Methods of integration

Solutions Main Menu

1 Which of the following is an expression for $\int \cos^2 x \sin x dx$?

Use the substitution $u = \cos x$.

(A) $2\cos x \sin x + c$

(B) $\cos^3 x + c$

(C) $\frac{1}{3}\cos^3 x + c$

(D) $-\frac{1}{3}\cos^3 x + c$

2 Which of the following is an expression for $\int (2x+1)^{10} dx$?

Use the substitution $u = 2x+1$.

(A) $\frac{1}{9}(2x+1)^9 + c$

(B) $\frac{1}{11}(2x+1)^{11} + c$

(C) $\frac{1}{18}(2x+1)^9 + c$

(D) $\frac{1}{22}(2x+1)^{11} + c$

3 Which of the following is an expression for $\int \frac{x}{\sqrt{x^2-2}} dx$?

Use the substitution $u = x^2 - 2$.

(A) $\sqrt{x^2-2} + c$

(B) $\left(\sqrt{x^2-2}\right)^3 + c$

(C) $2\sqrt{x^2-2} + c$

(D) $2\left(\sqrt{x^2-2}\right)^3 + c$

4 Which of the following is an expression for $\int x\sqrt{1-x^2} dx$?

Use the substitution $u = \sqrt{1-x^2}$.

(A) $-\frac{\left(1-x^2\right)^3}{3} + c$

(B) $\frac{\left(1-x^2\right)^3}{3} + c$

(C) $-\frac{\left(1-x^2\right)^{\frac{3}{2}}}{3} + c$

(D) $\frac{\left(1-x^2\right)^{\frac{3}{2}}}{3} + c$

5 Which of the following is an expression for $\int \frac{e^{-2x} dx}{e^{-x}+1}$?

Use the substitution $u = e^{-x} + 1$.

(A) $\frac{(e^{-x}+1)^2}{2} - e^{-x} + c$

(B) $\frac{(e^{-x}+1)^2}{2} + e^{-x} + c$

(C) $\log_e(e^{-x}+1) - e^{-x} + c$

(D) $\log_e(e^{-x}+1) + e^{-x} + c$

6 Which of the following is an expression for $\int \frac{e^x}{1+e^{2x}} dx$?

Use the substitution $u = e^x$.

(A) $\frac{-1}{2(1+e^x)^2} + c$

(B) $\frac{-e^x}{(1+e^x)^2} + c$

(C) $\tan^{-1} e^x + c$

(D) $\tan^{-1} e^{2x} + c$

7 Which of the following is an expression for $\int \frac{e^{3x}}{1+e^x}dx$?

Use the substitution $u = 1+e^x$.

(A) $\frac{(1+e^x)^3}{3}+(1+e^x)^2-\log_e(1+e^x)+c$

(B) $\frac{(1+e^x)^3}{3}-(1+e^x)^2+\log_e(1+e^x)+c$

(C) $\frac{(1+e^x)^2}{2}+2(1+e^x)-\log_e(1+e^x)+c$

(D) $\frac{(1+e^x)^2}{2}-2(1+e^x)+\log_e(1+e^x)+c$

8 What is the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x}dx$? Use the substitution $u = \tan x$.

(A) -0.6009

(B) 0.6913

(C) $\log_e\sqrt{3}$

(D) $\log_e 3$

9 What is the value of $\int_e^{e^2} \frac{1}{x\log x}dx$? Use the substitution $u = \log_e x$.

(A) $\log_e 0.5$

(B) $\log_e 2$

(C) $\log_e 4$

(D) 1

10 What is the exact value of $\int_{-1}^{1}\sqrt{4-x^2}\,dx$? Use the substitution $x = 2\sin\theta$.

(A) $\frac{\pi}{6}+\frac{\sqrt{3}}{4}$

(B) $\frac{\pi}{3}+\frac{\sqrt{3}}{2}$

(C) $\frac{2\pi}{3}+\sqrt{3}$

(D) $\frac{4\pi}{3}+2\sqrt{3}$

Primitive of $\sin^2 x$ and $\cos^2 x$

Solutions Main Menu

11 Which of the following is an expression for $\int \cos^2 2x\,dx$?

(A) $x-\frac{1}{4}\sin 4x+c$

(B) $x+\frac{1}{4}\sin 4x+c$

(C) $\frac{x}{2}-\frac{1}{8}\sin 4x+c$

(D) $\frac{x}{2}+\frac{1}{8}\sin 4x+c$

12 Which of the following is an expression for $\int \sin^2 6x\,dx$?

(A) $\frac{x}{2}-\frac{1}{24}\sin 6x+c$

(B) $\frac{x}{2}+\frac{1}{24}\sin 6x+c$

(C) $\frac{x}{2}-\frac{1}{24}\sin 12x+c$

(D) $\frac{x}{2}+\frac{1}{24}\sin 12x+c$

13 What is the indefinite integral for $\int\left(\cos^2 x+\sec^2 x\right)dx$?

(A) $\frac{1}{2}x+\frac{1}{4}\sin 2x+\frac{1}{2}\tan x+c$

(B) $\frac{1}{2}x-\frac{1}{4}\sin 2x+\frac{1}{2}\tan x+c$

(C) $\frac{1}{2}x+\frac{1}{4}\sin 2x+\tan x+c$

(D) $\frac{1}{2}x-\frac{1}{4}\sin 2x+\tan x+c$

14 What is the indefinite integral for $\int\left(\sin^2 x + x^2\right)dx$?

(A) $x - \frac{1}{2}\sin 2x + \frac{x^3}{3} + c$

(B) $\frac{1}{2}x - \frac{1}{4}\sin 2x + \frac{x^3}{3} + c$

(C) $x - \frac{1}{2}\sin 2x + 2x + c$

(D) $\frac{1}{2}x - \frac{1}{4}\sin 2x + 2x + c$

15 What is the indefinite integral for $\int\left(\cos 3x - \sin^2 x + 2\right)dx$?

(A) $\frac{1}{3}\sin 3x - \frac{3}{2}x - \frac{1}{4}\sin 2x + c$

(B) $\frac{1}{3}\sin 3x - \frac{3}{2}x + \frac{1}{4}\sin 2x + c$

(C) $\frac{1}{3}\sin 3x + \frac{3}{2}x + \frac{1}{4}\sin 2x + c$

(D) $\frac{1}{3}\sin 3x + \frac{3}{2}x - \frac{1}{4}\sin 2x + c$

16 What is the primitive function of $3\sin^2 \frac{2x}{5}$?

(A) $\frac{3x}{2} - \frac{6}{5}\sin\frac{4x}{5} + c$

(B) $\frac{3x}{2} - \frac{15}{8}\sin\frac{4x}{5} + c$

(C) $3x - \frac{15}{4}\sin\frac{4x}{5} + c$

(D) $3x - \frac{6}{5}\sin\frac{4x}{5} + c$

17 What is the exact value of the definite integral $\int_0^{\frac{\pi}{12}} \sin^2 x\,dx$?

(A) $\frac{\pi}{12} - \frac{1}{12}$ (B) $\frac{\pi}{12} - \frac{1}{6}$

(C) $\frac{\pi}{12} - \frac{1}{4}$ (D) $\frac{\pi}{12} - \frac{1}{2}$

18 What is the exact value of the definite integral $\int_{\frac{\pi}{2}}^{\pi} (\sin^2 x + x)dx$?

(A) $\frac{3\pi^2 + \pi + 2}{8}$

(B) $\frac{3\pi^2 + \pi}{8}$

(C) $\frac{3\pi^2 + 2\pi + 2}{8}$

(D) $\frac{3\pi^2 + 2\pi}{8}$

19 What is the exact value of the definite integral $\int_0^{\frac{\pi}{8}} \cos^2 x\,dx$?

(A) $\frac{\pi - 2\sqrt{2}}{16}$

(B) $\frac{\pi - 2\sqrt{2}}{8}$

(C) $\frac{\pi + 2\sqrt{2}}{16}$

(D) $\frac{\pi + 2\sqrt{2}}{8}$

20 What is the exact value of the definite integral $\int_0^{\pi} (\cos^2 x + 1)dx$?

(A) $\frac{3\pi}{2}$

(B) π

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

Objective Response Bank

Year 12 Mathematics Extension 1

Worked solutions

Methods of integration		Main Menu
	Solution	**Criteria**
1	$u=\cos x$ $\frac{du}{dx}=-\sin x$ $du=-\sin x dx$ $\int \cos^2 x \sin x dx=-\int u^2 du=-\frac{1}{3}u^3=-\frac{1}{3}\cos^3 x+c$	1 Mark: D
2	$u=2x+1$ $\frac{du}{dx}=2$ $du=2dx$ $\int (2x+1)^{10} dx=\frac{1}{2}\int u^{10} du$ $=\frac{1}{2}\times\frac{1}{11}u^{11}+c$ $=\frac{1}{22}(2x+1)^{11}+c$	1 Mark: D
3	$u=x^2-2$ $\frac{du}{dx}=2x$ $du=2xdx$ $\int \frac{x}{\sqrt{x^2-2}}dx=\frac{1}{2}\int \frac{1}{\sqrt{u}}du$ $=\frac{1}{2}\int u^{-\frac{1}{2}}du$ $=u^{\frac{1}{2}}+c=\sqrt{x^2-2}+c$	1 Mark: A
4	$u=\sqrt{1-x^2}$ $\frac{du}{dx}=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot -2x$ $du=\frac{-x}{(1-x^2)^{\frac{1}{2}}}dx$ $-(1-x^2)^{\frac{1}{2}}du=xdx$ or $-udu=xdx$ $\int x\sqrt{1-x^2}dx=\int u\times -udu$ $=\int -u^2 du$ $=-\frac{u^3}{3}+c$ $=-\frac{(1-x^2)^{\frac{3}{2}}}{3}+c$	1 Mark: C
5	$u=e^{-x}+1$ $\frac{du}{dx}=-e^{-x}$ $du=-e^{-x}dx$ Also $u=e^{-x}+1$ or $e^{-x}=u-1$ $\int \frac{e^{-2x}dx}{e^{-x}+1}=\int \frac{e^{-x}\times e^{-x}dx}{e^{-x}+1}$ $=\int \frac{-(u-1)du}{u}$ $=\int \left(\frac{1}{u}-1\right)du$ $=\log_e u-u+c$ $=\log_e(e^{-x}+1)-(e^{-x}+1)+c$ $=\log_e(e^{-x}+1)-e^{-x}+c$	1 Mark: C
6	$u=e^x$ $\frac{du}{dx}=e^x$ $du=e^x dx$ $\int \frac{e^x}{1+e^{2x}}dx=\int \frac{du}{1+u^2}$ $=\tan^{-1}u+c$ $=\tan^{-1}e^x+c$	1 Mark: C
7	$u=1+e^x$ $\frac{du}{dx}=e^x$ $du=e^x dx$ Also $u=1+e^x$ or $e^x=u-1$ $\int \frac{e^{3x}}{1+e^x}dx=\int \frac{e^{2x}\times e^x dx}{1+e^x}$ $=\int \frac{(u-1)^2\times du}{1+u-1}$ $=\int \frac{u^2-2u+1}{u}du$ $=\int u-2+\frac{1}{u}du$ $=\frac{u^2}{2}-2u+\log_e u+c=\frac{(1+e^x)^2}{2}-2(1+e^x)+\log_e(1+e^x)+c$	1 Mark: D

	Solution	Criteria
8	$u=\tan x$ and $du=\sec^2 x\,dx \qquad u=\tan\frac{\pi}{3}=\sqrt{3} \qquad u=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$ $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sec^2 x}{\tan x}dx=\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}\frac{1}{u}du$ $=\left[\log_e u\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$ $=\log_e\sqrt{3}-\log_e\frac{1}{\sqrt{3}}$ $=\log_e 3$	1 Mark: D
9	$u=\log_e x$ and $du=\frac{1}{x}dx \qquad u=\log_e e=1 \qquad u=\log_e e^2=2$ $\int_e^{e^2}\frac{1}{x\log x}dx=\int_1^2\frac{1}{u}du$ $=\left[\log_e u\right]_1^2$ $=\log_e 2-\log_e 1=\log_e 2$	1 Mark: B
10	$x=2\sin\theta,\quad \frac{dx}{d\theta}=2\cos\theta,\quad dx=2\cos\theta d\theta$ When $x=1 \quad 1=2\sin\theta$ When $x=-1 \quad -1=2\sin\theta$ $\sin\theta=\frac{1}{2}$ or $\theta=\frac{\pi}{6}$ $\qquad \sin\theta=-\frac{1}{2}$ or $\theta=-\frac{\pi}{6}$ $\int_{-1}^{1}\sqrt{4-x^2}\,dx=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\sqrt{4-(2\sin\theta)^2}\times 2\cos\theta d\theta$ $=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}2\sqrt{1-\sin^2\theta}\times 2\cos\theta d\theta$ $=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}2\cos\theta\times 2\cos\theta d\theta$ $=4\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2\theta d\theta$ $=2\left[x+\frac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$ $=2\left[\left(\frac{\pi}{6}+\frac{1}{2}\sin\frac{\pi}{3}\right)-\left(-\frac{\pi}{6}+\frac{1}{2}\sin-\frac{\pi}{3}\right)\right]$ $=2\left[\frac{\pi}{6}+\frac{1}{2}\times\frac{\sqrt{3}}{2}+\frac{\pi}{6}-\frac{1}{2}\times-\frac{\sqrt{3}}{2}\right]$ $=2\left[\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right]=\frac{2\pi}{3}+\sqrt{3}$	1 Mark: C

Primitive of $\sin^2 x$ and $\cos^2 x$ Main Menu

	Solution	Criteria
11	$\int\cos^2 2x\,dx=\int\frac{1}{2}(1+\cos 4x)dx$ $=\frac{1}{2}\left[x+\frac{1}{4}\sin 4x\right]+c$ $=\frac{x}{2}+\frac{1}{8}\sin 4x+c$	1 Mark: D
12	$\int\sin^2 6x\,dx=\int\frac{1}{2}(1-\cos 12x)dx$ $=\frac{1}{2}\left[x-\frac{1}{12}\sin 12x\right]+c$ $=\frac{x}{2}-\frac{1}{24}\sin 12x+c$	1 Mark: C
13	$\int\left(\cos^2 x+\sec^2 x\right)dx=\int\left(\frac{1}{2}(1+\cos 2x)+\sec^2 x\right)dx$ $=\frac{1}{2}x+\frac{1}{4}\sin 2x+\tan x+c$	1 Mark: C
14	$\int\left(\sin^2 x+x^2\right)dx=\int\frac{1}{2}(1-\cos 2x)+x^2dx$ $=\frac{1}{2}x-\frac{1}{4}\sin 2x+\frac{x^3}{3}+c$	1 Mark: B
15	$\int\left(\cos 3x-\sin^2 x+2\right)dx=\int\left(\cos 3x-\frac{1}{2}(1-\cos 2x)+2\right)dx$ $=\frac{1}{3}\sin 3x-\frac{1}{2}x+\frac{1}{4}\sin 2x+2x+c$ $=\frac{1}{3}\sin 3x+\frac{3}{2}x+\frac{1}{4}\sin 2x+c$	1 Mark: C
16	$\int 3\sin^2\frac{2x}{5}dx=3\int\frac{1}{2}(1-\cos\frac{4x}{5})dx$ $=\frac{3}{2}\left[x-\frac{5}{4}\sin\frac{4x}{5}\right]+c$ $=\frac{3x}{2}-\frac{15}{8}\sin\frac{4x}{5}+c$	1 Mark: B

17	$\int_0^{\frac{\pi}{12}} 2\sin^2 x dx = \int_0^{\frac{\pi}{12}} (1-\cos 2x)dx$ $= \left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{12}}$ $= \left[\left(\frac{\pi}{12} - \frac{1}{2}\sin\frac{\pi}{6}\right) - \left(0 - \frac{1}{2}\sin 0\right)\right]$ $= \left[\frac{\pi}{12} - \frac{1}{2}\times\frac{1}{2}\right] = \frac{\pi}{12} - \frac{1}{4}$	1 Mark: C
18	$\int_{\frac{\pi}{2}}^{\pi} (\sin^2 x + x)dx = \int_{\frac{\pi}{2}}^{\pi} [\frac{1}{2}(1-\cos 2x) + x]dx$ $= \left[\frac{1}{2}x - \frac{1}{4}\sin 2x + \frac{x^2}{2}\right]_{\frac{\pi}{2}}^{\pi}$ $= \left[\left(\frac{1}{2}\pi - \frac{1}{4}\sin 2\pi + \frac{\pi^2}{2}\right) - \left(\frac{1}{2}\times\frac{\pi}{2} - \frac{1}{4}\sin\pi + \frac{\pi^2}{8}\right)\right]$ $= \left[\frac{\pi}{2} + \frac{\pi^2}{2} - \frac{\pi}{4} - \frac{\pi^2}{8}\right]$ $= \frac{3\pi^2}{8} + \frac{\pi}{4} = \frac{3\pi^2 + 2\pi}{8}$	1 Mark: D
19	$\int_0^{\frac{\pi}{8}} \cos^2 x dx = \int_0^{\frac{\pi}{8}} \frac{1}{2}(1+\cos 2x)dx$ $= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{8}}$ $= \frac{1}{2}\left[\left(\frac{\pi}{8} + \frac{1}{2}\sin\frac{\pi}{4}\right) - \left(0 + \frac{1}{2}\sin 0\right)\right]$ $= \left[\frac{\pi}{16} + \frac{1}{4}\times\frac{1}{\sqrt{2}}\right]$ $= \frac{\pi}{16} + \frac{\sqrt{2}}{8} = \frac{\pi + 2\sqrt{2}}{16}$	1 Mark: C
20	$\int_0^{\pi} (\cos^2 x + 1)dx = \int_0^{\pi} [\frac{1}{2}(1+\cos 2x) + 1]dx$ $= \left[\frac{1}{2}x + \frac{1}{2}\sin 2x + x\right]_0^{\pi}$ $= \left[\left(\left[\frac{1}{2}\pi + \frac{1}{2}\sin 2\pi + \pi\right]\right) - \left[\frac{1}{2}\times 0 + \frac{1}{2}\sin 0 + 0\right]\right]$ $= \frac{\pi}{2} + \pi = \frac{3\pi}{2}$	1 Mark: A