Integration Solutions Main Menu

41 What is the value of $\int_0^1 xe^{-x^2}\,dx$?

(A) $\dfrac{1-e}{2e}$ (B) $\dfrac{e-1}{2e}$

(C) $\dfrac{2e-1}{e}$ (D) $\dfrac{1-2e}{e}$

42 What is the value of $\int_0^1 \dfrac{e^x}{1+e^x}\,dx$?

(A) $\log_e(1+e)$ (B) 1

(C) $\log_e\dfrac{(1+e)}{2}$ (D) $\log_e\dfrac{e}{2}-2$

43 Which of the following is an expression for $\int \dfrac{x}{\sqrt{16-x^2}}\,dx$?

(A) $-2\sqrt{16-x^2}+c$ (B) $-\sqrt{16-x^2}+c$

(C) $\dfrac{1}{2}\sqrt{16-x^2}+c$ (D) $-\dfrac{1}{2}\sqrt{16-x^2}+c$

44 Which of the following is an expression for $\int \dfrac{\sin x\cos x}{4+\sin x}\,dx$?

Use the substitution $u = 4+\sin x$.

(A) $-4\ln|4+\sin x|+c$ (B) $4\ln|4+\sin x|+c$

(C) $-\sin x-4\ln|4+\sin x|+c$ (D) $\sin x-4\ln|4+\sin x|+c$

45 Which of the following is an expression for $\int \cos^2 x\sin^7 x\,dx$?

Use the substitution $u=\cos x$.

(A) $-\dfrac{\cos^3 x}{3}+\dfrac{3\cos^5 x}{5}-\dfrac{3\cos^7 x}{7}+\dfrac{\cos^9 x}{9}+c$

(B) $-\cos^3 x+3\cos^5 x-3\cos^7 x+\cos^9 x+c$

(C) $\dfrac{\cos^3 x}{3}-\dfrac{3\cos^5 x}{5}+\dfrac{3\cos^7 x}{7}-\dfrac{\cos^9 x}{9}+c$

(D) $\cos^3 x-3\cos^5 x+3\cos^7 x-\cos^9 x+c$

46 Which of the following is an expression for $\int \dfrac{1}{\sqrt{x^2-6x+10}}\,dx$?

(A) $\ln\left(x-3-\sqrt{x^2-6x+10}\right)+c$

(B) $\ln\left(x+3-\sqrt{x^2-6x+10}\right)+c$

(C) $\ln\left(x-3+\sqrt{x^2-6x+10}\right)+c$

(D) $\ln\left(x+3+\sqrt{x^2-6x+10}\right)+c$

47 Which of the following is an expression for $\int \dfrac{1}{\sqrt{7-6x-x^2}}\,dx$?

(A) $\sin^{-1}\left(\dfrac{x-3}{2}\right)+c$ (B) $\sin^{-1}\left(\dfrac{x+3}{2}\right)+c$

(C) $\sin^{-1}\left(\dfrac{x-3}{4}\right)+c$ (D) $\sin^{-1}\left(\dfrac{x+3}{4}\right)+c$

48 Which of the following is an expression for $\int \dfrac{2}{x^2+4x+13}\,dx$?

(A) $\dfrac{1}{3}\tan^{-1}\dfrac{(x+2)}{3}+c$ (B) $\dfrac{2}{3}\tan^{-1}\dfrac{(x+2)}{3}+c$

(C) $\dfrac{1}{9}\tan^{-1}\dfrac{(x+2)}{9}+c$ (D) $\dfrac{2}{9}\tan^{-1}\dfrac{(x+2)}{9}+c$

49 Which of the following is an expression for $\int \dfrac{7x+4}{(x^2+1)(x+2)}\,dx$?

(A) $\ln(x^2+1)-\ln|x+2|+c$

(B) $\ln(x^2+1)-2\ln|x+2|+c$

(C) $\ln(x^2+1)+3\tan^{-1}x-\ln|x+2|+c$

(D) $\ln(x^2+1)+3\tan^{-1}x-2\ln|x+2|+c$

50 Which of the following is an expression for $\int \frac{x}{(x-1)(x+4)}dx$?

(A) $\frac{1}{5}\ln|x-1|+\frac{4}{5}\ln|x+4|+c$

(B) $-\frac{1}{5}\ln|x-1|+\frac{4}{5}\ln|x+4|+c$

(C) $\frac{1}{4}\ln|x-1|+\frac{5}{4}\ln|x+4|+c$

(D) $-\frac{1}{4}\ln|x-1|+\frac{5}{4}\ln|x+4|+c$

51 Which of the following is an expression for $\int \frac{4x^2-5x-1}{(x-3)(x^2+1)}dx$?

(A) $\ln(x-3)(x^2+1)+c$

(B) $\ln(x-3)^2(x^2+1)+c$

(C) $\ln(x-3)(x^2+1)+\tan^{-1}x+c$

(D) $\ln(x-3)^2(x^2+1)+\tan^{-1}x+c$

52 What is the value of $\int_0^{\frac{\pi}{2}} \frac{1}{\cos\theta+2\sin\theta+3}d\theta$? Use the substitution $t=\tan\frac{\theta}{2}$.

(A) 0.322 (B) 0.785

(C) 1.107 (D) 1.570

53 What is the value of $\int -\sec x\,dx$? Use the substitution $t=\tan\frac{x}{2}$.

(A) $\ln|(t-1)(t+1)|+c$ (B) $\ln\left|\frac{1-t}{t+1}\right|+c$

(C) $\ln|(1-t)(t+1)|+c$ (D) $\ln\left|\frac{t-1}{t+1}\right|+c$

54 What is the value of $\int_1^3 x(x-2)^5dx$? Use the substitution $u=x-2$.

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$

(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

55 Which of the following is an expression for $\int x\log_e x\,dx$? Use integration by parts.

(A) $\frac{x^2}{2}\log_e x-\frac{x^2}{4}+c$

(B) $\frac{x^2}{2}\log_e x-\frac{x}{2}+c$

(C) $x\log_e x-\frac{x^2}{4}+c$

(D) $x\log_e x-\frac{x}{2}+c$

56 Which of the following is an expression for $\int x^4\log_e x\,dx$? Use integration by parts.

(A) $\frac{x^4\log_e x}{4}-\frac{x^5}{20}+c$

(B) $\frac{x^4\log_e x}{4}-\frac{x^5}{5}+c$

(C) $\frac{x^5\log_e x}{5}-\frac{x^5}{20}+c$

(D) $\frac{x^5\log_e x}{5}-\frac{x^5}{5}+c$

57 What is the value of $\int_0^{\frac{\pi}{6}} x\cos x\,dx$? Use integration by parts.

(A) $\frac{\sqrt{3}\pi}{12}-\frac{1}{2}$ (B) $\frac{\sqrt{3}\pi}{12}+\frac{1}{2}$

(C) $\frac{\pi}{12}-\frac{\sqrt{3}}{2}+1$ (D) $\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$

58 Let $I_n=\int_0^x \cos^n t\,dt$, where $0\le x\le\frac{\pi}{2}$.

Which of the following is the correct expression for I_n?

(A) $I_n=\left(\frac{n-1}{n}\right)I_{n-2}$ with $n\ge 2$.

(B) $I_n=\left(\frac{n+1}{n}\right)I_{n-2}$ with $n\ge 2$.

(C) $I_n=n(n-1)I_{n-2}$ with $n\ge 2$.

(D) $I_n=n(n+1)I_{n-2}$ with $n\ge 2$.

59 Let $I_n = \int_0^{\pi} x^n \sin x\,dx$, where $0 \le x \le \frac{\pi}{2}$.

Which of the following is the correct expression for I_n?

(A) $\pi^n - n(n-1)I_{n-2}$

(B) $\pi^n + n(n-1)I_{n-2}$

(C) $\pi^n - n(n-2)I_{n-2}$

(D) $\pi^n + n(n-2)I_{n-2}$

60 Let $I_n = \int x^n e^{ax}\,dx$. Which of the following is the correct expression for I_n?

(A) $I_n = \dfrac{x^n e^{ax}}{a} - nI_{n-1}$

(B) $I_n = \dfrac{x^n e^{ax}}{a} - \dfrac{n}{a}I_{n-1}$

(C) $I_n = \dfrac{x^n e^{ax}}{a} + nI_{n-1}$

(D) $I_n = \dfrac{x^n e^{ax}}{a} + \dfrac{n}{a}I_{n-1}$

Integration		Main Menu
	Solution	**Criteria**
41	$\int_0^1 xe^{-x^2}dx = -\frac{1}{2}\int_0^1 -2xe^{-x^2}dx$ $= -\frac{1}{2}\left[e^{-x^2}\right]_0^1$ $= -\frac{1}{2}(e^{-1} - e^0)$ $= \frac{1}{2}(1 - \frac{1}{e})$ $= \frac{e-1}{2e}$	1 Mark: B
42	$\int_0^1 \frac{e^x}{1+e^x}dx = \left[\log_e\left(1+e^x\right)\right]_0^1$ $= \log_e(1+e) - \log_e 2$ $= \log_e \frac{(1+e)}{2}$	1 Mark: C
43	Let $u = 16 - x^2$ then $\frac{du}{dx} = -2x$ $xdx = -\frac{1}{2}du$ $\int \frac{x}{\sqrt{16-x^2}}dx = -\frac{1}{2}\int u^{-\frac{1}{2}}du$ $= -\frac{1}{2} \times 2u^{\frac{1}{2}}$ $= -\sqrt{16-x^2}$	1 Mark: B
44	Let $u = 4 + \sin x$ then $\frac{du}{dx} = \cos x$ Now $\sin x = u - 4$ $\int \frac{\sin x \cos x}{4 + \sin x}dx = \int \frac{(u-4)\cos x}{u} \frac{du}{\cos x}$ $= \int 1 - \frac{4}{u}\, du$ $= u - 4\ln\lvert u\rvert + c$ $= 4 + \sin x - 4\ln\lvert 4 + \sin x\rvert + c$ $= \sin x - 4\ln\lvert 4 + \sin x\rvert + c$	1 Mark: D

45	Let $u = \cos x$ then $-du = \sin x dx$ Now $\sin^6 x = (\sin^2 x)^3$ $= (1 - \cos^2 x)^3 = 1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x$ $\int \cos^2 x \sin^7 x dx = \int \cos^2 x \sin^6 x \sin x dx$ $= \int u^2(1 - 3u^2 + 3u^4 - u^6) - du$ $= \int -u^2 + 3u^4 - 3u^6 + u^8 du$ $= -\frac{u^3}{3} + \frac{3u^5}{5} - \frac{3u^7}{7} + \frac{u^9}{9} + c$ $= -\frac{\cos^3 x}{3} + \frac{3\cos^5 x}{5} - \frac{3\cos^7 x}{7} + \frac{\cos^9 x}{9} + c$	1 Mark: A
46	$\int \frac{dx}{\sqrt{x^2 - 6x + 10}} = \int \frac{dx}{\sqrt{x^2 - 6x + 9 + 1}} = \frac{dx}{\sqrt{(x-3)^2 + 1}}$ $= \ln\left(x - 3 + \sqrt{(x-3)^2 + 1}\right) + c$ $= \ln\left(x - 3 + \sqrt{x^2 - 6x + 10}\right) + c$	1 Mark: C
47	$\int \frac{1}{\sqrt{7 - 6x - x^2}}dx = \int \frac{1}{\sqrt{16 - 9 - 6x - x^2}}dx$ $== \int \frac{1}{\sqrt{16 - (x+3)^2}}dx$ $= \sin^{-1}\left(\frac{x+3}{4}\right) + c$	1 Mark: D
48	$\int \frac{2}{x^2 + 4x + 13}dx = 2\int \frac{dx}{(x+2)^2 + 3^2}$ $= \frac{2}{3}\tan^{-1}\frac{(x+2)}{3} + c$	1 Mark: B

49	$\frac{7x+4}{(x^2+1)(x+2)}=\frac{ax+b}{x^2+1}+\frac{c}{x+2}$ $7x+4=(ax+b)(x+2)+c(x^2+1)$ Let $x=-2$ and $x=0$ $-10=5c$ $\quad$ $4=b(0+2)-2(0^2+1)$ $c=-2$ $\quad$ $b=3$ Equating the coefficients of x^2 $\quad 0=a-2$ $a=2$ $\therefore a=2,\ b=3$ and $c=-2$ $\int\frac{7x+4}{(x^2+1)(x+2)}dx=\int(\frac{2x+3}{(x^2+1)}-\frac{2}{(x+2)})dx$ $=\int(\frac{2x}{(x^2+1)}+\frac{3}{(x^2+1)}-\frac{2}{(x+2)})dx$ $=\ln(x^2+1)+3\tan^{-1}x-2\ln\|x+2\|+c$	1 Mark: D
50	$\frac{x}{(x-1)(x+4)}=\frac{a}{x-1}+\frac{b}{x+4}$ $x=a(x+4)+b(x-1)$ Let $x=1$ and $x=-4$ $1=5a$ $\quad$ $-4=-5b$ $a=\frac{1}{5}$ $\quad$ $b=\frac{4}{5}$ $\int\frac{x}{(x-1)(x+4)}dx=\frac{1}{5}\int\frac{dx}{x-1}+\frac{4}{5}\int\frac{dx}{x+4}$ $=\frac{1}{5}\ln\|x-1\|+\frac{4}{5}\ln\|x+4\|+c$	1 Mark: A
51	$\frac{4x^2-5x-1}{(x-3)(x^2+1)}=\frac{a}{x-3}+\frac{bx+1}{x^2+1}$ $4x^2-5x-1=a(x^2+1)+(bx+1)(x-3)$ Let $x=0$ and $x=1$ $-1=a-3$ or $a=2$ $\quad$ $-2=2\times2+(b+1)\times-2$ or $b=2$ $\frac{4x^2-5x-1}{(x-3)(x^2+1)}dx=\int\left(\frac{2}{x-3}+\frac{2x+1}{x^2+1}\right)dx$ $=\int\left(\frac{2}{x-3}+\frac{2x}{x^2+1}+\frac{1}{x^2+1}\right)dx$ $=2\ \ln\|x-3\|+\ln\|x^2+1\|+\tan^{-1}x+c$ $=\ln(x-3)^2(x^2+1)+\tan^{-1}x+c$	1 Mark: D

52	$t=\tan\frac{\theta}{2}$ $dt=\frac{1}{2}\sec^2\frac{\theta}{2}d\theta$ $dt=\frac{1}{2}(1+t^2)d\theta$ or $d\theta=\frac{2}{1+t^2}dt$ When $\theta=0$ then $t=0$ and when $\theta=\frac{\pi}{2}$ then $t=1$ $\cos\theta+2\sin\theta+3=\frac{1-t^2+2(2t)+3(1+t^2)}{1+t^2}$ $=\frac{2(t^2+2t+2)}{1+t^2}$ $=\frac{2[1+(t+1)^2]}{1+t^2}$ $\int_0^{\frac{\pi}{2}}\frac{1}{\cos\theta+2\sin\theta+3}d\theta=\int_0^1\frac{1+t^2}{2[1+(t+1)^2]}\times\frac{2}{1+t^2}dt$ $=\int_0^1\frac{1}{1+(t+1)^2}dt$ $=\left[\tan^{-1}(t+1)\right]_0^1$ $=\tan^{-1}2-\frac{\pi}{4}$ $=0.322$	1 Mark: A
53	$t=\tan\frac{x}{2}$ $\quad$ $\cos x=\frac{1-t^2}{1+t^2}$ $\frac{dt}{dx}=\frac{1}{2}\sec^2\frac{x}{2}$ $\quad$ $\sec x=\frac{1+t^2}{1-t^2}$ $=\frac{1}{2}(1+t^2)$ $dx=\frac{2dt}{1+t^2}$ $\int-\sec x dx=-\int\frac{1+t^2}{1-t^2}\times\frac{2dt}{1+t^2}$ $=\int\frac{2dt}{t^2-1}$ $=\int\frac{a}{t-1}+\frac{b}{t+1}dt$ $=\int\frac{1}{t-1}-\frac{1}{t+1}dt$ $=\ln\|t-1\|-\ln\|t+1\|+c$ $=\ln\|\frac{t-1}{t+1}\|+c$	1 Mark: D

54	$u = x-2$ and $du = dx$ When $x=1$ then $u=-1$ and when $x=3$ then $u=1$ $\int_1^3 x(x-2)^5 dx = \int_{-1}^1 (u+2)u^5 du$ $= \int_{-1}^1 (u^6 + 2u^5) du$ $= \left[\frac{u^7}{7} + \frac{u^6}{3}\right]_{-1}^1$ $= \left[\frac{1}{7}+\frac{1}{3}\right] - \left[\frac{-1}{7}+\frac{1}{3}\right]$ $= \frac{2}{7}$	1 Mark: B
55	$\int x \log_e x dx = \frac{x^2}{2}\log_e x - \int \frac{x^2}{2} \times \frac{1}{x} dx$ $= \frac{x^2}{2}\log_e x - \int \frac{x}{2} dx$ $= \frac{x^2}{2}\log_e x - \frac{x^2}{4} + c$	1 Mark: A
56	$\int x^4 \log_e x dx = \log_e x \times \frac{x^5}{5} - \int \frac{x^5}{5}\frac{1}{x} dx$ $= \frac{x^5 \log_e x}{5} - \frac{1}{4}\int x^4 dx$ $= \frac{x^5 \log_e x}{5} - \frac{x^5}{20} + c$	1 Mark: C
57	$\int_0^{\frac{\pi}{6}} x \cos x dx = [x \sin x]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin x dx$ $= \frac{\pi}{12} + [\cos x]_0^{\frac{\pi}{6}}$ $= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$	1 Mark: D

58	$I_n = \int_0^{\pi} \cos^n x dx$ $= \int_0^{\frac{\pi}{2}} \cos^n x dx$ Integration by parts $I_n = \int_0^{\frac{\pi}{2}} \cos^{n-1} t \cos t dt$ $= \left[\cos^{n-1} t \sin t\right]_0^{\frac{\pi}{2}} + (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2} t \sin^2 t dt$ $= (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2} t \sin^2 t dt$ $= (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2} t(1-\cos^2 t) dt$ $= (n-1)\int_0^{\frac{\pi}{2}} (\cos^{n-2} t - \cos^n t) dt$ $= (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2} t dt - (n-1)\int_0^{\frac{\pi}{2}} \cos^n t dt$ Using the original integral $\int_0^{\frac{\pi}{2}} \cos^n t dt = (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2} t dt - n\int_0^{\frac{\pi}{2}} \cos^n t dt + \int_0^{\frac{\pi}{2}} \cos^n t dt$ $n\int_0^{\frac{\pi}{2}} \cos^n t dt = (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2} t dt$ $\int_0^{\frac{\pi}{2}} \cos^n t dt = \frac{(n-1)}{n}\int_0^{\frac{\pi}{2}} \cos^{n-2} t dt$ $I_n = \frac{(n-1)}{n} I_{n-2}$	1 Mark: A
59	$I_n = \int_0^{\pi} x^n \sin x dx$ $= [-x^n \cos x]_0^{\pi} + n\int_0^{\pi} x^{n-1} \cos x dx$ $= [-\pi^n \times -1 - 0] + n[[x^{n-1}\sin x]_0^{\pi} - (n-1)\int_0^{\pi} x^{n-2} \sin x dx$ $= \pi^n + n[0 - (n-1)\int_0^{\pi} x^{n-2} \sin x dx]$ $= \pi^n - n(n-1)I_{n-2}$	1 Mark: A
60	$I_n = \int x^n e^{ax} dx$ $= \frac{x^n e^{ax}}{a} - \int \frac{1}{a} e^{ax} n x^{n-1} dx$ $= \frac{x^n e^{ax}}{a} - \frac{n}{a}\int e^{ax} x^{n-1} dx$ $\therefore I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a} I_{n-1}$	1 Mark: A