Student Number:

MORIAH COLLEGE

2006

PRE-TRIAL EXAMINATION

Time allowed: 2 hours
+ 5 minutes reading time.

- Calculators may be used.
- Show all necessary working
- Start **each question** in a new booklet.

/84

TOTAL

MATHEMATICS

EXTENSION 1 (Additional)

EXTENSION 2 (Common)

EXAMINER: J Taylor

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ \text{if } n < 0.$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0.$$

$$\int e^{ax} dx = \frac{1}{a}eax,\ a \neq 0.$$

$$\int \cos ax dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left\{x+\sqrt{(x^2-a^2)}\right\},\ |x| > |a|$$

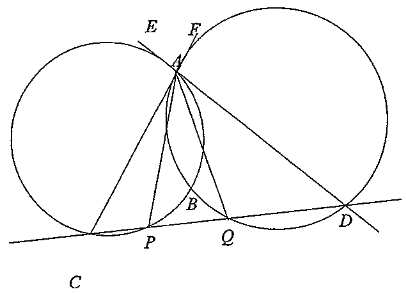
$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left\{x+\sqrt{(x^2+a^2)}\right\}$$

NOTE: $\ln x = \log_e x,\ x > 0.$

Question 1 (*Start a new booklet*).

a) M is the point (-1, 1), N is the point (a, b) and P is the point (6, 8). If P divides the interval MN externally in the ratio 7 : 2, find a and b. **3**

b) Two circles intersect at A and B. The tangents at A meet the two circles again at C and D. CD cuts the circles again at P and Q. **3**



Prove that $AP = AQ$.

c) Solve the inequality $\dfrac{3}{x-1} \le 2$. **3**

d) Find $\int 5^x \, dx$. **1**

e) Find $\dfrac{dy}{dx}$ if $y = \log(\log x)$. **2**

Question 2 (*Start a new booklet*).

a) Solve $|2x-3| \le x+1$. **3**

b) i) Find the acute angle between the lines $2y = x$ and $y = 3x$. Express your answer in radians in terms of π. **4**

ii) Find the area bounded by the above two lines and inside the circle $x^2 + y^2 = 4$. Leave your answer in terms of π.

c) Find $f^{-1}(x)$ if $f(x) = e^{2x+3}$. **2**

d) Evaluate $\displaystyle\int_{-\frac{3}{2}}^{\frac{3}{2}} \frac{dx}{3+4x^2}$. **3**

Question 3 (*Start a new booklet*).

a) Find $\displaystyle\lim_{x \to 0} \left[\frac{\tan 3x}{\sin 4x}\right]$ showing all working. **2**

b) Prove that if θ is small $\sin\left(\theta + \dfrac{\pi}{6}\right) \approx \dfrac{\theta\sqrt{3}+1}{2}$. **2**

c) Find $\int \sin^2 3x \, dx$. **3**

d) For what value of a does the equation $x^2 + a - \frac{1}{2}\cos(2x) = 0$ always have two solutions? Justify your answer. **2**

e) Show that $\dfrac{3x+2}{x-1} = 3 + \dfrac{5}{x-1}$ and hence find the exact value of $\displaystyle\int_2^3 \frac{3x+2}{x-1} dx$.

Question 4 (*Start a new booklet*).

a) i) Show that the curve $y = x^2\log_e x$ has one stationary point and determine its nature. **4**

ii) Determine whether the curve has any points of inflexion, and sketch the curve. **5**

b) By using the substitution $u = \tan x$ or otherwise, find $\displaystyle\int \frac{\sec^2 x}{\sqrt{1+2\tan x}}\,dx$. **3**

Question 5 (*Start a new booklet*).

a) i) For $f(x) = 5\sin^{-1}\left(\frac{3x}{2}\right)$, state the domain and range. **2**

ii) Sketch the curve $y = f(x)$. **2**

iii) Find $f'(x)$ in simplest form. **2**

b) i) Write down an expression for $\tan(\alpha - \beta)$ **1**

ii) Prove that $\tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239} = \frac{\pi}{4}$. **2**

c) i) Differentiate $x\tan^{-1}x$. **1**

ii) Hence find $\displaystyle\int \tan^{-1} x\,dx$. **2**

Question 6 (*Start a new booklet*).

a) P is the point $(2p, p^2)$ on the parabola $x^2 = 4y$.

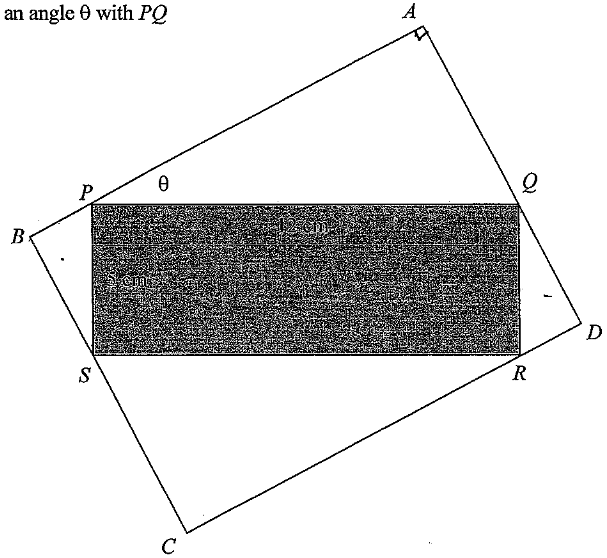
i) Find the equation of the chord through $P(2p, p^2)$ and $Q(2q, q^2)$. **3**

ii) If PQ passes through $(0, 2)$, prove that $pq = -2$. **1**

iii) The normals to the parabola P and Q intersect at R. You are given that R has coordinates $\left(2(p+q), p^2+q^2\right)$

Show that, as the chord PQ varies, R lies on the parabola $x^2 = 4(y - 4)$. **3**

b) $ABCD$ is a rectangle passing through the vertices of another rectangle $PQRS$, with AB making an angle θ with PQ **5**



The sides PQ and PS are 12 cm and 5 cm.

i) Show that the area of the rectangle $ABCD$ is given by

$$A = (5\cos\theta + 12\sin\theta)(5\sin\theta + 12\cos\theta).$$

ii) Show that $\dfrac{dA}{d\theta} = 169(\cos^2\theta - \sin^2\theta)$.

iii) Find the maximum area of the rectangle $ABCD$.

Question 7 (*Start a new booklet*).

a) Find the domain of the function $f(x)=\sqrt{\dfrac{-(2x^2+1)+\sqrt{8x^2+1}}{2}}$. **3**

b) i) Show that **2**

$$\sqrt{2}\cos\left(u+\frac{\pi}{4}\right)=\cos u-\sin u$$

ii) Prove that **3**

$$\frac{d}{dx}\left(e^x\cos x\right)=2^{1/2}e^x\cos\left(x+\frac{\pi}{4}\right).$$

iii) Prove, by the method of mathematical induction, that **4**

$$\frac{d^n}{dx^n}\left(e^x\cos x\right)=2^{n/2}e^x\cos\left(x+\frac{n\pi}{4}\right)\quad (n=1,2,..).$$

Extension 1 2006 PRETRIAL

Q1 a) $k:l = 7:-2$ 1

$x = \dfrac{lx_1 + kx_2}{k+l}$ $\qquad y = \dfrac{ly_1 + ky_2}{k+l}$

$6 = \dfrac{-2 + 7a}{5}$ $\qquad 8 = \dfrac{-2+7b}{5}$ 1

$\therefore a = 4$ $\qquad \therefore b = 6$ 1

b) $\angle APC = \angle CAE$ (alternate segment theorem)

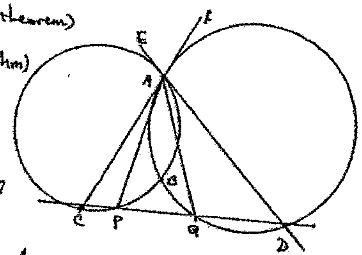
$\angle AQD = \angle DAF$ (alternate segment thm)

But $\angle CAE = \angle DAF$ (vert. opp. $\angle$s) } 1

$\therefore \angle APC = \angle AQD$

$\therefore \angle APQ = \angle AQP$ (supplements of equal angles) } 1

$\therefore AP = AQ$ (sides in a triangle opposite equal angles) } 1



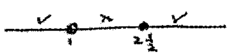
c) Method 1

Solve $\dfrac{3}{x-1} = 2$

$2x - 2 = 3$

$x = 2\frac{1}{2}$ (1)

Critical values : $x = 1$



T: 0 $\quad$ 2 $\quad$ 3

$-3 \le 2$ $\quad 3 \le 2$ $\quad \frac{3}{2} \le 2$ (1)

True $\quad$ False $\quad$ True

$\therefore x < 1$ or $x \ge 2\frac{1}{2}$ (1)

$x < 1$ or $x \ge 2\frac{1}{2}$ (1)

Method 2

Multiply by $(x-1)^2$

$3(x-1) \le 2(x-1)^2$ (1)

$2(x-1)^2 - 3(x-1) \ge 0$

$(x-1)(2x-2-3) \ge 0$

$(x-1)(2x-5) \ge 0$ (1)

Method 3

Put RHS = 0

$\dfrac{3}{x-1} - 2 \le 0$

$\dfrac{5-2x}{x-1} \le 0$ (1)

$\therefore (5-2x)(x-1) \le 0 \quad x \ne 1$ (1)

$\therefore x < 1$ or $x \ge 2\frac{1}{2}$ (1)

Marking depends on method

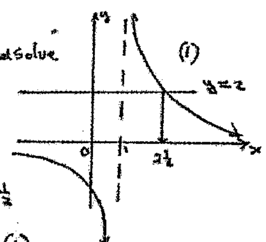
Method 4

Sketch and solve

$\dfrac{3}{x-1} = 2$

$x = 2\frac{1}{2}$ (1)

$\therefore x < 1$ or $x \ge 2\frac{1}{2}$



(1)

d) $\int 5^x \, dx = \dfrac{1}{\ln 5} 5^x + C$ 1

e) $\dfrac{dy}{dx} = \dfrac{1}{\ln x} \times \dfrac{1}{x}$ 1 for either

Question 2 a) $|2x-3| \le x+1$

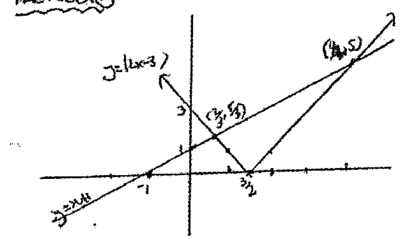
$|2x-3| = x+1$

when $2x-3 = x+1$ or $2x-3 = -x-1$

i.e. $x = 4$ or $x = 2/3$.

$\therefore$ To solve inequality:

method 1

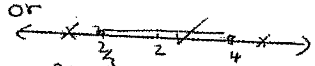


$y = |2x-3| \le y = x+1$, when $\frac{2}{3} \le x \le 4$

method 2

Test points:

or



0: $3 \le 1$? No.

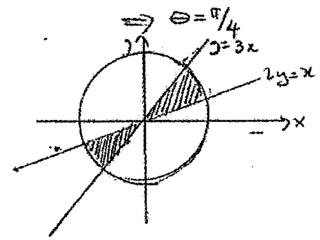
2: $1 \le 3$ ✓

10: $17 \le 11$? No

$\therefore$ Sol^n is $\frac{2}{3} \le x \le 4$

b) $m_1 = \frac{1}{2}$ $\quad m_2 = 3$. $\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\dfrac{3 - \frac{1}{2}}{1 + 3 \times \frac{1}{2}}\right| = \left|\dfrac{2\frac{1}{2}}{2\frac{1}{2}}\right| = 1$

$\Rightarrow \theta = \pi/4$



A = 2 sectors

$= 2 \times \frac{1}{2} \times r^2 \times \theta$

$= 4 \times \pi/4 = \pi \; u^2$

c) $y = e^{2x+3} \Rightarrow$

to find inverse do:-

$x = e^{2y+3}$.

$\ln x = 2y + 3$

$\therefore f^{-1}(x) = \dfrac{\ln x - 3}{2}$

(since $y = e^{2x+3}$ is always increasing it is not necessary to restrict the domain).

d) $\displaystyle\int_{-3/2}^{3/2} \frac{dx}{3+4x^2} = \frac{1}{2\sqrt{3}}\left[\tan^{-1}\frac{2x}{\sqrt{3}}\right]_{-3/2}^{3/2} = \frac{1}{2\sqrt{3}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}(-\sqrt{3})\right)$

$= \dfrac{1}{2\sqrt{3}}\left(\dfrac{\pi}{3} - \left(-\dfrac{\pi}{3}\right)\right) = \dfrac{\pi}{3\sqrt{3}}$.

Question 3

a) $\lim\limits_{x\to 0} \dfrac{\tan 3x}{\sin 4x} = \lim\limits_{x\to 0} \dfrac{3}{4} \times \dfrac{\tan 3x}{3x} \times \dfrac{4x}{\sin 4x}$

$= \dfrac{3}{4} \times 1 \times 1$

$= \dfrac{3}{4}$

b) $\sin(\theta + \frac{\pi}{6}) = \sin\theta\cos\frac{\pi}{6} + \cos\theta\sin\frac{\pi}{6}$

$= \sin\theta \dfrac{\sqrt{3}}{2} + \cos\theta \times \frac{1}{2}$

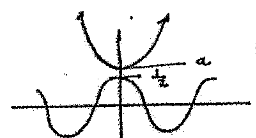
But, if θ is small $\sin\theta \doteqdot \theta$ and $\cos\theta \doteqdot 1$.

$\therefore \sin(\theta + \frac{\pi}{6}) \doteqdot \dfrac{\theta\sqrt{3}+1}{2}$

c) $\int \sin^2 3x\,dx = \frac{1}{2}\int 1 - \cos 6x\,dx$

$= \frac{1}{2}\left[x - \frac{1}{6}\sin 6x\right] + C$

d) the solutions are the same as the x values of the intersection of $y = x^2 + a$ and $y = \frac{1}{2}\cos 2x$



The parabola will cut the other curve twice when $a < \frac{1}{2}$

e) RHS $= 3 + \dfrac{5}{x-1}$

$= \dfrac{3x - 3 + 5}{x-1}$

$= \dfrac{3x+2}{x-1}$

$=$ LHS

$\therefore \int_2^3 \dfrac{3x+2}{x-1}\,dx = \int_2^3 3 + \dfrac{5}{x-1}\,dx$

$= \left[3x + 5\log|x-1|\right]_2^3$

$= \left[9 + 5\log 2 - 6 - 0\right]$

$= 3 + 5\log 2$

#4 a i) $y = x^2 \log x$

For S.P $y' = 0$.

$y' = (\log x)(2x) + (x^2)(\frac{1}{x})$

$y' = 2x\ln x + x$.

$y' = x(2\ln x + 1)$

$\therefore x = 0$ or $2\ln x + 1 = 0$

no sol^n ✓

$2\log_e x = -1$

$\log_e x = \frac{-1}{2}$.

$x = e^{-1/2}$

$x = \dfrac{1}{\sqrt{e}}$ ✓

$y = \left(\dfrac{1}{\sqrt{e}}\right)^2 \ln e^{-\frac{1}{2}}$

$y = \frac{1}{e} \cdot -\frac{1}{2} = -\frac{1}{2e}$ ✓

S.P $\left(\frac{1}{\sqrt{e}}, -\frac{1}{2e}\right)$.

nature: $y'' = (2\ln x + 1)(1) + (x)(\frac{2}{x})$

$y'' = 2\ln x + 1 + 2$

$y'' = 2\ln x + 3$

at $x = \frac{1}{\sqrt{e}}$

$y'' = 2\ln e^{-1/2} + 3$

$= 2$.

pos ∪ min ✓

4

ii) P.O.I. $y'' = 0$.

$2\ln x + 3 = 0$

$2\ln x = -3$

$\ln x = -\frac{3}{2}$

$x = e^{-3/2} = \dfrac{1}{\sqrt{e^3}}$

$y = \left(\dfrac{1}{\sqrt{e^3}}\right)^2 \ln e^{-\frac{3}{2}}$

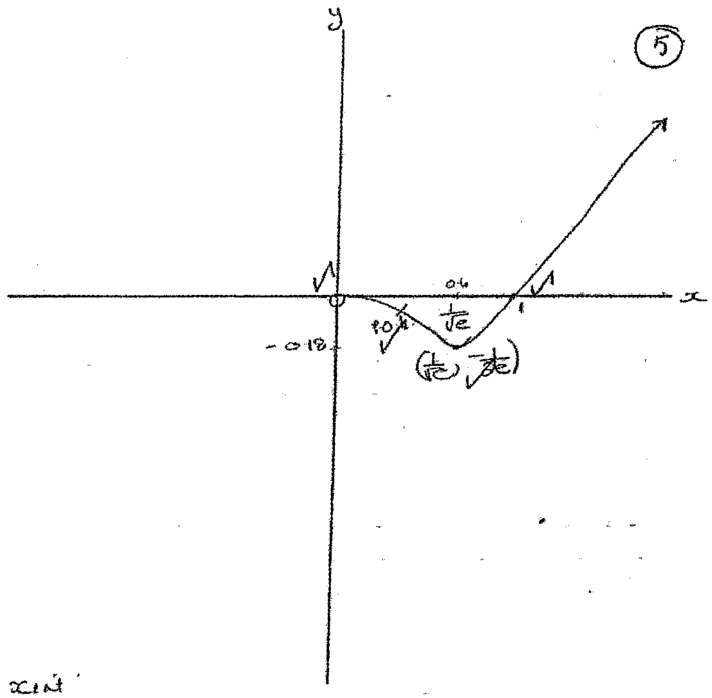
$y = \frac{1}{e^3} \times -\frac{3}{2} = -\frac{3}{2e^3}$

possible P.O.I. $\left(\frac{1}{\sqrt{e^3}}, -\frac{3}{2e^3}\right)$

check change in concavity:

x	$x=0.1$	$\frac{1}{\sqrt{e^3}}$ (0.2)	$x=1$
y''	-1.6	0	3
	∩		∪

∴ since concavity changes P.O.I.



5

x int
$\log x = 0$
$x = 1$.

#4 b) $\int \frac{\sec^2 x}{\sqrt{1+2\tan x}}\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\,dx$

$\int \frac{du}{\sqrt{1+2u}}$

$\frac{1}{2}\int 2(1+2u)^{-1/2}\,du$

$= \frac{1}{2}\frac{(1+2u)^{+1/2}}{\frac{1}{2}} + c.$

$= \sqrt{1+2\tan x} + c.$

3

OR!!

$\frac{1}{2}\int 2\sec^2 x\,(1+2\tan x)^{-1/2}\,dx$

$= \frac{1}{2}\frac{(1+2\tan x)^{1/2}}{\frac{1}{2}} + c$

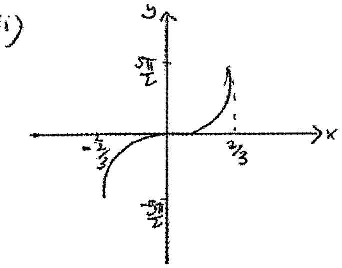
$= \sqrt{1+2\tan x} + c$

3

Question 5 a) i) Domain: $-1 \le \frac{3x}{2} \le 1$ Range: $-\frac{\pi}{2} \le \sin^{-1}\left(\frac{3x}{2}\right) \le \frac{\pi}{2}$

$\Rightarrow -\frac{2}{3} \le x \le \frac{2}{3}$. $\therefore -\frac{5\pi}{2} \le 5\sin^{-1}\left(\frac{3x}{2}\right) \le \frac{5\pi}{2}$

ii)



iii) $f'(x) = \frac{3}{2} \times \frac{5}{\sqrt{1 - 9x^2/4}}$

$= \frac{15}{\sqrt{4 - 9x^2}}$

b) i) $\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan\alpha \cdot \tan\beta}$.

ii) $\tan(LHS) = \tan\left(\tan^{-1}\left(\frac{120}{119}\right) - \tan^{-1}\left(\frac{1}{239}\right)\right)$

$= \frac{\tan\tan^{-1}\left(\frac{120}{119}\right) - \tan\tan^{-1}\left(\frac{1}{239}\right)}{1 + \tan\tan^{-1}\left(\frac{120}{119}\right) \cdot \tan\tan^{-1}\left(\frac{1}{239}\right)}$

$= \frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \cdot \frac{1}{239}} = \frac{120 \times 239 - 119}{119 \times 239 + 120} = \frac{28561}{28561} = 1$.

$\tan(LHS) = 1$

$\therefore LHS = \tan^{-1}(1) = \pi/4 = RHS$.

c) i) $\frac{d}{dx}(x\tan x) = (\text{product rule}) = \tan^{-1}x + \frac{x}{1+x^2}$.

ii) $\therefore \tan^{-1}x = \frac{d}{dx}(x\tan^{-1}x) - \frac{x}{1+x^2}$

$\int \tan^{-1}x\,dx = \int \frac{d}{dx}(x\tan^{-1}x) - \int \frac{x}{1+x^2}\,dx$.

$= x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$.

#6 a) $P(2p, p^2)$ normal: $x + py = p^3 + 2p$

i) $m(PQ) = \frac{q^2 - p^2}{2q - 2p}$ $\frac{(q-p)(q+p)}{2(q-p)}$ ✓

$= \frac{q+p}{2}$. 2.

$\therefore$ eq. of chord: $y - p^2 = \frac{q+p}{2}(x - 2p)$ ✓

$\binom{x\ \ y}{(0, 2)}$: $2 - p^2 = \frac{q+p}{2}(0 - 2p)$ 1.

$2 - p^2 = -qp - p^2$ ✓

$-2 = qp$.

ii) $(x)^2 = (2(p+q))^2$ ✓ $y = p^2 + q^2$ ✓ ⑥

$x^2 = 4(p^2 + 2pq + q^2)$ ✓ 3.

$x^2 = 4(y + 2(-2))$

$x^2 = 4(y - 4)$ ✓

b) In Δ APQ: $\cos\theta = \frac{AP}{12}$

$AP = 12\cos\theta$ ✓

In Δ BPS:

$\cos(90 - \theta) = \frac{BP}{5}$

$BP = 5\sin\theta$ ✓

$\angle BPS = 180 - (90 - \theta)$

$= 90 - \theta$.

and $\angle CSR = 90 - \theta$.

$AB = AP + BP$

$= 12\cos\theta + 5\sin\theta$. 2.

Similarly $\sin(90 - \theta) = \frac{BS}{5}$ ✓ and $\cos(90 - \theta) = \frac{SC}{12}$ ✓

$BS = 5\cos\theta$. $SC = 12\sin\theta$.

$BC = BS + SC = 5\cos\theta + 12\sin\theta$

$\therefore$ Area $= AB \times BC = (12\cos\theta + 5\sin\theta)(5\cos\theta + 12\sin\theta)$

$y' = vu' + uv'$

6 ii) $A = (\overset{u}{5\cos\theta + 12\sin\theta})(\overset{v}{5\sin\theta + 12\cos\theta})$

$\frac{dA}{d\theta} = (-5\sin\theta + 12\cos\theta)(5\sin\theta + 12\cos\theta) +$
$(5\cos\theta + 12\sin\theta)(5\cos\theta - 12\sin\theta)$ ✓

$= -25\sin^2\theta + 60\sin\theta\cos\theta - 60\sin\theta\cos\theta$
$+ 144\cos^2\theta + 25\cos^2\theta - 144\sin^2\theta$

$= -169\sin^2\theta + 169\cos^2\theta$ ✓ 1.
$= 169(\cos^2\theta - \sin^2\theta)$. ✓

or
$A = 25\sin\theta\cos\theta + 60\cos^2\theta + 60\sin^2\theta + \cancel{144}\sin\theta\cos\theta$

$A = 169\sin\theta\cos\theta + 60$. ✓
$A' = 169(\sin\theta(-\sin\theta) + \cos\theta(\cos\theta))$
$A' = 169(\cos^2\theta - \sin^2\theta)$.

To max area: $\frac{dA}{d\theta} = 0$.

$\frac{\cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta}$

$\tan^2\theta = 1$.
$\theta = 45°$. ✓

check: $\frac{d^2A}{d\theta^2} = 169(-2\cos\theta.\sin\theta - 2\sin\theta\cos\theta)$
$= 169(-4\sin\theta\cos\theta)$
$= -676 \times \frac{1}{2}$
$= -338$. 3.
$\frown$ max ✓

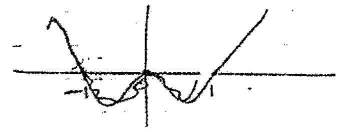
$\therefore$ max area $=$
$(5\cos 45 + 12\sin 45)(5\sin 45 + 12\cos 45)$
$= \left(5.\frac{1}{\sqrt{2}} + \frac{12}{\sqrt{2}}\right)\left(\frac{5}{\sqrt{2}} + \frac{12}{\sqrt{2}}\right)$

$= \frac{17}{\sqrt{2}} \times \frac{17}{\sqrt{2}}$

$= \frac{289}{2}$ ⑥

$= 144.5$ ✓

#7a) $-(2x^2+1) + \sqrt{8x^2+1} \geq 0$.
consider $\sqrt{8x^2+1} \geq 2x^2+1$
$8x^2+1 \geq 4x^4 + 4x^2 + 1$
$0 \geq 4x^4 - 4x^2$ ✓
$4x^2(x^2-1) \leq 0$.
$x=0 \quad x=1 \quad x=-1$. ✓ 3.



$-1 \leq x \leq 1$. ✓

b) i) LHS: $\sqrt{2}\cos(u + \pi/4)$
$= \sqrt{2}\left[\cos u \cos\frac{\pi}{4} - \sin u \sin\frac{\pi}{4}\right]$ ✓
$= \sqrt{2}\left[\cos u \frac{1}{\sqrt{2}} - \sin u . \frac{1}{\sqrt{2}}\right]$ ✓ 2.
$= \cos u - \sin u$ ✓
$=$ RHS.

ii) $\frac{d}{dx}(\overset{u}{e^x}\overset{v}{\cos x}) = (\cos x)(e^x) + (e^x)(-\sin x)$. ✓
$= e^x(\cos x - \sin x)$. ✓
$= e^x(\sqrt{2}\cos(x + \frac{\pi}{4}))$ ✓
$= 2^{\frac{1}{2}}.e^x\cos(x + \pi/4)$ ✓ 3.

prove true for $n=1$.

prove: $\frac{d}{dx}(e^x \cos x) = \sqrt{2}\, e^x \cos\left(x+\frac{\pi}{4}\right)$

proved above. ✓

assume true for $n=k$.

$\frac{d^k}{dx^k}(e^x \cos x) = 2^{\frac{k}{2}} e^x \cos\left(x + \frac{k\pi}{4}\right)$. ✓

prove true for $n=k+1$.

prove: $\frac{d^{k+1}}{dx^{k+1}}(e^x \cos x) = 2^{\frac{k+1}{2}} e^x \cos\left(x + \frac{(k+1)\pi}{4}\right)$.

LHS: $\frac{d}{dx}\left(\frac{d^k}{dx^k}(e^x \cos x)\right)$ ✓

$= \frac{d}{dx}\left(2^{k/2} e^x \cos\left(x + \frac{k\pi}{4}\right)\right)$

$= \cos\left(x + \frac{k\pi}{4}\right)\left(2^{k/2} e^x\right) + \left(2^{k/2} e^x\right)\left(-\sin\left(x + \frac{k\pi}{4}\right)\right)$ ✓

$= 2^{\frac{k}{2}} e^x \left(\cos\left(x + \frac{k\pi}{4}\right) - \sin\left(x + \frac{k\pi}{4}\right)\right)$ ✓

$= 2^{\frac{k}{2}} e^x \left(\sqrt{2} \cdot \cos\left(x + \frac{k\pi}{4} + \frac{\pi}{4}\right)\right)$

$= 2^{\frac{k}{2}} e^x\, 2^{\frac{1}{2}} \cos\left(x + \frac{(k+1)\pi}{4}\right)$

$= 2^{\frac{k+1}{2}} e^x \cos\left(x + \frac{(k+1)\pi}{4}\right)$. ✓ 4.

$=$ RHS.

1.