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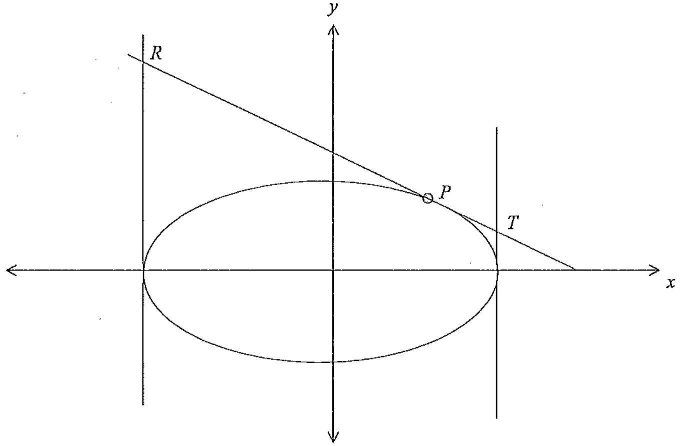
31 For the ellipse with the equation $\frac{x^2}{4}+\frac{y^2}{3}=1$. What is the eccentricity?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{3}{4}$ (D) $\frac{9}{16}$

32 The point P lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a>b>0$. The tangent at P meets the tangents at the ends of the major axis at R and T.



What is the equation of the tangent at P?

(A) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

(B) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

(C) $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$

(D) $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

33 The points $P(a\cos\theta,\ b\sin\theta)$ and $Q(a\cos\phi,\ b\sin\phi)$ lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the chord PQ subtends a right angle at $(0,0)$. Which of the following is the correct expression?

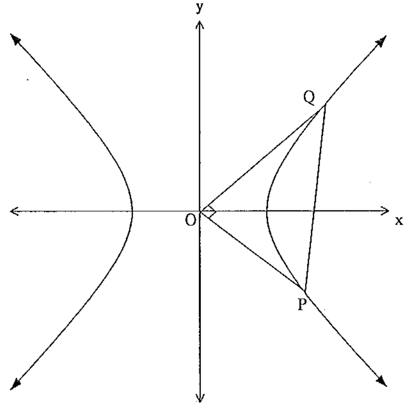
(A) $\tan\theta\tan\phi=-\frac{b^2}{a^2}$

(B) $\tan\theta\tan\phi=-\frac{a^2}{b^2}$

(C) $\tan\theta\tan\phi=\frac{b^2}{a^2}$

(D) $\tan\theta\tan\phi=\frac{a^2}{b^2}$

34 The diagram below shows the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ where $a>b>0$. The points $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\alpha, b\tan\alpha)$ lie on the hyperbola and the chord PQ subtends a right angle at the origin.



Use the parametric representation of the hyperbola to determine which of the following expressions is correct?

(A) $\sin\theta\sin\alpha=-\frac{a^2}{b^2}$

(B) $\sin\theta\sin\alpha=\frac{a^2}{b^2}$

(C) $\tan\theta\tan\alpha=-\frac{a^2}{b^2}$

(D) $\tan\theta\tan\alpha=\frac{a^2}{b^2}$

35 The points $P(cp, \frac{c}{p})$ and $Q(cq, \frac{c}{q})$ lie on the same branch of the hyperbola $xy = c^2$ ($p \neq q$). The tangents at P and Q meet at the point T. What is the equation of the normal to the hyperbola at P?

(A) $p^2x - py + c - cp^4 = 0$

(B) $p^3x - py + c - cp^4 = 0$

(C) $x + p^2y - 2c = 0$

(D) $x + p^2y - 2cp = 0$

36 Consider the hyperbola with the equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

What is the eccentricity of the hyperbola?

(A) $\frac{3}{4}$

(B) $\frac{5}{4}$

(C) $\frac{9}{16}$

(D) $\frac{25}{16}$

37 Consider the hyperbola with the equation $\frac{x^2}{144} - \frac{y^2}{25} = 1$.

What are the equations of the directrices?

(A) $x = \pm\frac{13}{144}$

(B) $x = \pm\frac{13}{25}$

(C) $x = \pm\frac{25}{13}$

(D) $x = \pm\frac{144}{13}$

38 Consider the hyperbola with the equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

What are the coordinates of the foci of the hyperbola?

(A) $(\pm 4, 0)$ (B) $(0, \pm 4)$

(C) $(0, \pm 5)$ (D) $(\pm 5, 0)$

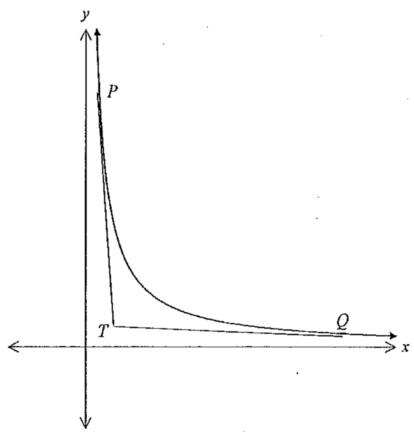
39 Consider the hyperbola with the equation $\frac{x^2}{4} - \frac{y^2}{3} = 1$.

What are the coordinates of the vertex of the hyperbola?

(A) $(\pm 2, 0)$ (B) $(0, \pm 2)$

(C) $(0, \pm 4)$ (D) $(\pm 4, 0)$

40 The points $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$, $p \neq q$, lie on the same branch of the hyperbola $xy = c^2$. The tangents at P and Q meet at the point T.



Which of the following expressions is the equation of the tangent to the hyperbola at Q?

(A) $x + q^2y = 2cq$

(B) $x + q^2y = 2c^2$

(C) $x + p^2y = 2cp$

(D) $x + p^2y = 2c^2$

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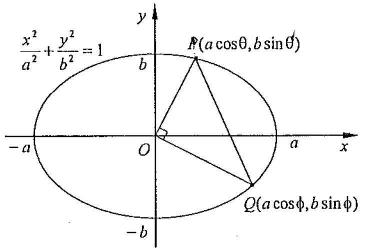
	Solution	**Criteria**
31	$b^2 = a^2(1-e^2)$ $3 = 4(1-e^2)$ $(1-e^2) = \frac{3}{4}$ or $e^2 = \frac{1}{4}$ or $e = \frac{1}{2}$	1 Mark: B
32	To find the equation of tangent through P $x = a\cos\theta \qquad y = b\sin\theta$ $\frac{dx}{d\theta} = -a\sin\theta \qquad \frac{dy}{d\theta} = b\cos\theta$ $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$ $= b\cos\theta \times \frac{1}{-a\sin\theta} = \frac{-b\cos\theta}{a\sin\theta}$ Equation of the tangent $y - y_1 = m(x - x_1)$ $y - b\sin\theta = \frac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$ $ay\sin\theta - ab\sin^2\theta = -bx\cos\theta + ab\cos^2\theta$ $bx\cos\theta + ay\sin\theta = ab(\sin^2\theta + \cos^2\theta)$ $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$	1 Mark: D
33	 POQ is a right-angled triangle. Therefore $OP^2 + OQ^2 = PQ^2$. $a^2\cos^2\theta + b^2\sin^2\theta + a^2\cos^2\varphi + b^2\sin^2\varphi$ $= a^2(\cos\theta - \cos\varphi)^2 + b^2(\sin\theta - \sin\varphi)^2$ $a^2(\cos^2\theta + \cos^2\varphi) + b^2(\sin^2\theta + \sin^2\varphi)$ $= a^2(\cos\theta - \cos\varphi)^2 + b^2(\sin\theta - \sin\varphi)^2$ Hence $\qquad 0 = -2a^2\cos\theta\cos\varphi - 2b^2\sin\theta\sin\varphi$ $2b^2\sin\theta\sin\varphi = -2a^2\cos\theta\cos\varphi$ $\frac{\sin\theta\sin\varphi}{\cos\theta\cos\varphi} = \frac{-2a^2}{2b^2}$ or $\tan\theta\tan\phi = -\frac{b^2}{a^2}$	1 Mark: B

34	Gradient of PO $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{b\tan\theta - 0}{a\sec\theta - 0}$ $= \frac{b\frac{\sin\theta}{\cos\theta}}{a\frac{1}{\cos\theta}} = \frac{b\sin\theta}{a}$ Gradient of QO $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{b\tan\alpha - 0}{a\sec\alpha - 0}$ $= \frac{b\frac{\sin\alpha}{\cos\alpha}}{a\frac{1}{\cos\alpha}} = \frac{b\sin\alpha}{a}$ PO and QO are at right angles $m_1m_2 = -1$ $\frac{b\sin\theta}{a} \times \frac{b\sin\alpha}{a} = -1$ or $\sin\theta\sin\alpha = -\frac{a^2}{b^2}$	1 Mark: A
35	To find the gradient of the tangent. $xy = c^2$ $x\frac{dy}{dx} + y = 0$ $\frac{dy}{dx} = -\frac{y}{x}$ At P $(cp, \frac{c}{p})$ $\frac{dy}{dx} = -\frac{\frac{c}{p}}{cp} = -\frac{1}{p^2}$ Gradient of the normal is p^2 $(m_1m_2 = -1)$ Equation of the normal at P $(cp, \frac{c}{p})$ $y - \frac{c}{p} = p^2(x - cp)$ $py - c = p^3x - cp^4$ $p^3x - py + c - cp^4 = 0$	1 Mark: B
36	$b^2 = a^2(e^2 - 1)$ $9 = 16(e^2 - 1)$ $(e^2 - 1) = \frac{9}{16}$ or $e^2 = \frac{25}{16}$ or $e = \frac{5}{4}$ $a^2 = 16$ and $b^2 = 9$. $a = 4 \qquad b = 3$	1 Mark: B
37	$b^2 = a^2(e^2 - 1)$ $25 = 144(e^2 - 1)$ $(e^2 - 1) = \frac{25}{144}$ or $e^2 = \frac{169}{144}$ or $e = \frac{13}{12}$ $a^2 = 144$ and $b^2 = 25$. $a = 12 \qquad b = 5$ Equation of the directrices are $x = \pm\frac{a}{e} = \pm\frac{144}{13}$.	1 Mark: D

38	$b^2 = a^2(e^2-1)$ $a^2 = 16$ and $b^2 = 9$. $9 = 16(e^2-1)$ $a = 4$ $b = 3$ $(e^2-1) = \frac{9}{16}$ or $e^2 = \frac{25}{16}$ or $e = \frac{5}{4}$ Foci of a hyperbola are $(\pm ae, 0)$ or $(\pm 5, 0)$.	1 Mark: D
39	$a^2 = 4$ $a = 2$ Vertex of a hyperbola are $(\pm a, 0)$ or $(\pm 2, 0)$.	1 Mark: A
40	To find the gradient of the tangent. $xy = c^2$ $x\frac{dy}{dx} + y = 0$ $\frac{dy}{dx} = -\frac{y}{x}$ At Q$(cq, \frac{c}{q})$ $\frac{dy}{dx} = -\frac{\frac{c}{q}}{cq} = -\frac{1}{q^2}$ Equation of the tangent at Q$(cq, \frac{c}{q})$ $y - \frac{c}{q} = \frac{-1}{q^2}(x - cq)$ $q^2y - cq = -x + cq$ $x + q^2y = 2cq$	1 Mark: A