MORIAH COLLEGE MATHEMATICS DEPARTMENT

Extension 1 Year 11 Sequences and Series

Name:.. Class:.....................

Question 1:

An arithmetic sequence is given by $-100, -96, -92, \ldots, 220$.

(a) Find an expression for the n^{th} term of this sequence.

(b) How many terms are in this sequence?

(c) Find the sum of the terms in this sequence.

(d) Find an expression for the sum S_n of the first n terms of this sequence.

(e) Find the value of n such that the sum of the first n terms equals zero.

(f) Find the largest term in the sequence such that the sum of the terms preceding it is less than the sum of the terms following it.

Question 2:

The sum of the first 5 terms of an arithmetic sequence is equal to the sum of the sixth and seventh terms. The sum of the eighth and ninth term is 94. Find the first term and the common difference.

Question 3:

The sum to n terms of a series is $4n^2 + 3n$. Prove that the series is an AP and find its common difference.

Question 4:

Express 0.58585858... as a GP and hence write it as a fraction.

Question 5

Which term of the series $2, 8, 32, \ldots$ is greater than 8000?

Question 6:

Evaluate $\sum_{n=1}^{20} 2^n + 2(n-1)$

Question 7:

For what values of x does the geometric series $1 + \frac{2x}{2-x} + \frac{4x^2}{(2-x)^2} + \ldots$ have a limiting sum?

Question 8:

For a particular geometric series it is known that the limiting sum is twice the sum of the first nine terms. Find the common ratio of the series.

Question 9:

An elastic ball is dropped to the ground from a height of 18 metres and rises $\frac{2}{3}$ of its previous height. If this ratio remains constant:

a) How high does it rise after 6 bounces?

b) Through what distance will the ball have moved before it comes to rest?

Question 10:

An A.P. is formed using the three terms $\frac{1}{q-p}, \frac{1}{2q}, \frac{1}{q-r}$ show that p, q, r are in G.P.

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Extension 1 Year 11 Sequences and Series

Name: Solns $/60$ Class: ....

Question 1:

An arithmetic sequence is given by $-100, -96, -92, \ldots, 220$.

(a) Find an expression for the n[th] term of this sequence.

$a = -100$ 1

$d = 4$ 1

$T_n = -100 + 4(n-1)$

$= 4n - 104.$ 1

(b) How many terms are in this sequence? $T_n = 220$.

$4n - 104 = 220$ 1

$4n = 324$

$n = 81$ 1

(c) Find the sum of the terms in this sequence.

$S_{81} = \frac{81}{2}\left[-100 + 220\right]$ 1

$= 4860$ 1

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(d) Find an expression for the sum S_n of the first n terms of this sequence.

$S_n = \frac{n}{2}\left[2 \times -100 + 4(n-1)\right]$ 1

$= \frac{n}{2}\left[4n - 204\right]$

$= 2n^2 - 102n.$ 1

(e) Find the value of n such that the sum of the first n terms equals zero.

$2n(n-51) = 0.$ 1

$n = 0, \quad n = 51$

$\therefore n = 51$ 1

(f) Find the largest term in the sequence such that the sum of the terms preceding it is less than the sum of the terms following it.

$-100, -96, \ldots, T_{n-1}, T_n, T_{n+1}, \ldots, 220$

$S_{n-1} < S_{81} - S_n$ 1

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$2(n-1)^2 - 102(n-1) < 4860 - 2n^2 + 102n$

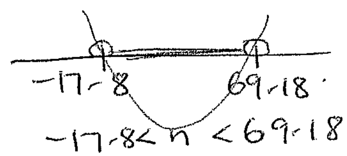
$2n^2 - 4n + 2 - 102n + 102 < 4860 - 2n^2 + 102n$

$4n^2 - 208n - 4756 < 0.$

$n^2 - 52n - 1189 < 0.$

$n^2 - 52n - 1189 = 0$

$n = \frac{52 \pm \sqrt{7460}}{2}$



$-17.8 < n < 69.18$ 1 $\Rightarrow n = 69$

$T_{69} = 4(69) - 104$

$= 172.$ 1

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Question 2:

The sum of the first 5 terms of an arithmetic sequence is equal to the sum of the sixth and seventh terms. The sum of the eighth and ninth term is 94. Find the first term and the common difference.

$S_5 = T_6 + T_7$

$\frac{5}{2}(2a+4d) = a+5d+a+6d$ |

$5a+10d = 2a+11d.$

$3a = d$ — ①|

$T_8 + T_9 = 94$

$a+7d+a+8d = 94$ |

$2a+15d = 94$ — ②

Sub ① in ②:

$2a+15(3a) = 94$

$47a = 94$

$a = 2$ |

$\therefore d = 6.$ |

Question 3:

The sum to n terms of a series is $4n^2+3n$. Prove that the series is an AP and find its common difference.

$S_n = 4n^2+3n$.

$T_n = S_n - S_{n-1}$

$= 4n^2+3n - [4(n-1)^2+3(n-1)]$ |

$= 4n^2+3n - [4n^2-8n+4+3n-3]$ |

$= 8n-1.$ |

$T_1 = 7,\ T_2 = 15,\ T_3 = 23.$

AP: $T_2 - T_1 = T_3 - T_2 = \dots = 8.$

$15-7 = 23-15 = \dots = 8.$

$\therefore$ AP where $a=7,\ d=8$ |

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Question 4:

Express 0.58585858. . . as a GP and hence write it as a fraction.

$0.5858\dots = \frac{58}{100} + \frac{58}{10^4} + \frac{58}{10^6} + \dots$ |

$a = \frac{58}{100} \quad r = \frac{1}{100}.$ |

$S_\infty = \frac{a}{1-r}$

$= \frac{\frac{58}{100}}{1-\frac{1}{100}}$ |

$= \frac{58}{99}$ |

Question 5

Which term of the series 2, 8, 32, . . . is greater than 8000?

$a = 2 \quad r = 4$

$T_n > 8000.$

$ar^{n-1} > 8000$ |

$2\times 4^{n-1} > 8000$

$4^{n-1} > 4000$ |

$(n-1)\log 4 > \log 4000$

$n-1 > \frac{\log 4000}{\log 4}$

$n > \frac{\log 4000}{\log 4} + 1$

$n > 6.98$ |

$\therefore n = 7$ |

$\therefore$ the 7th term is greater. |

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Question 6:

Evaluate $\sum_{n=1}^{20} 2^n + 2(n-1)$

$n=1$: $= 2^1 + 0 +$
$n=2$: $2^2 + 2 +$
$n=3$: $2^3 + 4 +$
$\vdots$
$n=20$: $2^{20} + 38$

1

$= 2^1 + 2^2 + \ldots + 2^{20} + 0 + 2 + \ldots + 38$ 2

$= \dfrac{2(2^{20}-1)}{2-1} + \dfrac{20}{2}[0+38]$

$= 2(2^{20}-1) + 10[38]$ 2

$= 2^{21} - 2 + 380$

$= 2^{21} + 378$. $(= 2097530)$

Question 7:

For what values of x does the geometric series $1 + \frac{2x}{2-x} + \frac{4x^2}{(2-x)^2} + \ldots$ have a limiting sum?

$r = \dfrac{2x}{2-x}$. 1

for limiting sum $-1 < r < 1$.

ie $-1 < \dfrac{2x}{2-x} < 1$ 1

consider: $-1 < \dfrac{2x}{2-x}$ and $\dfrac{2x}{2-x} < 1$ 1

(i) $x \neq 2$
(ii) $-1 = \dfrac{2x}{2-x}$ 1
$x - 2 = 2x$
$x = 2$
(iii) $-2 < x < 2$

(i) $x \neq 2$
(ii) $\dfrac{2x}{2-x} = 1$
$2x = 2 - x$ 1
$3x = 2$
$x = 2/3$
(iii) $x < 2/3$, $x > 2$

both must be true (hold)

$\therefore$ $\boxed{-2 < x < \frac{2}{3}}$ 1

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Question 8:

For a particular geometric series it is known that the limiting sum is twice the sum of the first nine terms. Find the common ratio of the series.

$S_\infty = 2 \times S_9$

$\dfrac{a}{1-r} = \dfrac{2a(1-r^9)}{1-r}$ — (1) 1

(i) $a = 0$, $r =$ any value.

(ii) $a \neq 0$ ($\div a$ in (1))

$1 = 2(1 - r^9)$ 1

$r^9 = \frac{1}{2}$

$r = \sqrt[9]{1/2}$ 1

Question 9:

An elastic ball is dropped to the ground from a height of 18 metres and rises $\frac{2}{3}$ of its previous height. If this ratio remains constant:

a) How high does it rise after 6 bounces?

18 $\downarrow$ $\uparrow \frac{2}{3} \times 18$ $\downarrow$ $\uparrow \frac{2}{3} \times \frac{2}{3} \times 18$ $\downarrow$ ....

$a = 12$ $r = 2/3$. 1

$\therefore T_6 = ar^5$

$= 12\left(\frac{2}{3}\right)^5$ 1

$\doteqdot 1.58$m. (to 2 dp) 1

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b) Through what distance will the ball have moved before it comes to rest?

rest $\Rightarrow S_\infty$ (no movement)

$\downarrow 18 \uparrow 12 \downarrow 12 \ldots$ 1

$S_\infty = 18 + 2\left(\frac{12}{1-2/3}\right)$ 1

$= 18 + 72$

$= 90m$ 1

Question 10:

An A.P. is formed using the three terms $\frac{1}{q-p}, \frac{1}{2q}, \frac{1}{q-r}$ show that p, q, r are in G.P.

$T_2 - T_1 = T_3 - T_2$

$\frac{1}{2q} - \frac{1}{q-p} = \frac{1}{q-r} - \frac{1}{2q}$ 1

$\frac{q-p-2q}{2q(q-p)} = \frac{2q-q+r}{(q-r)2q}$

$(-q-p)(2q^2-2qr) = (2q^2-2pq)(q+r)$

$-2q^3+2q^2r-2pq^2+2qpr = 2q^3+2q^2r-2pq^2-2pqr$

$-2q^3-2q^3+2qpr+2qpr=0$

$-4q^3+4pqr=0$

$4q(-q^2+pr)=0$

$q=0$, ie. $q^2=pr$

which it can't due to $\frac{1}{2q}$ is defined 1

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For GP. p, q, r.

$\frac{q}{p} = \frac{r}{q}$ 1

$\Rightarrow q^2=pr$

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