TOPICS - QUADRATICS, CALCULUS + LOCUS

YEAR 12 REVISION **2005**

Question 1.

(i) Differentiate with respect to x:

(a) $3x^2 - 5x + 7$

(b) $\sqrt{x^2 + 4}$

(c) $\dfrac{2x+1}{x+1}$

(d) $x^3(x+7)^8$

(ii) Differentiate the following functions using the method of differentiation from first principles:

(a) $f(x) = 2x^2 + 6x + 3$

(iii) Show that the equation of the normal to the curve $y = x^3 + 3x$ is $x + 15y - 212 = 0$ at the point where $x = 2$.

Question 2. (15 marks)

(i) Solve the following equations:

(a) $3x^2 + 6x + 3 = 0$

(b) $x^2 - 6x + 4 = 0$

(c) $3x^4 - 13x^2 + 4 = 0$

(d) $x^2 + x - \dfrac{2}{x^2 + x} = 1$

(ii) State whether the roots of the following equation are equal or unequal, giving reasons for your answer:

$$x^2 + 4x + 2 = 0$$

(iii) Find the values of m for which the equation

$$x^2 + (m-1)x + 3 = 0$$

has:

(a) equal roots.

(b) no real roots.

Question 3.

(i) Let α and β be the roots of the equation $x^2 - 7x + 3 = 0$. Find the values of :

(a) $\alpha + \beta$

(b) $\alpha\beta$

(c) $(\alpha + 1)(\beta + 1)$

(ii) Find the equation of the quadratic with roots $3 + \sqrt{11}$, $3 - \sqrt{11}$.

(iii) Find the values of g for which the equation $3x^2 - (2g+1)x + (3g - 5) = 0$ has:

(a) the sum of its roots equal to the product of its roots.

(b) the roots as reciprocals of each other.

(iv) Express $3x^2 - x + 20$ in the form $A(x-1)^2 + B(x+2) + C$.

Question 4.

(i) Let A and B be the fixed points (-4, 3) and (5, 3) respectively and let P be the variable point (x, y).

(a) Write down expressions for PA^2 and PB^2 in terms of x and y.

(b) Suppose that P moves so that $PA = 2PB$. Show that locus of P is a circle.

(c) Find the centre and radius of the circle.

(ii) Find the equation of the parabola with focus at (-2, 2) and whose directrix is given by the equation $y = 3$.

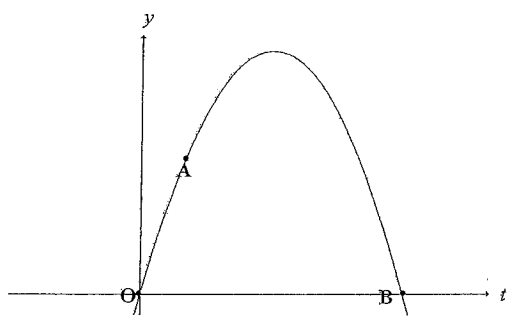
(iii) Find the coordinates of the vertex and the focus, and the equation of the directrix, of the parabola with equation:

$$y^2 - 4y + 8x + 20 = 0.$$

(iv) Hence, sketch the parabola including all relevant information.

Question 5. (7 marks)

A ball is thrown through the air and its motion is described by the parabola below.



The ball is thrown upward from **O** when $t = 0$.
After 1 second the ball reaches a height of 10m at **A** and then it returns to the ground after 3 seconds at **B**.

(a) Show that the equation of the height, y, of the ball in terms of the time, t is given as $y = -5t^2 + 15t$.

(b) Calculate the time, t, at which the ball reaches its maximum height and find the maximum height reached by the ball.

(c) The velocity of the ball is given as $\frac{dy}{dt}$. Find the velocity of the ball when it hits the ground at **B**.

YEAR 12 2U ASSESSMENT 1 NOVEMBER 2004

Q1 a) If $y = 3x^2 - 5x + 7$

$\therefore \frac{dy}{dx} = 6x - 5$

b) Let $y = \sqrt{x^2+4}$

$= (x^2+4)^{\frac{1}{2}}$

$\therefore \frac{dy}{dx} = \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \times 2x$

$= \frac{x}{\sqrt{x^2+4}}$

c) Let $y = \frac{2x+1}{x+1}$

$\therefore \frac{dy}{dx} = \frac{2(x+1) - (2x+1)\times 1}{(x+1)^2}$

$= \frac{1}{(x+1)^2}$

d) Let $y = x^3(x+7)^8$

$\therefore \frac{dy}{dx} = 3x^2(x+7)^8 + x^3 \cdot 8(x+7)^7$

$= x^2(x+7)^7\left[3(x+7) + 8x\right]$

$= x^2(x+7)^7\left[11x + 21\right]$

ii) $f(x+h) - f(x) = 2(x+h)^2 + 6(x+h) + 3 - (2x^2 + 6x + 3)$

$= 2x^2 + 4xh + 2h^2 + 6x + 6h + 3 - 2x^2 - 6x - 3$

$= 4xh + 2h^2 + 6h$

$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} 4x + 2h + 6$

$= 4x + 6$

iii) $\frac{dy}{dx} = 3x^2 + 3$

$= 15$ at $x = 2$

$\therefore$ At $x = 2$, $y = 14$ and the gradient is 15

$\therefore$ The equation of the normal is

$y - 14 = -\frac{1}{15}(x - 2)$

$15y - 210 = -x + 2$

or $x + 15y - 212 = 0$ as required.

Q2.

i) a) $3(x^2+2x+1)=0$

$\therefore (x+1)^2=0$

$\therefore x=-1$

b) $x^2-6x+4=0$

$x^2-6x\boxed{+9}=-4\boxed{+9}$

$(x-3)^2=5$

$x=3\pm\sqrt{5}$

c) Let $m=x^2$

$\therefore 3m^2-13m+4=0$

$(3m-1)(m-4)=0$

$\therefore m=\frac{1}{3}$ or 4

$\therefore x^2=\frac{1}{3}$ or 4

$\therefore x=\pm\frac{1}{\sqrt{3}}$ or $x=\pm 2$

d) Let $m=x^2+x$

$\therefore m-\frac{2}{m}=1$

$m^2-m-2=0$

$(m-2)(m+1)=0$

$\therefore m=2$ or -1

$\therefore x^2+x=2$ or $x^2+x=-1$

$\therefore x^2+x-2=0$ or $x^2+x+1=0$

$\therefore x=1$ or $x=-2$ no solution

ii) $\Delta=16+8$

$=8$

Since $\Delta>0$ the roots are real and unequal

iii) $\Delta=(m-1)^2-12$

$=m^2-2m-11$

a) For equal roots $\Delta=0$

$\therefore m^2-2m-11=0$

$m^2-2m\boxed{+1}=11\boxed{+1}$

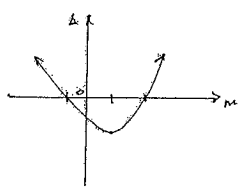
$(m-1)^2=12$

$\therefore m=1\pm\sqrt{12}$

or $m=1\pm 2\sqrt{3}$

b) For no real roots $\Delta<0$

$\therefore m^2-2m-11<0$



$\therefore 1-2\sqrt{3}<m<1+2\sqrt{3}$

3) i) a) $\alpha+\beta=-\frac{b}{a}$

$=-\frac{-7}{1}$

$=7$

b) $\alpha\beta=\frac{c}{a}$

$=\frac{3}{1}$

$=3$

c) $(\alpha+1)(\beta+1)=\alpha\beta+(\alpha+\beta)+1$

$=3+7+1$

$=11$

ii) $\alpha+\beta=3+\sqrt{11}+3-\sqrt{11}$

$\alpha+\beta=6$

$\alpha\beta=(3+\sqrt{11})(3-\sqrt{11})$

$\alpha\beta=9-3\sqrt{11}+3\sqrt{11}-11$

$\alpha\beta=9-11$

$\alpha\beta=-2$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-6x-2=0$ or $y=a(x^2-6x-2)$

iii) a)

$\alpha+\beta=\alpha\beta$

$\frac{2g+1}{3}=\frac{3g-5}{3}$

$2g+1=3g-5$

$g=6$

b) $\alpha=\frac{1}{\beta}$

$\alpha\beta=1$

$\frac{3g-5}{3}=1$

$3g-5=3$

$3g=8$

$g=\frac{8}{3}$

iv) $3x^2-x+20=A(x-1)^2+B(x+2)+C$

$=A(x^2-2x+1)+Bx+2B+C$

$=Ax^2+(B-2A)x+(A+2B+C)$

By comparing coefficients of x.

$A=3$

$B-2A=-1$

$B-6=-1$

$B=5$

$A+2B+C=20$

$3+10+C=20$

$C=7$

$3x^2-x+20=3(x-1)^2+5(x+2)+7$

4i) a) $PA^2 = (x+4)^2 + (y-3)^2$
$PB^2 = (x-5)^2 + (y-3)^2$

1

CRITERIA	MARK
Correct expressions	1

4i) b) **2**

Squaring both sides of the expression and obtaining a factor of 4 on the right hand side	1 out of 2
OR An expression of the form $(x-a)^2 + (y-b)^2 = r^2$	1 out of 2
Correct expression of the circle	2

$PA = 2PB$
$\sqrt{(x+4)^2 + (y-3)^2} = 2\sqrt{(x-5)^2 + (y-3)^2}$
$(x+4)^2 + (y-3)^2 = 4\{(x-5)^2 + (y-3)^2\}$
$x^2 + 8x + 16 + y^2 - 6y + 9 = 4(x^2 - 10x + 25 + y^2 - 6y + 9)$
$x^2 - 16x + y^2 - 6y = -37$
$x^2 - 16x + 64 + y^2 - 6y + 9 = -37 + 64 + 9$
$(x-8)^2 + (y-3)^2 = 36$

Correct centre consistent with expression found in 4 i) b)	1 out of 2
Correct radius consistent with expression found in 4 i) b)	1 out of 2

This is a circle as it is of the form $(x-h)^2 + (y-k)^2 = r^2$

4i) c) *center* $= (8, 3)$
radius $= 6$

2

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4 ii) $(x+2)^2 = -2\left(y - \frac{5}{2}\right)$

3

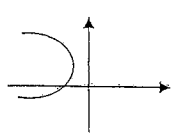
CRITERIA		MARK
$(x+2)^2 = -2\left(y - \frac{5}{2}\right)$		3
$a = \frac{1}{2}$	1 mark each	1 out of 2
Vertex $= \left(-2, 2\frac{1}{2}\right)$	1 mark each	1 out of 2

4 iii) $y^2 - 4y + 4 = 4 - 8x - 20$
$(y-2)^2 = -8x - 16$
$(y-2)^2 = -8(x+2)$
Vertex $= (-2, 2)$
Focus $= (-4, 2)$
Directrix is $x = 0$

3

Correct Vertex	1
Correct Focus **or** Focus consistent with Vertex and graph and the value of a	1
Correct Directrix **or** Directrix consistent with Vertex and graph and the value of a	1

4 iv)



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Parabola correct position and orientation	2
Parabola with axis parallel to the x axis with vertex consistent with answer in 4 iii)	1
Parabola in the correct direction with vertex consistent with answer in 4 iii)	1

Question 5

(a) $t=0, y=0$ $\quad t=1, y=10$, $t=3, y=0$

form of parabola: $y = at^2 + bt + c$

At $(0,0)$: $0 = 0+0+c$

$\therefore c = 0$

At $(1,10)$: $10 = a(1)^2 + b(1)$

$a+b = 10$

$a = 10-b$

At $(3,0)$: $a(3)^2 + b(3) = 0$

$9a + 3b = 0$

$3a + b = 0$

$3(10-b) + b = 0$

$30 - 3b + b = 0$

$-2b = -30$

$b = 15$

$a = 10-15$

$= -5$

$\therefore y = -5t^2 + 15t$

OR $y = -at(t-3)$ roots at $(0,0)$ & $(3,0)$ concave down

at $(1,10)$: $10 = -a(1-3)$

$10 = -a(-2)$

$a = 5$

$\therefore y = -5t(t-3)$

$y = -5t^2 + 15t$

(b) maximum height at vertex

$$t = \frac{-b}{2a}$$

$$= \frac{-15}{2\times -5}$$

$$= \frac{-15}{-10} = \frac{3}{2}$$

At $t = \frac{3}{2}$, $y = -5\left(\frac{3}{2}\right)^2 + 15\left(\frac{3}{2}\right)$

$= 11\frac{1}{4}$ OR 11.25

$\therefore$ ball reaches maximum height of 11.25 m when $t = \frac{3}{2}$ (1.5) sec

(c) $\frac{dy}{dt} = -10t + 15$

At $B(3,0)$: $\frac{dy}{dt} = -10(3)+15$

$= -30+15$

$= -15$

$\therefore$ the velocity of the belt is -15 m/s.