Polynomials Solutions Main Menu

80 Let α, β and γ be roots of the equation $x^3+x^2-2x-5=0$. Which of the following polynomial equations have roots $\alpha-2$, $\beta-2$ and $\gamma-2$?

(A) $x^3+7x^2+14x+3=0$

(B) $x^3+7x^2+21x+3=0$

(C) $x^3+x^2-6x+9=0$

(D) $x^3+2x^2-6x+9=0$

81 The polynomial equation $x^3-5x^2+6=0$ has roots α, β and γ. Which of the following polynomial equations have roots $\alpha-1$, $\beta-1$ and $\gamma-1$?

(A) $x^3-8x^2+13x=0$

(B) $x^3-8x^2-7x=0$

(C) $x^3-3x^2-7x+2=0$

(D) $x^3-2x^2-7x+2=0$

82 The polynomial equation $x^3-3x^2-x+2=0$ has roots α, β and γ. Which of the following polynomial equations have roots $2\alpha+\beta+\gamma$, $\alpha+2\beta+\gamma$ and $\alpha+\beta+2\gamma$?

(A) $x^3-6x^2+44x-49=0$

(B) $x^3-12x^2+44x-49=0$

(C) $x^3+3x^2+36x+5=0$

(D) $x^3+6x^2+36x+5=0$

83 Let α, β and γ be roots of the equation $x^3+3x^2+4=0$. Which of the following polynomial equations have roots α^2, β^2 and γ^2?

(A) $x^3-9x^2-24x-4=0$

(B) $x^3-9x^2-12x-4=0$

(C) $x^3-9x^2-24x-16=0$

(D) $x^3-9x^2-12x-16=0$

84 The polynomial equation $x^3-3x^2-x+2=0$ has roots α, β and γ. Which of the following polynomial equations have roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$?

(A) $x^3-x^2-3x+1=0$

(B) $x^3-2x^2-3x+1=0$

(C) $2x^3-x^2-3x+1=0$

(D) $2x^3-2x^2-3x+1=0$

85 The polynomial equation $P(x)=2x^4+3x^3-2x^2+7x-3$ has roots α, β, γ and δ. Which of the following polynomial equations have roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$, $\frac{1}{\gamma}$ and $\frac{1}{\delta}$?

(A) $2x^4-3x^3+x^2-5x-4$

(B) $2x^4+3x^3+x^2-5x-4$

(C) $3x^4-7x^3+2x^2-3x-2$

(D) $3x^4+7x^3+2x^2-3x-2$

86 The polynomial equation $x^3+x^2-2x-5=0$ has roots α, β and γ. Which of the following polynomial equations have roots α^2, β^2 and γ^2?

(A) $x^3-5x^2-6x-25=0$

(B) $x^3-5x^2+14x-25=0$

(C) $x^3-4x^2+5x-1=0$

(D) $x^3+4x^2+5x-1=0$

87 The polynomial equation $x^3-5x^2+6=0$ has roots α, β and γ. Which of the following polynomial equations have roots α^2, β^2 and γ^2?

(A) $x^3-25x^2+60x-36=0$

(B) $x^3-25x^2+60x-12=0$

(C) $x^3-x^2+12x-36=0$

(D) $x^3-x^2+12x-12=0$

88 The equation $24x^3 - 12x^2 - 6x + 1$ has roots α, β and γ.
What is the value of α if $\alpha = \beta + \gamma$?

(A) $-\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

89 What are the zeros of the equation $x^4 + x^2 + 6x + 4 = 0$ over the complex field given that it has a rational zero of multiplicity 2?

(A) -1, $1+\sqrt{5}i$ and $1-\sqrt{5}i$

(B) -1, $1+\sqrt{3}i$ and $1-\sqrt{3}i$

(C) $+1$, $1+\sqrt{5}i$ and $1-\sqrt{5}i$

(D) $+1$, $1+\sqrt{3}i$ and $1-\sqrt{3}i$

90 The polynomial $P(x) = x^4 + ax^2 + bx + 28$ has a double root at $x = 2$.
What are the values of a and b?

(A) $a = -11$ and $b = -12$

(B) $a = -5$ and $b = -12$

(C) $a = -11$ and $b = 12$

(D) $a = -5$ and $b = 12$

91 The polynomial $P(x) = x^5 - 3x^4 + 4x^3 - 4x^2 + 3x - 1$ has $x = 1$ as a root of multiplicity 3 and $x = i$ as a root. Which of the following expressions is a factorised form of $P(x)$ over the complex numbers?

(A) $P(x) = (x+1)^3(x-1)(x+1)$

(B) $P(x) = (x-1)^3(x-1)(x+1)$

(C) $P(x) = (x+1)^3(x-i)(x+i)$

(D) $P(x) = (x-1)^3(x-i)(x+i)$

92 What are the values of real numbers p and q such that $1-i$ is a root of the equation $z^3 + pz + q = 0$?

(A) $p = -2$ and $q = -4$

(B) $p = -2$ and $q = 4$

(C) $p = 2$ and $q = 4$

(D) $p = 2$ and $q = 4$

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	Solution	Criteria
80	If α, β and γ are zeros of $p(x)=x^3+x^2-2x-5$ then $p(\alpha)=\alpha^3+\alpha^2-2\alpha-5=0$ $p(\beta)=\beta^3+\beta^2-2\beta-5=0$ $p(\gamma)=\gamma^3+\gamma^2-2\gamma-5=0$ $p(\alpha-2)=(\alpha+2)^3+(\alpha+2)^2-2(\alpha+2)-5=0$ $p(\beta-2)=(\beta+2)^3+(\beta+2)^2-2(\beta+2)-5=0$ $p(\gamma-2)=(\gamma+2)^3+(\gamma+2)^2-2(\gamma+2)-5=0$ Polynomial equation is $(x+2)^3+(x+2)^2-2(x+2)-5=0$ $(x+2)(x^2+4x+4)+(x^2+4x+4)-2x-4-5=0$ $x^3+4x^2+4x+2x^2+8x+8+x^2+4x+4-2x-4-5=0$ $x^3+7x^2+14x+3=0$	1 Mark: A
81	The equation $x^3-5x^2+6=0$ has roots α,β,γ. Therefore the equation $(x+1)^3-5(x+1)^2+6=0$ has roots $\alpha-1$, $\beta-1$ and $\gamma-1$. $(x+1)^3-5(x+1)^2+6=0$ $(x+1)(x^2+2x+1)-5(x^2+2x+1)+6=0$ $x^3+2x^2+x+x^2+2x+1-5x^2-10x-5+6=0$ $x^3-2x^2-7x+2=0$ $x^3-2x^2-7x+2=0$ is the equation	1 Mark: D
82	The equation $x^3-3x^2-x+2=0$ has roots α,β,γ. $\alpha+\beta+\gamma=3$ $x=2\alpha+\beta+\gamma=\alpha+3$ $x=\alpha+2\beta+\gamma=\beta+3$ $x=\alpha+\beta+2\gamma=\gamma+3$ $\alpha=x-3$ satisfies $x^3-3x^2-x+2=0$ $(x-3)^3-3(x-3)^2-(x-3)+2=0$ 1 $x^3-12x^2+44x-49=0$ is the equation	1 Mark: B
83	If α, β and γ are zeros of $x^3+3x^2+4=0$ then Polynomial equation is $(\sqrt{x})^3+3(\sqrt{x})^2+4=0$ $(\sqrt{x})^3=-(3x+4)$ $x^3=9x^2+24x+16$ $x^3-9x^2-24x-16=0$	1 Mark: C
84	$x=\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$ $\alpha=\frac{1}{x}$ satisfies $x^3-3x^2-x+2=0$ $\left(\frac{1}{x}\right)^3-3\left(\frac{1}{x}\right)^2-\frac{1}{x}+2=0$ $1-3x-x^2+2x^3=0$ $2x^3-x^2-3x+1=0$	1 Mark: C
85	$x=\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma},\frac{1}{\delta}$ $\alpha=\frac{1}{x}$ satisfies $2x^4+3x^3-2x^2+7x-3$ $2\left(\frac{1}{x}\right)^4+3\left(\frac{1}{x}\right)^3-2\left(\frac{1}{x}\right)^2+7\left(\frac{1}{x}\right)-3=0$ $2+3x-2x^2+7x^3-3x^4=0$ $3x^4-7x^3+2x^2-3x-2=0$	1 Mark: C
86	If α, β and γ are zeros of $p(x)=x^3+x^2-2x-5$ then $p(\alpha)=\alpha^3+\alpha^2-2\alpha-5=0$ $p(\beta)=\beta^3+\beta^2-2\beta-5=0$ $p(\gamma)=\gamma^3+\gamma^2-2\gamma-5=0$ $p(\alpha^2)=(\sqrt{\alpha})^3+(\sqrt{\alpha})^2-2(\sqrt{\alpha})-5=0$ $p(\beta^2)=(\sqrt{\beta})^3+(\sqrt{\beta})^2-2(\sqrt{\beta})-5=0$ $p(\gamma^2)=(\sqrt{\gamma})^3+(\sqrt{\gamma})^2-2(\sqrt{\gamma})-5=0$ Polynomial equation is $(\sqrt{x})^3+(\sqrt{x})^2-2(\sqrt{x})-5=0$ $\sqrt{x}(x-2)=5-x$ $x(x^2-4x+4)=25-10x+x^2$ $x^3-5x^2+14x-25=0$	1 Mark: B

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87	The equation $x^3-5x^2+6=0$ has roots α,β,γ. Therefore the equation $(\sqrt{x})^3-5(\sqrt{x})^2+6=0$ has roots α^2, β^2 and γ^2 $(\sqrt{x})^3-5(\sqrt{x})^2+6=0$ $x\sqrt{x}-5x+6=0$ $x\sqrt{x}=5x-6$ $x^3=(5x-6)^2$ $x^3=25x^2-60x+36$ $x^3-25x^2+60x-36=0$ $x^3-25x^2+60x-36=0$ is the equation	1 Mark: A
88	Sum of the roots $\alpha+\beta+\gamma=-\frac{b}{a}=-\frac{-12}{24}$ $\alpha+(\beta+\gamma)=\frac{1}{2}$ $\alpha+(\alpha)=\frac{1}{2}$ or $\alpha=\frac{1}{4}$	1 Mark: B
89	$P(x)=x^4+x^2+6x+4$ $P'(x)=4x^3+2x+6$ $=2(2x^3+x+3)$ To determine the roots of $2x^3+x+3$ $P'(-1)=2(2(-1)^3+(-1)+3)$ $=0$ Therefore -1 is a zero of multiplicity 2 of $P(x)$ $P(x)=x^4+x^2+6x+4$ $=(x+1)^2(x^2+bx+c)$ $=(x^2+2x+1)(x^2+bx+c)$ $=x^4+bx^3+cx^2+2x^3+2bx^2+2cx+x^2+bx+c$ $=x^4+(b+2)x^3+(c+2b+1)x^2+(b+2c)x+4$ Hence $c=4$, $b=-2$ Therefore $P(x)=x^4+x^2+6x+4$ $=(x+1)^2(x^2-2x+4)$ $=(x+1)^2((x-1)^2-1+4)$ $=(x+1)^2((x-1)^2+3)$ $=(x+1)^2(x-1+\sqrt{3}i)(x-1-\sqrt{3}i)$ Zeros of $P(x)$ are -1, $1+\sqrt{3}i$ and $1-\sqrt{3}i$	1 Mark: B

90	$P(x)=x^4+ax^2+bx+28$ $P'(x)=4x^3+2ax+b$ Root at $x=2$ $P(2)=2^4+a\times2^2+b\times2+28=0$ $44+4a+2b=0$ (1) $P'(2)=4\times2^3+2a\times2+b=0$ $32+4a+b=0$ (2) Eqn (1) − (2) $12+b=0$ $b=-12$ Subsitute $b=-12$ into Eqn (2) $32+4a-12=0$ $a=-5$ Therefore $a=-5$ and $b=-12$	1 Mark: B
91	Since all coefficients are real then $x=-i$ is also a root of $P(x)$ $P(x)=(x-1)^3(x-i)(x+i)$	1 Mark: D
92	Using the conjugate root theorem $1+i$ and $1-i$ are both roots of the equation $z^3+pz+q=0$. $(1+i)+(1-i)+\alpha=0$ (sum of the roots) $\alpha=-2$ $(1+i)\times(1-i)\times-2=-q$ (product of the roots) $(1+1)\times-2=-q$ $q=4$ $(1+i)(1-i)+(1-i)-2+(1+i)-2=p$ $p=-2$ Therefore $p=-2$ and $q=4$	1 Mark: B