MORIAH COLLEGE MATHEMATICS DEPARTMENT

Year 11 Extension 1 Common Test March, 2006

- Answer these questions on separate sheets with your name on top of each sheet.
- Start each question on a new sheet.

Question 1 – *Use a separate answer sheet*

a) Let $f(x) = x^2 - x + 3$ and $g(x) = 7x - 5$:

i) Evaluate $f(-2)$.

ii) Find $f(g(2))$

iii) Find $f(x) + g(x-2)$.

b) State the natural **domain** and **range** for each of the following functions:

i) $y = 3^x$

ii) $y = \sqrt{9 - x^2}$

iii) $y = \dfrac{1}{x-1}$

c) i) Show that $f(x) = x^3 - 16x$ is an odd function.

ii) Describe the symmetry property associated with this type of function

Question 2 – *Use a separate answer sheet*

Simplify:

i) $\sqrt{(p^2 - 1)^2 + 4p^2}$

ii) $\dfrac{1}{\sqrt{1-a} + \sqrt{1+a}} + \dfrac{1}{\sqrt{1-a} - \sqrt{1+a}}$

b) Find the values of x and y if

$$x + y\sqrt{5} = \frac{\sqrt{5}}{3 + \sqrt{5}}$$

c) Solve for x:

i) $x^2 \le 2x$

ii) $\dfrac{4}{|x|} < 3$

iii) $\dfrac{2x}{x-3} > -1$

Question 3 – *Use a separate answer sheet*

a) Sketch the following including all essential features.

i) $y=(x^2-4)(x+2)$

ii) $(x+3)^2+(y-1)^2=9$

iii) $y=2^{-(x+2)}$

iv) $y=\dfrac{x-3}{x+2}$

b) A function $f(x)$ is defined by

$$f(x)=\begin{cases} x^2-1 & \text{for } x\le -1 \\ 2x-4 & \text{for } x>-1 \end{cases}$$

i) Sketch the function

ii) Find the range of the function.

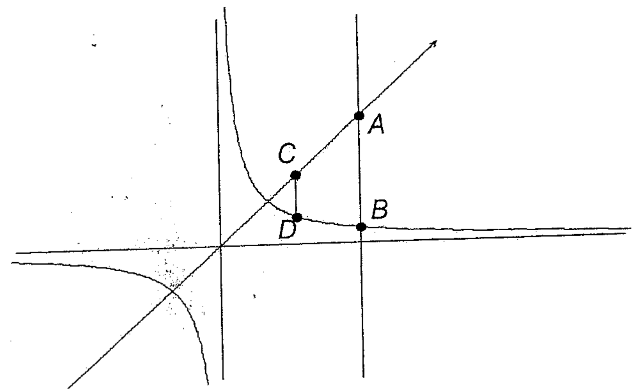
c) i) Sketch the graphs of $y=|x-1|$ and $y=(x+1)^2$ on the same number plane.

ii) Use your sketch to help you solve the equation $|x-1|=(x+1)^2$.

iii) On your graph, shade the region where $y\le|x-1|$ and $y\ge(x+1)^2$.

Question 4 – *Use a separate answer sheet*

a) The diagram below shows the hyperbola $y=\dfrac{1}{x}$, the line $y=x$ and the line $x=3$ all on the same number plane.



i) Find the points of intersection of the hyperbola $y=\dfrac{1}{x}$ and the line $y=x$.

ii) Hence, or otherwise, solve the inequality $\dfrac{1}{x}\ge x$.

iii) By finding the coordinates of A and B, find the length of the vertical line segment AB.

iv) The vertical line segment CD, shown in the diagram, has a length of 1 unit. Find the x-coordinate of C.

v) Hence, find the y-coordinate of D, giving your answer in exact form.

Michael Hazan

Q1.

$f(x) = x^2 - x + 3$

a)

i) $f(-2) = (-2)^2 - (-2) + 3$
$= 4 + 2 + 3$
$= 9$ ✓

ii) $g(2) = 14 - 5$
$= 9$ ✓
$\therefore f(g(2)) = f(9)$
$= (9)^2 - 9 + 3$ ✓
$= 75$

iii) $f(x) + g(x-2) = x^2 - x + 3 + 7(x-2) - 5$
$= x^2 - x + 3 + 7x - 14 - 5$ ✓
$= x^2 + 6x - 16$

(13)

b) i) $y = 3^x$

R: $y : y > 0$ ✓ D: $x : x \in \mathbb{R}$ ✓

ii) $y = \sqrt{9 - x^2}$

D: $-3 \leq x \leq 3$ ✓ R: y $3 \leq y \leq 0$ ✗

iii) $y = \frac{1}{x-1}$

D: $x : x \in R$ $x \neq 1$ ✓
R: $y : y \in \mathbb{R}$, $y \neq 0$ ✓

Michael Hazan

c) i) $f(x) = x^3 - 16x$

$f(-x) = -x^3 + 16x$ ✓

$-f(x) = -x^3 + 16x$ ✓

$\therefore$ since $f(-x) = -f(x)$ above is an odd function.

ii) This function has point symmetry. ✓
ie. it is symmetrical, rotated 180°'s around origin.

Michael Hazan

Q2

i) $\sqrt{(p^2-1)^2+4p^2} = \sqrt{p^4-2p^2+1+4p^2}$

$=\sqrt{p^4+2p^2+1}$

$=\sqrt{(p^2+1)^2}$

$=p^2+1$

ii) $\dfrac{1}{\sqrt{1-a}+\sqrt{1+a}} + \dfrac{1}{\sqrt{1-a}-\sqrt{1+a}}$

$= \dfrac{\sqrt{1-a}-\sqrt{1+a}+\sqrt{1-a}+\sqrt{1+a}}{(\sqrt{1+a}+\sqrt{1-a})(\sqrt{1-a}-\sqrt{1+a})}$

$= \dfrac{\sqrt{1-a}+\sqrt{1-a}}{(1+a)+(1-a)}$

$= \dfrac{2\sqrt{1-a}}{2}$

$=\sqrt{1-a}$

b) $x+y\sqrt{5} = \dfrac{\sqrt{5}}{3+\sqrt{5}}$

~~$x=\frac{3}{4}$~~ ✓

$x=-\dfrac{5}{4}$

$y=\dfrac{3}{4}$

~~$=\sqrt{5}(3-\sqrt{5})$~~ $= \dfrac{\sqrt{5}(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$

$= \dfrac{3\sqrt{5}-5}{9-5}$

$= \dfrac{3\sqrt{5}-5}{4}$

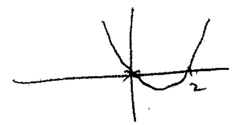
Michael Hazan

Q2.

c)

i) $x^2-2x\leq 0$

$x(x-2)\leq 0$



$0\leq x\leq 2$

ii) $\dfrac{4}{|x|}<3$

$4<3|x|$

$\dfrac{4}{3}<|x|$

$x>\dfrac{4}{3}$ ✓ $x<-\dfrac{4}{3}$ ✓

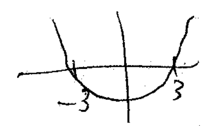
iii) $\dfrac{2x}{x-3}>-1$ $x\neq 3$

$\dfrac{2x-(x-3)}{x-3}>0$

$\dfrac{2x-x+3}{x-3}>0$

$\dfrac{x+3}{x-3}>0$

same sign as $(x+3)(x-3)$



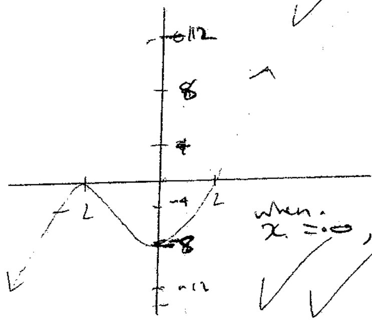
Sol^n: $x<-3$, or $x>3$

16/17

Michael Hazan

Q3. a) i) $y = (x^2 - 4)(x+2)$

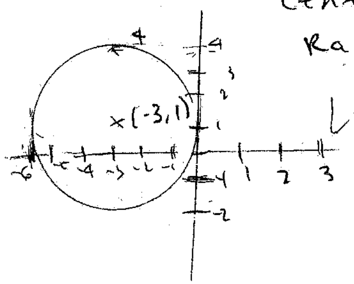
$y = (x+2)(x-2)(x+2)$
$= (x+2)^2(x-2)$



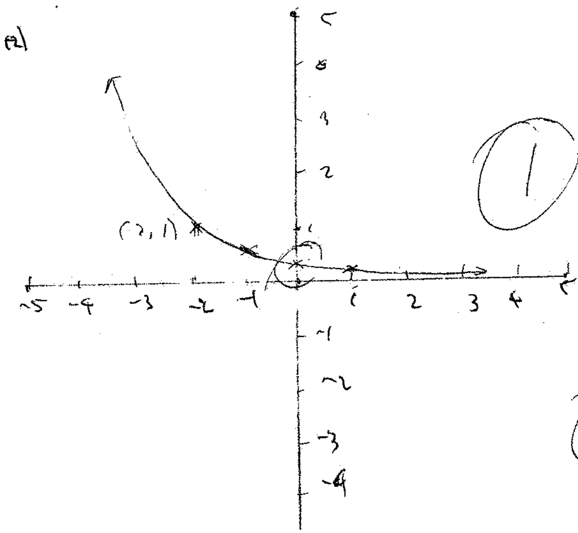
when: $x = 0$, $y = -8$

ii) $(x+3)^2 + (y-1)^2 = 9$

Center: $(-3, 1)$
Radius: 3



iii) $y = 2^{-(x+1)}$



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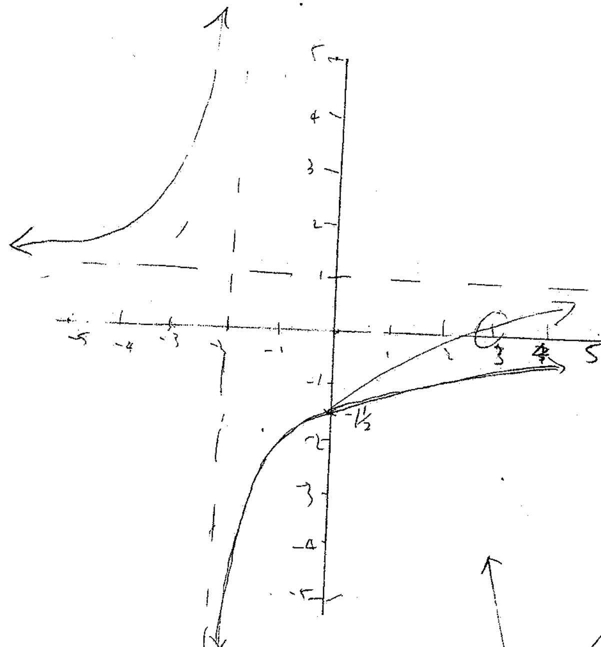
Q3

iv) $y = \frac{x-3}{x+2}$ $x \neq -2$

$y = \frac{x+2-5}{x+2}$

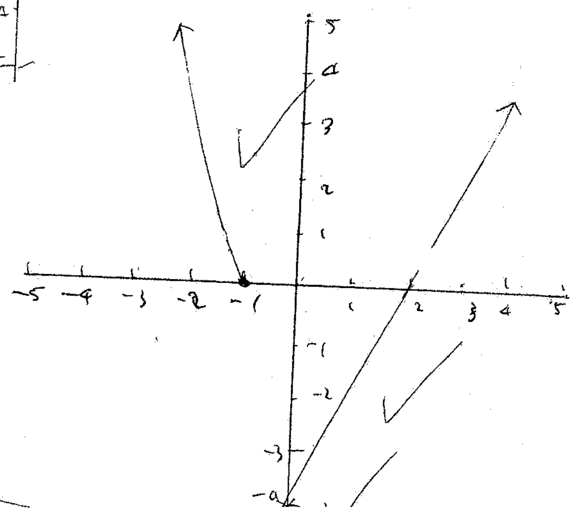
$= \frac{-5}{x+2} + 1$



b) i)

ii) R: $y > -6$

open circle $(-1, -6)$



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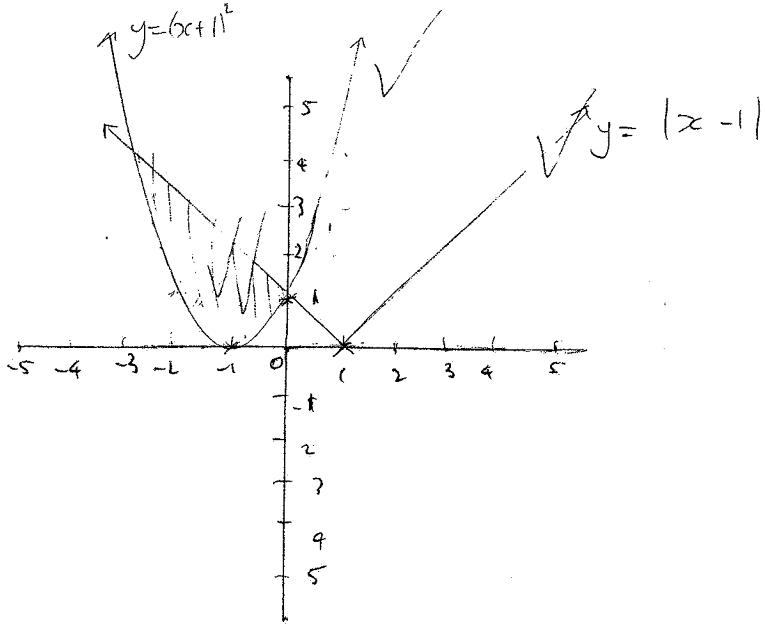
Michael Herzan.

c i)



ii) $|x-1| = (x+1)^2$

$x = 0 \quad y = 1$

P (0, 1) ✓

iii) above.

(4)

(9)

Michael Herzan.

Q4

a) i) Points of int

$x = 1$, $y = 1$ ✓ $x = -1$, $y = -1$ ✓   (2)

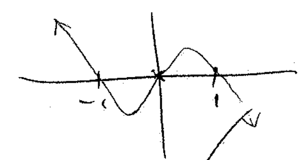
(ii) $\frac{1}{x} \geq x$

$\frac{1}{x} - x \geq 0$

$\frac{1-x^2}{x} \geq 0$ ✓

$\frac{(1+x)(1-x)}{x} \geq 0$

same sign as



$x \leq -1$ ✓ $0 < x \leq 1$ ✓   ($1\frac{1}{2}$)

iii) int of ~~curve~~: $y = x$, $x = 3$ ✓

A : $y = 3$, $x = 3$ ✓

B : $y = \frac{1}{3}$ $x = 3$ ✓

∴ length AB = 8/3 units. ✓   (2)

Michael Hazan

iv)

$y = x$

$x = \frac{1}{x} + 1$ ✓

$x = \frac{1+x}{x}$ ✓

$x^2 = 1 + x$

$x^2 - x - 1 = 0$ ✓

$x = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times -1}}{2}$ ✗

$= \frac{-1 \pm \sqrt{5}}{2}$

$= \frac{-1 \pm \sqrt{5}}{2}$

$x = \frac{-1 + \sqrt{5}}{2}$

since C is positive

x coordinate of C $= \frac{-1 + \sqrt{5}}{2}$ ✓ H

(1½)

v) x coordinate of D $= \frac{-1 + \sqrt{5}}{2}$

(2) H

y coordinate of D: $y = \frac{\sqrt{5}+1}{2}$

$y = \frac{1}{\frac{-1+\sqrt{5}}{2}}$

$y = \frac{2}{-1+\sqrt{5}}$

$y = \frac{2(\sqrt{5}+1)}{5-1}$

$y = \frac{2(\sqrt{5}+1)}{4}$

$y = \frac{\sqrt{5}+1}{2}$