Inverse functions and inverse trig functions   <u>Solutions</u>   <u>Main Menu</u>

55 If $f(x) = e^{x+2}$ what is the inverse function $f^{-1}(x)$?

(A) $f^{-1}(x) = e^{y-2}$

(B) $f^{-1}(x) = e^{y+2}$

(C) $f^{-1}(x) = \log_e x - 2$

(D) $f^{-1}(x) = \log_e x + 2$

56 If $f(x) = 1 - \cos\dfrac{x}{2}$ what is the inverse function $f^{-1}(x)$?

(A) $f^{-1}(x) = 2\cos^{-1}(1-x)$

(B) $f^{-1}(x) = \dfrac{1}{2}\cos^{-1}(1-x)$

(C) $f^{-1}(x) = \dfrac{1}{2}\cos^{-1}(1+x)$

(D) $f^{-1}(x) = 2\cos^{-1}(1+x)$

57 What is the domain and range of $y = \cos^{-1}(\dfrac{3x}{2})$?

(A) Domain: $-\dfrac{2}{3} \le x \le \dfrac{2}{3}$. Range: $0 \le y \le \pi$

(B) Domain: $-1 \le x \le 1$. Range: $0 \le y \le \pi$

(C) Domain: $-\dfrac{2}{3} \le x \le \dfrac{2}{3}$. Range: $-\pi \le y \le \pi$

(D) Domain: $-1 \le x \le 1$. Range: $-\pi \le y \le \pi$

58 What is the derivative of $y = \cos^{-1}(\dfrac{1}{x})$ with respect to x?

(A) $\dfrac{-1}{\sqrt{x^2 - 1}}$

(B) $\dfrac{-1}{x\sqrt{x^2 - 1}}$

(C) $\dfrac{1}{\sqrt{x^2 - 1}}$

(D) $\dfrac{1}{x\sqrt{x^2 - 1}}$

59 What is the value of $f'(x)$ if $f(x) = 2x^2 \cos^{-1} 2x$?

(A) $\dfrac{-8x}{\sqrt{1 - 2x^2}}$

(B) $\dfrac{-8x}{\sqrt{1 - 4x^2}}$

(C) $\dfrac{-4x^2}{\sqrt{1 - 2x^2}} + 4x\cos^{-1} 2x$

(D) $\dfrac{-4x^2}{\sqrt{1 - 4x^2}} + 4x\cos^{-1} 2x$

60 What is the value of $f'(x)$ if $f(x) = \tan^{-1} x + x\tan^{-1} x$?

(A) $\dfrac{1}{1 + x^2}$

(B) $\dfrac{x+1}{1 + x^2}$

(C) $\tan^{-1} x + \dfrac{1}{1 + x^2}$

(D) $\tan^{-1} x + \dfrac{x+1}{1 + x^2}$

61 Which of the following is the correct expression for $\displaystyle\int \dfrac{dx}{\sqrt{36 - x^2}}$?

(A) $\cos^{-1}\dfrac{x}{6} + c$

(B) $\cos^{-1} 6x + c$

(C) $\sin^{-1}\dfrac{x}{6} + c$

(D) $\sin^{-1} 6x + c$

62 Which of the following is the correct expression for $\displaystyle\int \dfrac{dx}{\sqrt{49 - x^2}}$?

(A) $-\cos^{-1}\dfrac{x}{7} + c$

(B) $-\cos^{-1} 7x + c$

(C) $-\sin^{-1}\dfrac{x}{7} + c$

(D) $-\sin^{-1} 7x + c$

63 What is the exact value of the definite integral $\int_{\frac{2}{\sqrt{3}}}^{2\sqrt{3}} \dfrac{dx}{x^2+4}\,dx$?

(A) $\dfrac{\pi}{12}$ (B) $\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{2}$

64 What is the exact value of the definite integral $\int_0^1 \left(\dfrac{1}{1+x} + e^{-x} + \dfrac{1}{\sqrt{1-x^2}} \right) dx$?

(A) $\log_e 2 - \dfrac{1}{e} + \dfrac{\pi}{2} + 1$

(B) $\log_e 2 - \dfrac{1}{e} + \dfrac{\pi}{2} + 2$

(C) $\log_e 2 - \dfrac{1}{e} + \pi + 1$

(D) $\log_e 2 - \dfrac{1}{e} + \pi + 2$

65 Which of the following is an expression for $\int \dfrac{e^x}{1+e^{2x}}\,dx$?

Use the substitution $u = e^x$.

(A) $e^x \tan^{-1} e^x + c$ (B) $e^x \tan^{-1} e^{2x} + c$

(C) $\tan^{-1} e^x + c$ (D) $\tan^{-1} e^{2x} + c$

66 What is the solution to the equation $5(\cos(x + \tan^{-1}\dfrac{4}{3})) = 0.5$ for $0 \le x \le 2\pi$?

(A) $x = -7.21$ or $x = 0.12$

(B) $x = -5.67$ or $x = 2.82$

(C) $x = 0.54$ or $x = 3.89$

(D) $x = 1.95$ or $x = 4.12$

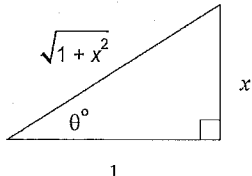67 Which of the following expressions is correct?

(A) $\tan^{-1} x = \cos^{-1} \dfrac{1}{\sqrt{1-x^2}}$ (B) $\tan^{-1} x = \cos^{-1} \dfrac{1}{\sqrt{1+x^2}}$

(C) $\tan^{-1} x = \cos^{-1} \dfrac{x}{\sqrt{1-x^2}}$ (D) $\tan^{-1} x = \cos^{-1} \dfrac{x}{\sqrt{1+x^2}}$

	Inverse functions and inverse trigonometric functions	Main Menu
	Solution	**Criteria**
55	$f(x) = e^{x+2}$ or $y = e^{x+2}$ Inverse function is $x = e^{y+2}$ $$\log_e x = y + 2$$ $$y = \log_e x - 2$$ $$f^{-1}(x) = \log_e x - 2$$	1 Mark: C
56	$f(x) = 1 - \cos\dfrac{x}{2}$ or $y = 1 - \cos\dfrac{x}{2}$ Inverse function is $\quad x = 1 - \cos\dfrac{y}{2}$ $$\cos\dfrac{y}{2} = 1 - x$$ $$y = 2\cos^{-1}(1-x)$$ $$f^{-1}(x) = 2\cos^{-1}(1-x)$$	1 Mark: A
57	Domain: $-1 \le \dfrac{3x}{2} \le 1$ or $-\dfrac{2}{3} \le x \le \dfrac{2}{3}$. Range: $0 \le y \le \pi$	1 Mark: A
58	$\dfrac{d}{dx}\cos^{-1}\left(\dfrac{1}{x}\right) = -\dfrac{1}{\sqrt{1 - \dfrac{1}{x^2}}} \times \left(-\dfrac{1}{x^2}\right)$ $$= \dfrac{-x}{\sqrt{x^2 - 1}} \times \left(-\dfrac{1}{x^2}\right)$$ $$= \dfrac{1}{x\sqrt{x^2 - 1}}$$	1 Mark: D
59	$f(x) = 2x^2 \cos^{-1} 2x$ $$= 2x^2 \dfrac{-1}{\sqrt{\dfrac{1}{4} - x^2}} + 4x\cos^{-1} 2x$$ $$= \dfrac{-4x^2}{\sqrt{1 - 4x^2}} + 4x\cos^{-1} 2x$$	1 Mark: D
60	$f(x) = \tan^{-1} x + x\tan^{-1} x$ $f'(x) = \dfrac{1}{1+x^2} + x \times \dfrac{1}{1+x^2} + \tan^{-1} x \times 1$ $$= \tan^{-1} x + \dfrac{x+1}{1+x^2}$$	1 Mark: D
61	$\displaystyle\int \dfrac{dx}{\sqrt{36 - x^2}} = \sin^{-1}\dfrac{x}{6} + c$	1 Mark: C
62	$\displaystyle\int \dfrac{dx}{\sqrt{49 - x^2}} = -\cos^{-1}\dfrac{x}{7} + c$	1 Mark: A
63	$\displaystyle\int_{\frac{2}{\sqrt{3}}}^{2\sqrt{3}} \dfrac{dx}{x^2 + 4}dx = \left[\dfrac{1}{2}\tan^{-1}\dfrac{x}{2}\right]_{\frac{2}{\sqrt{3}}}^{2\sqrt{3}}$ $$= \dfrac{1}{2}\left(\tan^{-1}\sqrt{3} - \tan^{-1}\dfrac{1}{\sqrt{3}}\right)$$ $$= \dfrac{1}{2}\left(\dfrac{\pi}{3} - \dfrac{\pi}{6}\right) = \dfrac{\pi}{12}$$	1 Mark: A
64	$\displaystyle\int_0^1 \left(\dfrac{1}{1+x} + e^{-x} + \dfrac{1}{\sqrt{1-x^2}}\right)dx = \left[\log_e(1+x) - e^{-x} + \sin^{-1} x\right]_0^1$ $$= \log_e 2 - e^{-1} + \sin^{-1} 1 - \left(\log_e 1 - e^0 + \sin^{-1} 0\right)$$ $$= \log_e 2 - \dfrac{1}{e} + \dfrac{\pi}{2} + 1$$	1 Mark: A
65	$u = e^x$ $\dfrac{du}{dx} = e^x$ $du = e^x dx$ $\displaystyle\int \dfrac{e^x}{1 + e^{2x}}dx = \int \dfrac{du}{1 + u^2}$ $$= \tan^{-1} u + c$$ $$= \tan^{-1} e^x + c$$	1 Mark: C
66	$5(\cos(x + \tan^{-1}\dfrac{4}{3})) = 0.5$ $\cos(x + \tan^{-1}\dfrac{4}{3}) = 0.1$ $x + \tan^{-1}\dfrac{4}{3} = 1.470628906,\ (2\pi - 1.470628906)$ $x = 0.543336876,\ 3.885261184$ $= 0.54,\ 3.89$	1 Mark: C
67	Let $\theta = \tan^{-1} x$ $\cos\theta = \dfrac{1}{\sqrt{1+x^2}}$ $\cos^{-1}\dfrac{1}{\sqrt{1+x^2}} = \theta$ $\tan^{-1} x = \cos^{-1}\dfrac{1}{\sqrt{1+x^2}}$	1 Mark: B