Mechanics Solutions Main Menu

70 A particle of mass m falls from rest under gravity and the resistance to its motion is mkv^2, where v is its speed and k is a positive constant. Which of the following is the correct expression for square of the velocity where x is the distance fallen?

(A) $v^2 = \frac{g}{k}\left(1-e^{-2kx}\right)$

(B) $v^2 = \frac{g}{k}\left(1+e^{-2kx}\right)$

(C) $v^2 = \frac{g}{k}\left(1-e^{2kx}\right)$

(D) $v^2 = \frac{g}{k}\left(1+e^{2kx}\right)$

71 A rock is projected vertically upwards from ground level. Assume air resistance is kv, where v is the velocity of the rock and k is a positive constant. The rock falls back to ground level under the influence of g, the acceleration due to gravity. Consider the rock's motion starting from maximum height. Let y be the displacement and t be the time elapsed after the rock has reached maximum height. Assume the rock has a unit mass. Which of the following is the correct expression for velocity?

(A) $v = \frac{g}{k}(e^{kt}-1)$

(B) $v = \frac{g}{k}(e^{kt}+1)$

(C) $v = \frac{k}{g}(e^{kt}-1)$

(D) $v = \frac{k}{g}(e^{kt}+1)$

72 A particle of mass m is moving in a straight line under the action of a force.

$$F = \frac{m}{x^3}(6-10x)$$

What of the following is an expression for its velocity in any position, if the particle starts from rest at $x=1$?

(A) $v = \pm\frac{1}{x}\sqrt{(-3+10x-7x^2)}$

(B) $v = \pm x\sqrt{(-3+10x-7x^2)}$

(C) $v = \pm\frac{1}{x}\sqrt{2(-3+10x-7x^2)}$

(D) $v = \pm x\sqrt{2(-3+10x-7x^2)}$

73 A particle of mass m is projected vertically upwards with an initial velocity of u ms^{-1} in a medium in which the resistance to the motion is proportional to the square of the velocity v ms^{-1} of the particle or mkv^2. Let x be the displacement in metres of the particle above the point of projection, O, so that the equation of motion is $\ddot{x} = -\left(g+kv^2\right)$ where g ms^{-2} is the acceleration due to gravity. Assume $k = 10$ and the acceleration due to gravity is 10 ms^{-2}.
Which of the following gives the correct expression for the time taken?

(A) $t = \frac{1}{10}\left(\tan^{-1}u - \tan^{-1}v\right)$

(B) $t = \frac{1}{10}\left(\tan^{-1}v - \tan^{-1}u\right)$

(C) $t = \frac{1}{10}\left(\tan^{-1}u + \tan^{-1}v\right)$

(D) $t = \frac{1}{10}\left(\tan^{-1}v + \tan^{-1}u\right)$

74 A conical pendulum consists of a body P of mass m kg and a string of length l metres. Point A is fixed and the body P rotates in a horizontal circle of radius r and centre O at a constant angular velocity of ω radians per second. OA is vertical and has a length of h metres. The angle OAP is θ radians. The body, P, is subject to a gravitational force of mg newtons. The tension in the string is T newtons. Which of the following gives the correct resolution of forces on P in the horizontal and vertical directions?

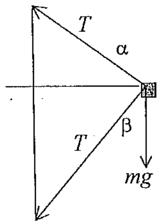
(A) $T\sin\theta - mg = 0$ and $T\cos\theta = mr\omega^2$

(B) $T\sin\theta + mg = 0$ and $T\cos\theta = mr\omega^2$

(C) $T\cos\theta - mg = 0$ and $T\sin\theta = mr\omega^2$

(D) $T\cos\theta + mg = 0$ and $T\sin\theta = mr\omega^2$

75 Two light inextensible strings are attached to a particle of mass m. The particle describes a horizontal circle with constant angular velocity ω. Which of the following gives the correct resolution of forces in the horizontal and vertical directions?



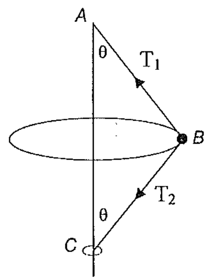
(A) $T\sin\alpha - T\sin\beta = m\omega^2 r$ and $T\cos\alpha - T\cos\beta = mg$

(B) $T\sin\alpha + T\cos\beta = m\omega^2 r$ and $T\sin\alpha - T\cos\beta = mg$

(C) $T\sin\alpha + T\sin\beta = m\omega^2 r$ and $T\cos\alpha - T\cos\beta = mg$

(D) $T\sin\alpha - T\cos\beta = m\omega^2 r$ and $T\sin\alpha - T\cos\beta = mg$

76 A body of mass m kg is attached by two light rods AB and BC. Both rods are l metres in length. Rod AB is hinged at point A and makes an angle θ with the vertical shaft. Rod BC is attached to a ring that can slide freely along the vertical shaft.



What are the tensions in the rods?

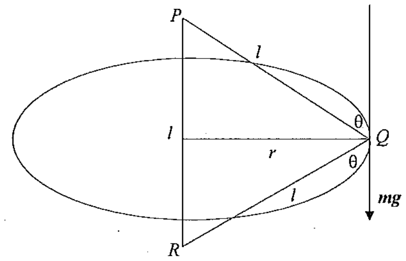
(A) $T_1 = \frac{1}{2}\left(mg\sec\theta + ml\omega^2\right)$ and $T_2 = \frac{1}{2}\left(ml\omega^2 - mg\sec\theta\right)$

(B) $T_1 = \frac{1}{2}\left(mg\sin\theta + ml\omega^2\right)$ and $T_2 = \frac{1}{2}\left(ml\omega^2 - mg\sin\theta\right)$

(C) $T_1 = \frac{1}{2}\left(mg\sec\theta - ml\omega^2\right)$ and $T_2 = \frac{1}{2}\left(ml\omega^2 + mg\sec\theta\right)$

(D) $T_1 = \frac{1}{2}\left(mg\sin\theta - ml\omega^2\right)$ and $T_2 = \frac{1}{2}\left(ml\omega^2 + mg\sin\theta\right)$

77 Two light inextensible strings PQ and QR each of length l are attached to a particle of mass m at Q. The other ends P and R are fixed to two points in a vertical line such that P is a distance l above R. The particle describes a horizontal circle with constant angular velocity ω.



What is the tension in the strings?

(A) $T_1 = \frac{m}{2}(lw^2 + 2g)$ and $T_2 = \frac{m}{2}(lw^2 - g)$

(B) $T_1 = \frac{m}{2}(lw^2 - 2g)$ and $T_2 = \frac{m}{2}(lw^2 + g)$

(C) $T_1 = m(lw^2 - 2g)$ and $T_2 = m(lw^2 + g)$

(D) $T_1 = m(lw^2 + 2g)$ and $T_2 = m(lw^2 - g)$

78 What is the angle at which a road must be banked so that a car may round a curve with a radius of 200 metres at 100 km/h without sliding? Assume that the road is smooth.

(A) 21.49° (B) 22.49°

(C) 23.49° (D) 24.49°

79 A conical pendulum consists of a body P of mass m kg and a string of length l metres. A is fixed and the body P rotates in a horizontal circle of radius r and centre O at a constant angular velocity of ω radians per second. OA is vertical and $OA = h$ metres. The angle OAP is α. The body, P, is subject to a gravitational force of mg newtons. The tension in the string is T newtons. What is the angular velocity?

(A) $\sqrt{\frac{g}{h}}$

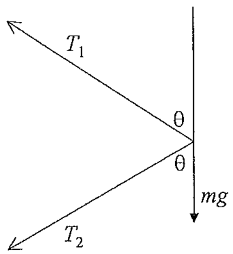
(B) $\sqrt{\frac{h}{g}}$

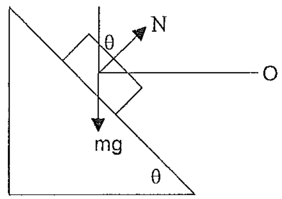
(C) $2\pi\sqrt{\frac{g}{h}}$

(D) $2\pi\sqrt{\frac{h}{g}}$

Mechanics		Main Menu
	Solution	**Criteria**
70	$\dot{v} = g - kv^2$ $\frac{1}{2}\frac{dv^2}{dx} = g - kv^2$ $2\,dx = \frac{dv^2}{g - kv^2}$ $-2k\,dx = \frac{-k\,dv^2}{g - kv^2}$ $-2kx + c = \log_e\left\|g - kv^2\right\|$ Initial conditions $t = 0$, $v = 0$ and $x = 0$ or $c = \log_e g$ $-2kx = \log_e\left\|1 - \frac{k}{g}v^2\right\|$ $v^2 = \frac{g}{k}\left(1 - e^{-2kx}\right)$	1 Mark: A
71	$\ddot{y} = kv - g$ $\frac{dv}{dt} = kv - g$ $\frac{dt}{dv} = \frac{1}{kv - g}$ $\int \frac{dt}{dv}\,dv = \int \frac{1}{kv - g}\,dv$ $t = \frac{1}{k}\log_e(kv - g) + c$ Initial conditions $t = 0$ and $v = 0$ $0 = \frac{1}{k}\log_e(-g) + c$ or $c = -\frac{1}{k}\log_e g$ $t = \frac{1}{k}\log_e(kv - g) - \frac{1}{k}\log_e g$ $= \frac{1}{k}\log_e(\frac{kv - g}{g})$ $kt = \log_e(\frac{kv - g}{g})$ $e^{kt} = \frac{kv - g}{g}$ $= \frac{kv}{g} - 1$ $v = \frac{g}{k}(e^{kt} + 1)$	1 Mark: B

72	$F = \frac{m}{x^3}(6 - 10x)$ $ma = \frac{m}{x^3}(6 - 10x)$ $v\frac{dv}{dx} = \frac{6}{x^3} - \frac{10}{x^2}$ $\int v\,dv = \int(\frac{6}{x^3} - \frac{10}{x^2})dx$ $\frac{1}{2}v^2 = (\frac{6x^{-2}}{-2} - \frac{10x^{-1}}{-1}) + c$ $\frac{1}{2}v^2 = (\frac{-3}{x^2} + \frac{10}{x}) + c$ When $v = 0$ and $x = 1$ $\frac{1}{2}0^2 = (\frac{-3}{1^2} + \frac{10}{1}) + c$ $c = -7$ $\frac{1}{2}v^2 = (\frac{-3}{x^2} + \frac{10}{x}) - 7$ $v^2 = (\frac{-6}{x^2} + \frac{20}{x}) - 14$ $= \frac{-6 + 20x - 14x^2}{x^2}$ $v = \pm\frac{1}{x}\sqrt{2(-3 + 10x - 7x^2)}$	1 Mark: C
73	$\ddot{x} = -\left(g + kv^2\right)$ $\ddot{x} = -\left(10 + 10v^2\right)$ $\frac{dv}{dt} = -10\left(1 + v^2\right)$ $\frac{dv}{1 + v^2} = -10dt$ $\tan^{-1}v = -10t + c$ When $t = 0$, $v = u$ and $c = \tan^{-1}u$ $\tan^{-1}v = -10t + \tan^{-1}u$ or $t = \frac{1}{10}\left(\tan^{-1}u - \tan^{-1}v\right)$	1 Mark: A
74	Body moving in a horizontal circle. $T\cos\theta - mg = 0$ and $T\sin\theta = mr\omega^2$	1 Mark: C
75	The horizontal component is $mr\omega^2$. $T\sin\alpha + T\sin\beta = m\omega^2 r$ The vertical component is zero $T\cos\alpha - T\cos\beta = mg$	1 Mark: C

<table>
<tr><td>76</td><td>Resolving the forces vertically
$T_1\cos\theta - T_2\cos\theta - mg = 0$ (1)
Resolving the forces horizontally
$T_1\sin\theta + T_2\sin\theta = mr\omega^2$
Now $\sin\theta = \frac{r}{l}$ or $r = l\sin\theta$
$T_1\sin\theta + T_2\sin\theta = ml\sin\theta\omega^2$ (2)
Simplifying Eqn (1)
$T_1 - T_2 = mg\sec\theta$ (3)
Simplifying Eqn (2)
$T_1 + T_2 = ml\omega^2$ (4)
Eqn (3) + Eqn (4)
$2T_1 = mg\sec\theta + ml\omega^2$
$T_1 = \frac{1}{2}\left(mg\sec\theta + ml\omega^2\right)$
Eqn (4) - Eqn (3)
$2T_2 = ml\omega^2 - mg\sec\theta$
$T_2 = \frac{1}{2}\left(ml\omega^2 - mg\sec\theta\right)$</td><td>1 Mark: A</td></tr>
<tr><td>77</td><td>Horizontal components
$T_1\sin\theta + T_2\sin\theta = mrw^2$


Also $\sin\theta = \frac{r}{l}$
$T_1\frac{r}{l} + T_2\frac{r}{l} = mrw^2$
$T_1 + T_2 = mlw^2$ (1)
Vertical components
$T_1\cos\theta = T_2\cos\theta + mg$
Also $\cos\theta = \frac{\frac{1}{2}l}{l} = \frac{1}{2}$
$T_1\frac{1}{2} - T_2\frac{1}{2} = mg$ or $T_1 - T_2 = 2mg$ (2)
Equations (1) + (2) $2T_1 = mlw^2 + 2mg$
$T_1 = \frac{m}{2}(lw^2 + 2g)$
Substitute $\frac{m}{2}(lw^2 + 2g)$ for T_1 into equation (2)
$\frac{m}{2}(lw^2 + 2g) - T_2 = 2mg$
$T_2 = \frac{m}{2}lw^2 + mg - 2mg = \frac{m}{2}(lw^2 - g)$
Therefore $T_1 = \frac{m}{2}(lw^2 + 2g)$ and $T_2 = \frac{m}{2}(lw^2 - g)$</td><td>1 Mark: A</td></tr>
<tr><td>78</td><td>

Consider the forces acting vertically and radially.
$N\cos\theta = mg$ $N\sin\theta = \frac{mv^2}{r}$
Dividing these two equations
$\tan\theta = \frac{v^2}{rg}$
Now $v = 100$ km/h
$= \frac{100 \times 1000}{3600}$ m/s
$= \frac{250}{9}$ m/s
$\tan\theta = \frac{v^2}{rg}$
$= \frac{(250/9)^2}{200 \times 9.8}$
$= 0.393675988$
$\theta = 21.49^\circ$</td><td>1 Mark: A</td></tr>
<tr><td>79</td><td>Body moving in a horizontal circle.
$T\cos\theta - mg = 0$ and $T\sin\theta = mr\omega^2$
Dividing the two equations
$\frac{T\sin\alpha}{T\cos\alpha} = \frac{mr\omega^2}{mg}$
$\tan\alpha = \frac{r\omega^2}{g}$
$\tan\alpha = \frac{r}{h}$
$\frac{r\omega^2}{g} = \frac{r}{h}$
$\omega^2 = \frac{g}{h}$
$\omega = \sqrt{\frac{g}{h}}$</td><td>1 Mark: A</td></tr>
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