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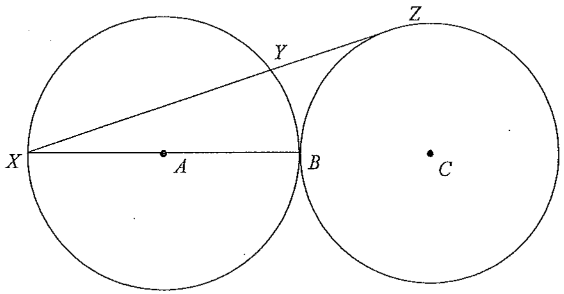
93 Two equal circles touch externally at B. XB is a diameter of one circle. XZ is the tangent from X to the other circle and cuts the first circle at Y.



Which is the correct expression that relates XZ to XY?

(A) $3XZ = 4XY$

(B) $XZ = 2XY$

(C) $2XZ = 3XY$

(D) $2XZ = 5XY$

94 What is the derivative of $\sin^{-1} x - \sqrt{1-x^2}$?

(A) $\dfrac{\sqrt{1+x}}{\sqrt{1-x}}$

(B) $\dfrac{\sqrt{1+x}}{1-x}$

(C) $\dfrac{1+x}{\sqrt{1-x}}$

(D) $\dfrac{1+x}{1-x}$

95 Using the binomial theorem $(1+x)^n = {}^nC_0 + {}^nC_1x^1 + {}^nC_2x^2 + \ldots + {}^nC_nx^n = \sum_{k=0}^{n} {}^nC_kx^k$ which of the following expressions is correct?

(A) $(1+\frac{1}{n})^n = \sum_{k=0}^{n} \frac{n(n-1)(n-2)\ldots(n-k)}{n^k} \times \frac{1}{k!}$

(B) $(1+\frac{1}{n})^n = \sum_{k=0}^{n} \frac{n(n-1)(n-2)\ldots(n-k)}{n^k} \times \frac{1}{(k+1)!}$

(C) $(1+\frac{1}{n})^n = \sum_{k=0}^{n} \frac{n(n-1)(n-2)\ldots(n-k+1)}{n^k} \times \frac{1}{k!}$

(D) $(1+\frac{1}{n})^n = \sum_{k=0}^{n} \frac{n(n-1)(n-2)\ldots(n-k+1)}{n^k} \times \frac{1}{(k+1)!}$

96 The labor party conducted a survey for the 2010 election. The ratio of the votes in three seats X, Y and Z was 4:3:2 respectively. The percentage of votes for Ms Gillard in these seats was 60%, 30% and 90% respectively. Ten voters were chosen at random, what is the probability that Ms Gillard gained at least eight votes?

(A) 0.1672897536

(B) 0.2509346304

(C) 0.3345795072

(D) 0.418224384

97 A coin is tossed 20 times. What is the probability of obtaining at most 3 heads?

(A) 0.0000029

(B) 0.0002

(C) 0.0013

(D) 0.0059

98 What is the solution to the equation $\tan^{-1}(4x) - \tan^{-1}(3x) = \tan^{-1}\left(\frac{1}{7}\right)$?

(A) $x = \frac{1}{7}$ or $x = \frac{2}{7}$

(B) $x = \frac{1}{3}$ or $x = \frac{2}{3}$

(C) $x = \frac{1}{3}$ or $x = \frac{1}{4}$

(D) $x = 3$ or $x = 4$

99 What is the solution to the inequation $2\sin 3x \geq 1$ if $0 \leq x \leq 2\pi$?
Hint: Use a sketch.

(A) $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}, \frac{13\pi}{6} \leq x \leq \frac{17\pi}{6}, \frac{25\pi}{6} \leq x \leq \frac{29\pi}{6}$

(B) $\frac{\pi}{6} \leq x \leq \frac{7\pi}{6}, \frac{13\pi}{6} \leq x \leq \frac{20\pi}{6}, \frac{25\pi}{6} \leq x \leq \frac{31\pi}{6}$

(C) $\frac{\pi}{18} \leq x \leq \frac{5\pi}{18}, \frac{13\pi}{18} \leq x \leq \frac{17\pi}{18}, \frac{25\pi}{18} \leq x \leq \frac{29\pi}{18}$

(D) $\frac{\pi}{18} \leq x \leq \frac{7\pi}{18}, \frac{13\pi}{18} \leq x \leq \frac{20\pi}{18}, \frac{25\pi}{18} \leq x \leq \frac{31\pi}{18}$

100 What is the solution to the inequation $\frac{x(5-x)}{x-4} \geq -3$?

(A) $2 \leq x < 4$ or $x \geq 6$

(B) $1 \leq x < 4$ or $x \geq 5$

(C) $4 < x \leq 6$ or $x \leq 2$

(D) $4 > x \leq 5$ or $x \leq 1$

101 A rock is projected to just clear two poles of height h metres at distances of b and c metres from the point of projection. If v is the velocity of the projection at an angle θ to the horizontal. Which of the following is the correct expression for square of the velocity?

(A) $v^2 = \frac{2(b-c)\tan\theta}{g\sec^2\theta}$

(B) $v^2 = \frac{2(b+c)\tan\theta}{g\sec^2\theta}$

(C) $v^2 = \frac{(b-c)g\sec^2\theta}{2\tan\theta}$

(D) $v^2 = \frac{(b+c)g\sec^2\theta}{2\tan\theta}$

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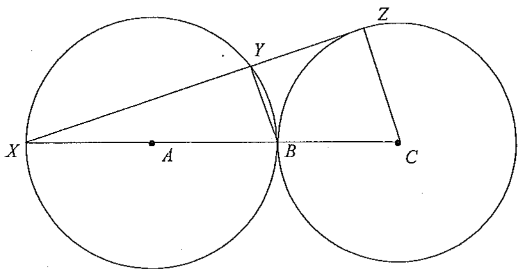
	Solution	Criteria

93



Construction: Join BY, produce XB to C, join CZ.

Proof:

$\angle XYB = 90^0$ (angle in a semicircle is a right angle)

$\angle XZC = 90^0$ (angle in a semicircle is a right angle)

$\angle BYX = \angle CZX$ (corresponding angles are equal)

$\Delta XYB \parallel\mid \Delta XZC$ (equiangular)

$\frac{XY}{XZ} = \frac{XB}{XC}$ (corresponding sides of similar triangles)

However $\frac{XB}{XC} = \frac{2}{3}$ $\quad (BC = \frac{1}{2}XB)$

$\frac{XY}{XZ} = \frac{2}{3}$

$\therefore 2XZ = 3XY$

1 Mark: C

94

$$y = \sin^{-1} x - \sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \times -2x$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}$$

$$= \frac{1+x}{\sqrt{1-x^2}}$$

$$= \frac{1+x}{\sqrt{(1+x)(1-x)}}$$

$$= \frac{\sqrt{1+x}}{\sqrt{1-x}}$$

Result defined for $-1 \le x \le 1$

1 Mark: A

95

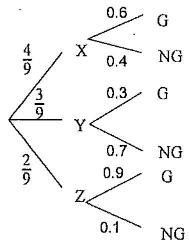
$$(1+\frac{1}{n})^n = \sum_{k=0}^{n} {}^nC_k (\frac{1}{n})^k$$

$$= \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} (\frac{1}{n})^k$$

$$= \sum_{k=0}^{n} \frac{n(n-1)(n-2)...(n-k+1)}{n^k} \frac{1}{k!}$$

1 Mark: C

96



The probability that Ms Gillard will gain this vote.

$$P = \frac{4}{9} \times 0.6 + \frac{3}{9} \times 0.3 + \frac{2}{9} \times 0.9 = 0.6$$

The probability that Ms Gillard will receive at least eight votes.

P(at least 8 votes)

$$= {}^{10}C_8 (0.6)^8 (0.4)^2 + {}^{10}C_9 (0.6)^9 (0.4)^1 + {}^{10}C_{10} (0.6)^{10} (0.4)^0$$

$$= 0.1672897536$$

1 Mark: A

97

$$\text{P(most 3 heads)} = \left(\frac{1}{2}\right)^{20} + {}^{20}C_1 \left(\frac{1}{2}\right)^{19} \left(\frac{1}{2}\right)^1 + {}^{20}C_2 \left(\frac{1}{2}\right)^{18} \left(\frac{1}{2}\right)^2 + {}^{20}C_3 \left(\frac{1}{2}\right)^{17} \left(\frac{1}{2}\right)^3$$

$$= \left(\frac{1}{2}\right)^{20} (1 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3)$$

$$= 0.0012884... \approx 0.0013$$

1 Mark: C

98

Let $\tan A = 4x$ and $\tan B = 3x$

$$A - B = \tan^{-1}\left(\frac{1}{7}\right) \text{ or } \tan(A-B) = \frac{1}{7}$$

$$\frac{\tan A - \tan B}{1 + \tan a \tan B} = \frac{1}{7}$$

$$\frac{4x - 3x}{1 + 4x \times 3x} = \frac{1}{7}$$

$$7x = 1 + 12x^2$$

$$12x^2 - 7x + 1 = 0 \text{ or } x = \frac{1}{3} \text{ or } \frac{1}{4}$$

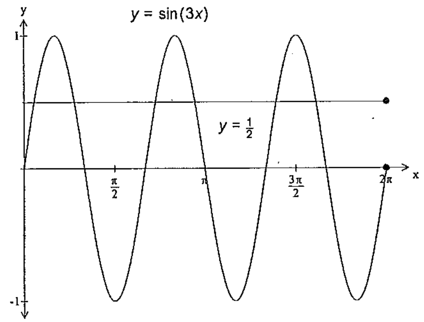
1 Mark: C

99 — 1 Mark: C

$2\sin 3x \geq 1$ for $0 \leq x \leq 2\pi$.

$\sin 3x \geq \frac{1}{2}$

Graph $y = \sin 3x$ and $y = 1/2$



$3x = \frac{\pi}{6} + 2k\pi$ or $3x = \frac{5\pi}{6} + 2k\pi$

$x = \frac{\pi}{18} + \frac{2k\pi}{3}$ or $x = \frac{5\pi}{18} + \frac{2k\pi}{3}$

$x = \frac{\pi}{18}, \frac{13\pi}{18}, \frac{25\pi}{18}$ or $x = \frac{5\pi}{18}, \frac{17\pi}{18}, \frac{29\pi}{18}$

Therefore

$\frac{\pi}{18} \leq x \leq \frac{5\pi}{18}, \frac{13\pi}{18} \leq x \leq \frac{17\pi}{18}, \frac{25\pi}{18} \leq x \leq \frac{29\pi}{18}$

100 — 1 Mark: C

Multiply both sides by $(x-4)^2$ with $x \neq 4$

$$\frac{x(5-x)}{x-4} \geq -3$$

$$\frac{(x-4)^2 x(5-x)}{x-4} \geq -3(x-4)^2$$

$$(x-4)x(5-x) \geq -3(x-4)^2$$

$$(x-4)x(5-x) + 3(x-4)^2 \geq 0$$

$$(x-4)[x(5-x) + 3(x-4)] \geq 0$$

$$(x-4)[+5x - x^2 + 3x - 12] \geq 0$$

$$(x-4)(-x^2 + 8x - 12) \geq 0$$

$$(4-x)(x^2 - 8x + 12) \geq 0$$

$$(4-x)(x-6)(x-2) \geq 0$$

Critical points 2, 4 and 6.

Solution: $4 < x \leq 6$ or $x \leq 2$

101 — 1 Mark: D

Cartesian equation of path

$$y = x\tan\theta - \frac{gx^2}{2v^2}.\sec^2\theta$$

Passes through (b,c) and (c,h)

$$h = b\tan\theta - \frac{gb^2}{2v^2}\sec^2\theta \quad (1)$$

$$h = c\tan\theta - \frac{gc^2}{2v^2}\sec^2\theta \quad (2)$$

$$\therefore b\tan\theta - \frac{gb^2}{2v^2}\sec^2\theta = c\tan\theta - \frac{gc^2}{2v^2}\sec^2\theta$$

$$(b-c)\tan\theta = (b^2 - c^2)\frac{g}{2v^2}\sec^2\theta$$

$$\tan\theta = (b+c)\frac{g}{2v^2}\sec^2\theta$$

$$v^2 = \frac{(b+c)g\sec^2\theta}{2\tan\theta}$$