2+1$

ii) $\dfrac{1}{\sqrt{1-a}+\sqrt{1+a}} + \dfrac{1}{\sqrt{1-a}-\sqrt{1+a}}$

$= \dfrac{\sqrt{1-a}-\cancel{\sqrt{1+a}}+\sqrt{1-a}+\cancel{\sqrt{1+a}}}{(\sqrt{1+a}+\sqrt{1-a})(\sqrt{1-a}-\sqrt{1+a})}$

$= \dfrac{\sqrt{1-a}+\sqrt{1-a}}{(1+a)+(1-a)}$

$= \dfrac{2\sqrt{1-a}}{2}$

$= \sqrt{1-a}$

b) $x+y\sqrt{5} = \dfrac{\sqrt{5}}{3+\sqrt{5}}$

$\cancel{x = \frac{3\sqrt{5}}{4}}$ ✓

$= \dfrac{\sqrt{5}(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$

$x = -\dfrac{5}{4}$

$y = \dfrac{3}{4}$

$= \dfrac{3\sqrt{5}-5}{9-5}$

$= \dfrac{3\sqrt{5}-5}{}$

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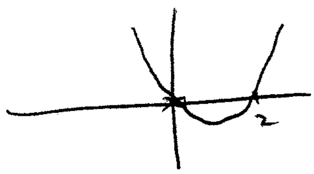
Q2.

c)

i) $x^2 - 2x \leq 0$

$x(x-2) \leq 0$



$0 \leq x \leq 2$

ii) $\frac{4}{|x|} < 3$

$4 < 3|x|$

$\frac{4}{3} < |x|$

$x > \frac{4}{3}$ ✓ $\quad x < -\frac{4}{3}$ ✓

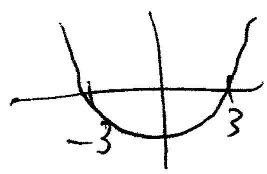
iii) $\frac{2x}{x-3} > -1 \qquad x \neq 3$

$\frac{2x - (x-3)}{x-3} > 0$

$\frac{2x - x + 3}{x-3} > 0$

$\frac{x+3}{x-3} > 0$

same sign as $(x+3)(x-3)$



Sol^n: $x < -3$, or $x > 3$

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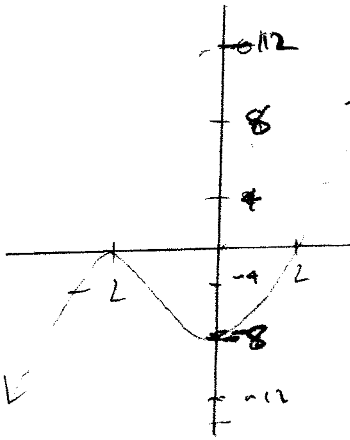
Q3. a) i) $y = (x^2-4)(x+2)$

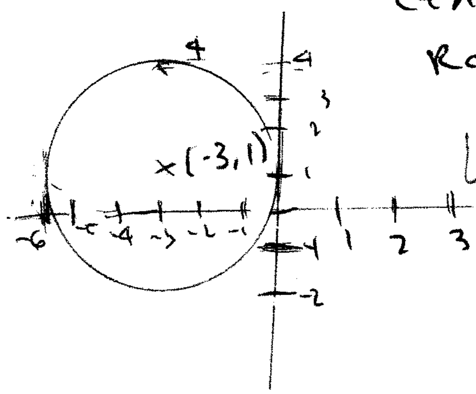
$y = (x+2)(x-2)(x+2)$
$= (x+2)^2(x-2)$



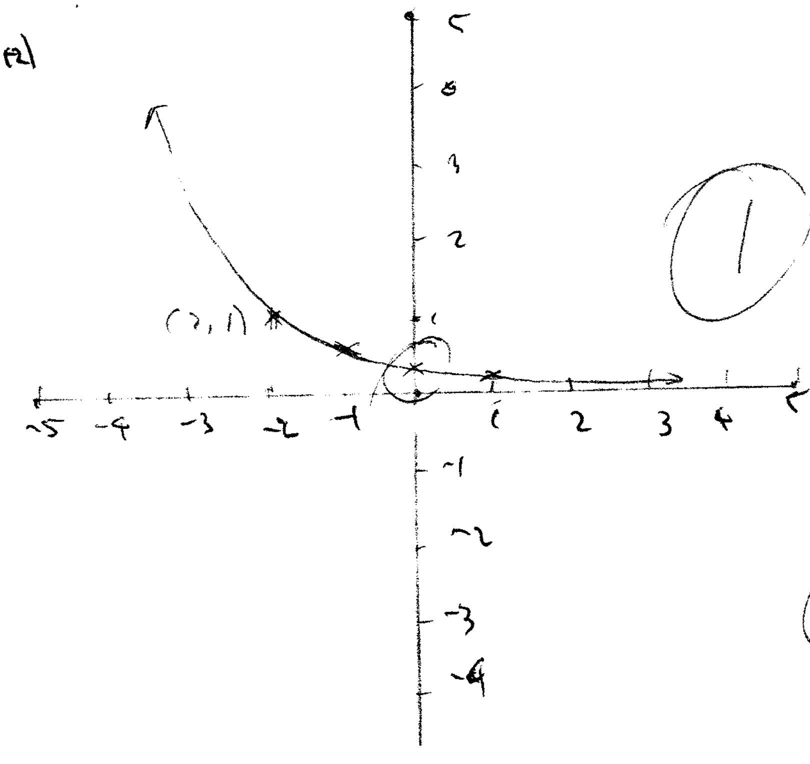
when $x = 0$, $y = -8$

ii) $(x+3)^2 + (y-1)^2 = 9$

center: $(-3, 1)$
Radius: 3



iii) $y = 2^{-(x+2)}$



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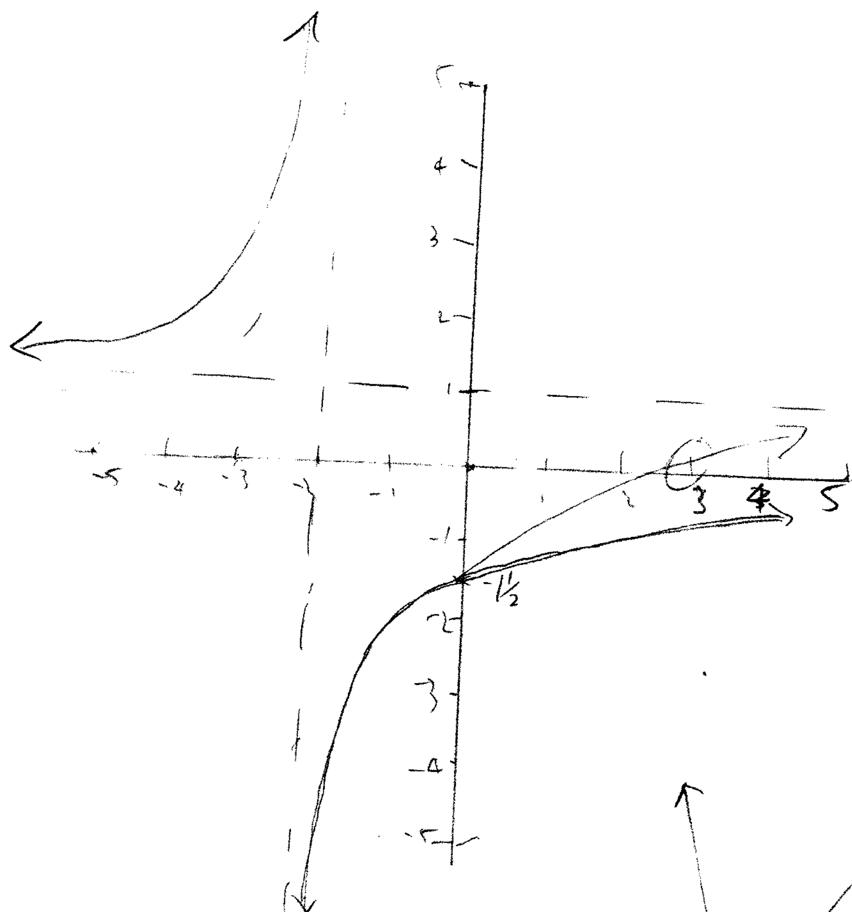
Q3

iv) $y = \dfrac{x-3}{x+2}$ $\qquad x \neq -2$

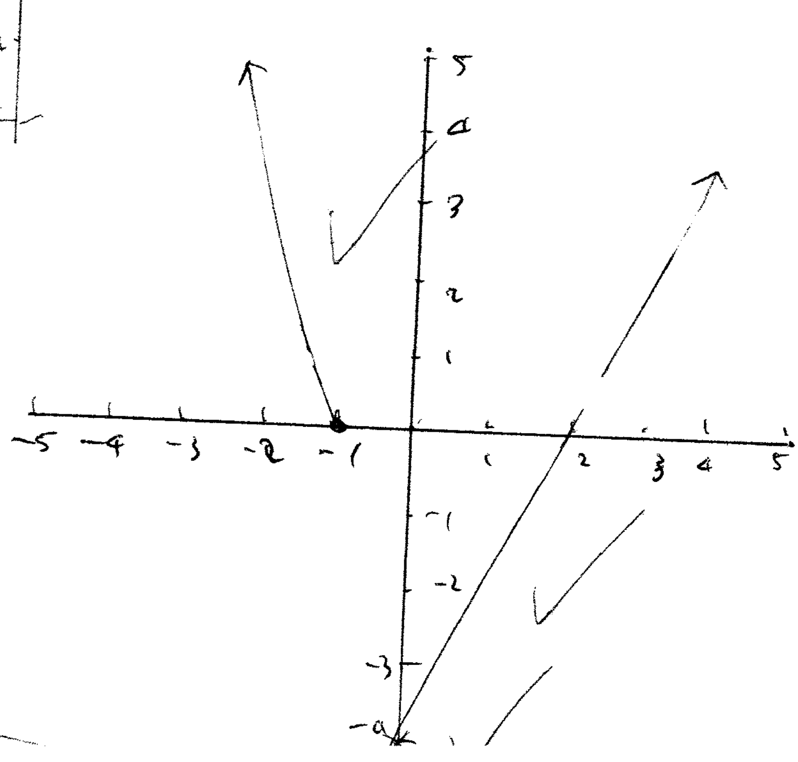
$y = \dfrac{x+2-5}{x+2}$

$= \dfrac{-5}{x+2} + 1$



b) i)

ii) R: $y > -6$



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open circle $(-1, -6)$

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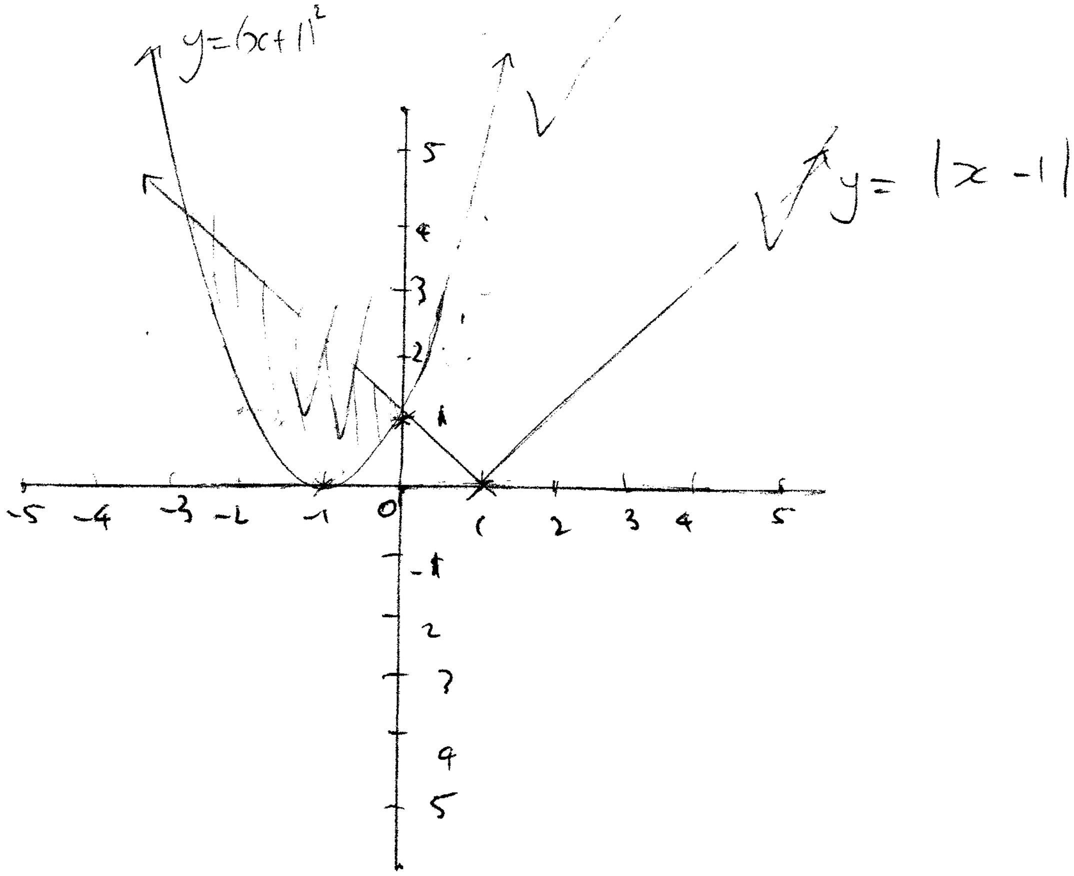
c i)



ii) $|x-1| = (x+1)^2$

~~x=1~~ $x=0$ $y=1$

P(0, 1)

iii) above.

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Q4

a) i) Points of int

$\boxed{\begin{matrix} x = 1 \\ y = 1 \end{matrix}}$ $\boxed{\begin{matrix} x = -1 \\ y = -1 \end{matrix}}$

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ii) $\frac{1}{x} \geq x$

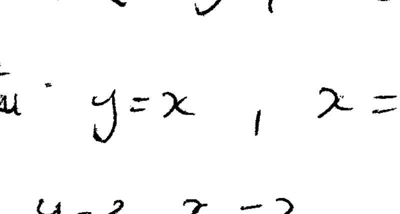
$\frac{1}{x} - x \geq 0$

$\frac{1 - x^2}{x} \geq 0$

$\frac{(1+x)(1-x)}{x} \geq 0$

same sign as

$1\frac{1}{2}$



$x \leq -1$, ~~...~~ $0 \leq x \leq 1$

iii) int of ~~...~~: $y = x$, $x = 3$

A: $y = 3$, $x = 3$

B. $y = \frac{1}{3}$ $x = 3$

$\therefore$ length AB $= 8/3$ units.

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iv)

$y = x$

$x = \frac{1}{x} + 1$

$x = \frac{1+x}{x}$

$x^2 = 1 + x$

$x^2 - x - 1 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times -1}}{2}$

$= \frac{-1 \pm \sqrt{5}}{2}$

$= \frac{-1 \pm \sqrt{5}}{2}$

$x = \frac{-1 + \sqrt{5}}{2}$

since C is positive

x coordinate of C $= \frac{-1 + \sqrt{5}}{2}$

$1\frac{1}{2}$

v) x coordinate of D $= \frac{-1 + \sqrt{5}}{2}$

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y coordinate of D: $y = \frac{\sqrt{5} + 1}{2}$

$y = \frac{1}{\frac{-1 + \sqrt{5}}{2}}$

$y = \frac{2}{-1 + \sqrt{5}}$

$y = \frac{2(\sqrt{5} + 1)}{5 - 1}$

$y = \frac{2(\sqrt{5} + 1)}{4}$

$y = \frac{\sqrt{5} + 1}{2}$