Complex numbers Solutions Main Menu

11 Let $z = 1 + i$ and $w = 1 - 2i$. What is the value of zw?

(A) $-1 - i$
(B) $-1 + i$
(C) $3 - i$
(D) $3 + i$

12 Let $z = 3 - 4i$ and $w = \sqrt{3} + i$. What is the value of $\frac{z}{w}$?

(A) $\frac{3\sqrt{3}+4}{4} + \frac{(-4\sqrt{3}-3)i}{4}$
(B) $\frac{3\sqrt{3}-4}{4} + \frac{(-4\sqrt{3}-3)i}{4}$
(C) $\frac{3\sqrt{3}+4}{2} + \frac{(-4\sqrt{3}-3)i}{2}$
(D) $\frac{3\sqrt{3}-4}{2} + \frac{(-4\sqrt{3}-3)i}{2}$

13 Let $z = 1 + 2i$ and $w = -2 + i$. What is the value of $\frac{5}{iw}$?

(A) $-1 - 2i$
(B) $-1 + 2i$
(C) $1 - 2i$
(D) $1 + 2i$

14 Let $z = 3 - i$. What is the value of $\overline{iz}$?

(A) $-1 - 3i$
(B) $-1 + 3i$
(C) $1 - 3i$
(D) $1 + 3i$

15 Let $z = 2 + i$ and $w = 1 - i$. What is the value of $3\bar{z} + iw$?

(A) $5 - 4i$
(B) $5 + 4i$
(C) $7 - 4i$
(D) $7 + 4i$

Year 12 Mathematics Extension 2

16 What is the value of $\arg \bar{z}$ given the complex number $z = 1 - i\sqrt{3}$?

(A) $-\frac{\pi}{3}$
(B) $-\frac{2\pi}{3}$
(C) $-\frac{\pi}{3}$
(D) $\frac{\pi}{3}$

17 What is the value of $\frac{z_1}{z_2}$ given the complex numbers $z_1 = -2 + 2i$ and $z_2 = 1 + i\sqrt{3}$?

(A) $\frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$
(B) $\frac{1-\sqrt{3}}{2} - \frac{\sqrt{3}+1}{2}i$
(C) $\frac{\sqrt{3}-1}{4} + \frac{\sqrt{3}+1}{4}i$
(D) $\frac{1-\sqrt{3}}{4} - \frac{\sqrt{3}+1}{4}i$

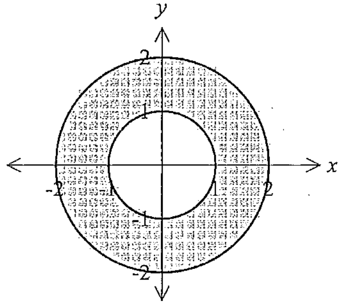
18 It is given that $3 + i$ is a root of $P(z) = z^3 + az^2 + bz + 10$ where a and b are real numbers. Which expression factorises $P(z)$ over the real numbers?

(A) $(z-1)(z^2 + 6z - 10)$
(B) $(z-1)(z^2 - 6z - 10)$
(C) $(z+1)(z^2 + 6z + 10)$
(D) $(z+1)(z^2 - 6z + 10)$

19 What is the solution to the equation $z^2 = i\bar{z}$?

(A) $(0,0)$ and $(0,1)$
(B) $(0,0)$ and $(0,-1)$
(C) $(0,0)$, $(0,-1)$, $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ and $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$
(D) $(0,0)$, $(0,1)$, $(\frac{\sqrt{3}}{2}, \frac{1}{2})$ and $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

20 Consider the Argand diagram below.



Which inequality could define the shaded area?

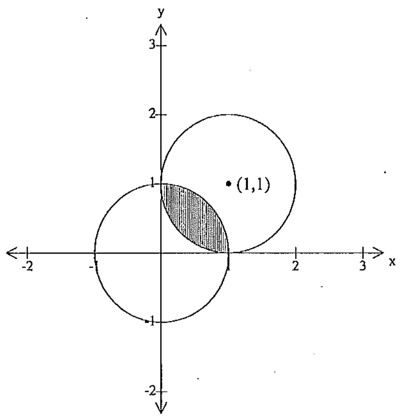
(A) $0 \le |z| \le 2$

(B) $1 \le |z| \le 2$

(C) $0 \le |z-1| \le 2$

(D) $1 \le |z-1| \le 2$

21 Consider the Argand diagram below.



Which inequality could define the shaded area?

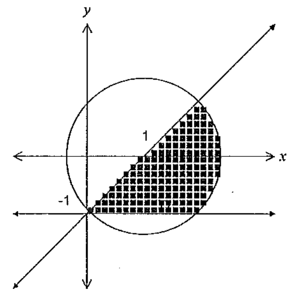
(A) $|z| \le 1$ and $|z-(1-i)| \ge 1$

(B) $|z| \le 1$ and $|z-(1+i)| \ge 1$

(C) $|z| \le 1$ and $|z-(1-i)| \le 1$

(D) $|z| \le 1$ and $|z-(1+i)| \le 1$

22 Consider the Argand diagram below.



Which inequality could define the shaded area?

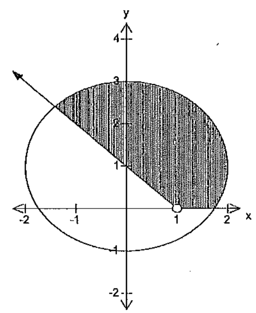
(A) $|z-1| \le \sqrt{2}$ and $0 \le \arg(z-i) \le \frac{\pi}{4}$

(B) $|z-1| \le \sqrt{2}$ and $0 \le \arg(z+i) \le \frac{\pi}{4}$

(C) $|z-1| \le 1$ and $0 \le \arg(z-i) \le \frac{\pi}{4}$

(D) $|z-1| \le 1$ and $0 \le \arg(z+i) \le \frac{\pi}{4}$

23 Consider the Argand diagram below.



Which inequality could define the shaded area?

(A) $|z-i| \le 2$ and $0 \le \arg(z-1) \le \frac{3\pi}{4}$

(B) $|z+i| \le 2$ and $0 \le \arg(z-1) \le \frac{3\pi}{4}$

(C) $|z-i| \le 2$ and $0 \le \arg(z-1) \le \frac{\pi}{4}$

(D) $|z+i| \le 2$ and $0 \le \arg(z-1) \le \frac{\pi}{4}$

24 What is $-1+i$ expressed in modulus-argument form?

(A) $(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$

(B) $\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$

(C) $(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})$

(D) $\sqrt{2}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})$

25 What is $-\sqrt{3}+i$ expressed in modulus-argument form?

(A) $\sqrt{2}(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$

(B) $2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$

(C) $\sqrt{2}(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6})$

(D) $2(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6})$

26 What is $-2+2\sqrt{3}i$ expressed in modulus-argument form?

(A) $2(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$

(B) $4(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$

(C) $2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$

(D) $4(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$

27 What is $(1+\sqrt{3}i)^{-1}$ expressed in modulus-argument form?

(A) $\frac{1}{4}(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3})$

(B) $\frac{1}{4}(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$

(C) $\frac{1}{2}(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3})$

(D) $\frac{1}{2}(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$

28 What are the three roots of $z^3-1=0$ in modulus argument form?

(A) $\text{cis}0,\ \text{cis}\frac{2\pi}{3}$

(B) $\text{cis}0,\ \text{cis}\frac{2\pi}{3},\ \text{cis}-\frac{2\pi}{3}$

(C) $\text{cis}0,\ \text{cis}\frac{\pi}{3}$

(D) $\text{cis}0,\ \text{cis}\frac{\pi}{3},\ \text{cis}-\frac{\pi}{3}$

29 Which of the following complex numbers equals $(\sqrt{3}+i)^4$?

(A) $-2+\frac{2}{\sqrt{3}}i$

(B) $-8+\frac{8}{\sqrt{3}}i$

(C) $-2+2\sqrt{3}i$

(D) $-8+8\sqrt{3}i$

30 Let the point R represent the complex number z on an Argand diagram. Which of the following describes the locus of R specified by $|z|=|z-4|$?

(A) Perpendicular bisector of (0,0) and (−4,0)

(B) Perpendicular bisector of (0,0) and (4,0)

(C) Circle with a centre (0,0) and radius of 2

(D) Circle with a centre (0,0) and radius of 4

Complex numbers		Main Menu
	Solution	**Criteria**
11	$zw=(1+i)(1-2i)$ $=1-i-2i^2$ $=3-i$	1 Mark: C
12	$\frac{z}{w}=\frac{(3-4i)}{(\sqrt{3}+i)}\times\frac{(\sqrt{3}-i)}{(\sqrt{3}-i)}$ $=\frac{3\sqrt{3}-4}{4}+\frac{(-4\sqrt{3}-3)i}{4}$	1 Mark: B
13	$\frac{5}{iw}=\frac{5}{-2i+i^2}$ $=\frac{5}{-1-2i}\times\frac{-1+2i}{-1+2i}$ $=\frac{-5+10i}{1+4}$ $=-1+2i$	1 Mark: B
14	$\overline{iz}=\overline{i(3-i)}$ $=\overline{3i+1}$ $=1-3i$	1 Mark: C
15	$3z+iw=3(2+i)+i(1-i)$ $=6+3i+i-i^2$ $=7+4i$	1 Mark: D
16	$\overline{z}=1+i\sqrt{3}$ $\arg\overline{z}=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$	1 Mark: D
17	$\frac{z_1}{z_2}=\frac{-2+2i}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$ $=\frac{-2+2i\sqrt{3}+2i+2\sqrt{3}}{1+3}$ $=\frac{2(-1+\sqrt{3})+i(1+\sqrt{3})}{4}$ $=\frac{\sqrt{3}-1}{2}+\frac{\sqrt{3}+1}{2}i$	1 Mark: A

18	Roots are $3+i$, $3-i$ and α $(3+i)(3-i)\alpha=-\frac{10}{1}$ $(9-i^2)\alpha=-10$ $10\alpha=-10$ $\alpha=-1$ $P(z)=(z--1)[z-(3+i)][z-(3-i)]$ $=(z+1)(z^2-6z+10)$	1 Mark: D
19	Let $z=x+iy$ and $\overline{z}=x-iy$ $z^2=i\overline{z}$ $(x+iy)^2=i(x-iy)$ $x^2-y^2+2xyi=y+ix$ Equating the real and imaginary parts $x^2-y^2=y$ (1) $2xy=x$ (2) Rearranging eqn (2) $x(2y-1)=0$ $x=0$ or $y=\frac{1}{2}$ Substitute $x=0$ into eqn (1) $-y^2=y$ $y(y+1)=0$ $y=0$ or $y=-1$ Substitute $y=\frac{1}{2}$ into eqn (1) $x^2-\frac{1}{4}=\frac{1}{2}$ $x^2=\frac{3}{4}$, $x=\pm\frac{\sqrt{3}}{2}$ Solution is $(0,0)$, $(0,-1)$, $(\frac{\sqrt{3}}{2},\frac{1}{2})$ and $(-\frac{\sqrt{3}}{2},\frac{1}{2})$	1 Mark: C
20	$\lvert z\rvert\leq 1$ represents a region with a centre is (0, 0) and radius is greater than or equal to 1. $\lvert z\rvert\leq 2$ represents a region with a centre is (0, 0) and radius is less than or equal to 1. $1\leq\lvert z\rvert\leq 2$	1 Mark: B

21	$\lvert z \rvert \le 1$ represents a region with a centre is (0, 0) and radius is greater than or equal to 1. $\lvert z-(1+i) \rvert \le 1$ represents a region with a centre is (1, 1) and radius is less than or equal to 1. $\lvert z \rvert \le 1$ and $\lvert z-(1+i) \rvert \le 1$	1 Mark: D
22	$\lvert z-1 \rvert \le \sqrt{2}$ represents a region with a centre is (1, 0) and radius is less than or equal to $\sqrt{2}$. $0 \le \arg(z+i) \le \frac{\pi}{4}$ represents a region between angle 0 and $\frac{\pi}{4}$ whose vertex is $(-1,0)$ not including the vertex $\lvert z-1 \rvert \le \sqrt{2}$ and $0 \le \arg(z+i) \le \frac{\pi}{4}$	1 Mark: B
23	$\lvert z-i \rvert \le 2$ represents a region with a centre is (0, 1) and radius is less than or equal to 2. $0 \le \arg(z-1) \le \frac{3\pi}{4}$ represents a region between angle 0 and $\frac{3\pi}{4}$ whose vertex is (1, 0), not including the vertex $\lvert z-i \rvert \le 2$ and $0 \le \arg(z-1) \le \frac{3\pi}{4}$	1 Mark: A
24	$\tan\theta = \frac{1}{-1}$ $r^2 = x^2 + y^2$ $\theta = \frac{3\pi}{4}$ $= 1^2 + 1^2$ $r = \sqrt{2}$ $-1+i = \sqrt{2}(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$	1 Mark: D
25	$\tan\theta = \frac{1}{-\sqrt{3}}$ $r^2 = x^2 + y^2$ $\theta = \frac{5\pi}{6}$ $= (\sqrt{3})^2 + 1^2$ $r = 2$ $-\sqrt{3}+i = 2(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$	1 Mark: D
26	$-2+2\sqrt{3}i = 2(-1+i\sqrt{3})$ $= 4(-\frac{1}{2} + \frac{i\sqrt{3}}{2})$ $= 4(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$	1 Mark: B

27	$(1+\sqrt{3}i)^{-1} = \frac{1}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$ $= \frac{1-\sqrt{3}i}{4}$ $= \frac{1}{4} - \frac{\sqrt{3}i}{4}$ $= \frac{1}{2}(\frac{1}{2} - \frac{\sqrt{3}i}{2})$ $= \frac{1}{2}(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3})$	1 Mark: C
28	$z^3 - 1 = 0$ $z^3 = 1$ $z^3 = r^3\text{cis}3\theta$ $r^3\text{cis}3\theta = 1\text{cis}0$ or $1\text{cis}\pm 2\pi$ $r = 1$ and $3\theta = 0, \pm 2\pi$ $\theta = 0, \frac{\pm 2\pi}{3}$ $\text{cis}0,\ \text{cis}\frac{2\pi}{3},\ \text{cis}-\frac{2\pi}{3}$	1 Mark: B
29	$\sqrt{3}+i = 2(\frac{\sqrt{3}}{2} + \frac{1}{2}i)$ $= 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ $(\sqrt{3}+i)^4 = 2^4(\cos 4\times\frac{\pi}{6} + i\sin 4\times\frac{\pi}{6})$ $= 16(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$ $= -8+8\sqrt{3}i$	1 Mark: D
30	The locus of $\lvert z \rvert = \lvert z-4 \rvert$ consists of the set of points that are an equal distance from the origin as they are from the point (4, 0). It is the perpendicular bisector of (0,0) and (4,0) on the Argand diagram.	1 Mark: B