Volumes Solutions Main Menu

61 The parabola $y=x^3$ is rotated about the y axis $\{x:0\le x\le 2\}$ to form a solid.
What is the volume of this solid using the method of slicing?

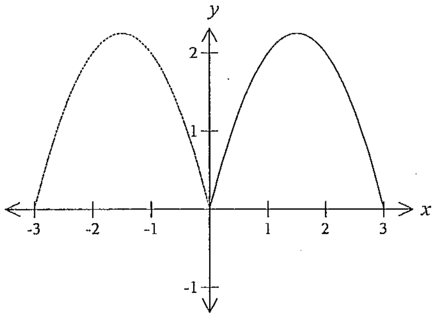
(A) $\frac{2\pi}{5}$ cubic units

(B) $\frac{3\pi}{5}$ cubic units

(C) $\frac{93\pi}{5}$ cubic units

(D) $\frac{96\pi}{5}$ cubic units

62 The area between the curve $y=3x-x^2$, the x-axis, $x=0$ and $x=3$, is rotated about the y-axis to form a solid.



What is the volume of this solid using the method of slicing?

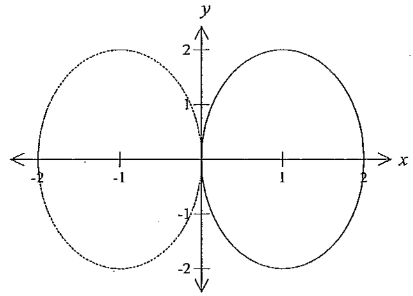
(A) $\frac{9\pi}{4}$ cubic units

(B) $\frac{9\pi}{2}$ cubic units

(C) $\frac{27\pi}{4}$ cubic units

(D) $\frac{27\pi}{2}$ cubic units

63 The region enclosed by the ellipse $(x-1)^2+\frac{y^2}{4}=1$ is rotated about the y axis to form a solid.



What is the correct expression for volume of this solid using the method of slicing?

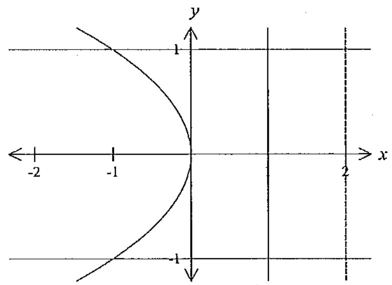
(A) $V=\int_{-2}^{2}\pi\sqrt{1-y^2}\,dy$

(B) $V=\int_{-2}^{2}2\pi\sqrt{1-y^2}\,dy$

(C) $V=\int_{-2}^{2}\pi\sqrt{4-y^2}\,dy$

(D) $V=\int_{-2}^{2}2\pi\sqrt{4-y^2}\,dy$

64 The region is bounded by the lines $x=1$, $y=1$, $y=-1$ and by the curve $x=-y^2$. The region is rotated through 360° about the line $x=2$ to form a solid. What is the correct expression for volume of this solid?



(A) $V=\int_{-1}^{1}\pi\left(y^4-4y^2+3\right)dy$

(B) $V=\int_{-1}^{1}\pi\left(y^4+4y^2+3\right)dy$

(C) $V=\int_{-1}^{1}\pi\left(y^4-4y^2+4\right)dy$

(D) $V=\int_{-1}^{1}\pi\left(y^4+4y^2+4\right)dy$

65 What is the volume of the solid formed when the region bounded by the curves $y = 2x^3$ and $y = 2\sqrt{x}$ is rotated about the x-axis? Use the method of slicing.

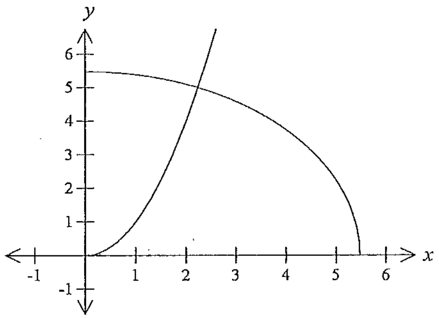
(A) $\frac{5\pi}{14}$ cubic units

(B) $\frac{10\pi}{14}$ cubic units

(C) $\frac{5\pi}{7}$ cubic units

(D) $\frac{10\pi}{7}$ cubic units

66 What is the volume of the solid formed when the region bounded by the curves $y = x^2$, $y = \sqrt{30 - x^2}$ and the y-axis is rotated about the y-axis? Use the method of slicing.



What is the correct expression for volume of this solid using the method of cylindrical shells?

(A) $V = \int_0^{\sqrt{5}} 2\pi\left(x^2 - \sqrt{30 - x^2}\right)dx$

(B) $V = \int_0^{\sqrt{5}} 2\pi x\left(x^2 - \sqrt{30 - x^2}\right)dx$

(C) $V = \int_0^{\sqrt{5}} 2\pi\left(\sqrt{30 - x^2} - x^2\right)dx$

(D) $V = \int_0^{\sqrt{5}} 2\pi x\left(\sqrt{30 - x^2} - x^2\right)dx$

67 The region bounded by $y \le 4x^2 - x^4$ and $0 \le x \le 2$ is rotated about the y axis to form a solid.
What is the volume of this solid using the method of cylindrical shells?

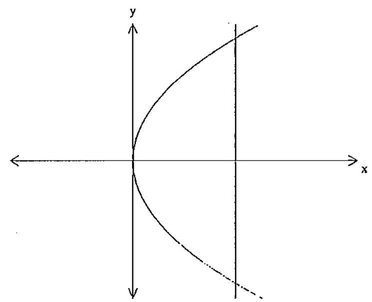
(A) $\frac{16\pi}{3}$ units^3

(B) $\frac{8\pi}{3}$ units^3

(C) $\frac{20\pi}{3}$ units^3

(D) $\frac{32\pi}{3}$ units^3

68 A solid is formed by rotating the region enclosed by the parabola $y^2 = 4ax$, its vertex (0,0) and the line $x = a$, about the x-axis.



What is the volume of this solid using the method of cylindrical shells?

(A) $\frac{7\pi a^3}{4}$ units^3

(B) $\frac{7\pi a^3}{8}$ units^3

(C) $\frac{7\pi a^3}{16}$ units^3

(D) $2\pi a^3$ units^3

69 The region enclosed by $y = \sin x$, $y = 0$ and $x = \frac{\pi}{2}$ is rotated around the y-axis to produce a solid. What is the volume of this solid using the method of cylindrical shells?

(A) π units^3

(B) $\frac{\pi}{2}$ units^3

(C) $\frac{3\pi}{2}$ units^3

(D) 2π units^3

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	Solution	**Criteria**
61	Area of the slice is a circle radius is x and height y $A=\pi x^2$ $=\pi(y^{\frac{1}{3}})^2=\pi y^{\frac{2}{3}}$ $\delta V=\delta A.\delta y$ $V=\lim_{\delta y\to 0}\sum_{y=0}^{8}\pi y^{\frac{2}{3}}\delta y$ $=\int_0^8 \pi y^{\frac{2}{3}}dy$ $=\pi\left[\frac{3}{5}y^{\frac{5}{3}}\right]_0^8$ $=\frac{3\pi}{5}\times 8^{\frac{5}{3}}=\frac{96\pi}{5}$ cubic units	1 Mark: D
62	The base is an annulus. $A=\pi(r_2^2-r_1^2)$ $=\pi(r_2+r_1)(r_2-r_1)$ Now the r_1 and r_2 are the roots $y=3x-x^2$ $r^2-3r+y=0$ $r_2+r_1=3$ (sum of the roots) $r=\frac{3\pm\sqrt{3^2-4y}}{2}$ (quadratic formula) $r_2-r_1=2\times\frac{\sqrt{9-4y}}{2}$ $=\sqrt{9-4y}$ Area of the annulus $A=\pi(r_2+r_1)(r_2-r_1)$ $=3\pi\sqrt{9-4y}$ $V=\lim_{\delta y\to 0}\sum_{y=0}^{\frac{9}{4}}3\pi\sqrt{9-4y}\,\delta y$ $=\int_0^{\frac{9}{4}}3\pi\sqrt{9-4y}\,dy$ $=3\pi\int_0^{\frac{9}{4}}(9-4y)^{\frac{1}{2}}dy$ $=3\pi\left[\frac{2}{3}\times-\frac{1}{4}(9-4y)^{\frac{3}{2}}\right]_0^{\frac{9}{4}}$ $=-\frac{\pi}{2}\left[(0)^{\frac{3}{2}}-(9)^{\frac{3}{2}}\right]=\frac{27\pi}{2}$	1 Mark: D

63	The base is an annulus. $A=\pi(r_2^2-r_1^2)$ $=\pi(r_2+r_1)(r_2-r_1)$ $(r-1)^2+\frac{y^2}{4}=1$ $r^2-2r+\frac{y^2}{4}=0$ $r=\frac{2\pm\sqrt{(-2)^2-4\times1\times\frac{y^2}{4}}}{2}$ $r=1\pm\sqrt{1-\frac{y^2}{4}}$ Therefore $r_2+r_1=2$ and $r_2-r_1=2\sqrt{1-\frac{y^2}{4}}$ $V=\lim_{\delta y\to 0}\sum_{y=-2}^{2}\pi\times2\times2\sqrt{1-\frac{y^2}{4}}\,\delta y$ $=\int_{-2}^{2}2\pi\sqrt{4-y^2}\,dy$	1 Mark: D
64	Area of the slice is an annulus Inner radius is 1 and outer radius is $2+y^2$ and height y $A=\pi\left(R^2-r^2\right)$ $=\pi\left((2+y^2)^2-1^2\right)$ $=\pi\left(4+4y^2+y^4-1\right)$ $=\pi\left(y^4+4y^2+3\right)$ $\delta V=\delta A.\delta y$ $V=\lim_{\delta y\to 0}\sum_{y=-1}^{1}\pi\left(y^4+4y^2+3\right)\delta y$ $=\int_{-1}^{1}\pi\left(y^4+4y^2+3\right)dy$	1 Mark: B

65	Slices are taken perpendicular to the axis of rotation (x-axis). The base is an annulus. $A=\pi(r_2^{\ 2}-r_1^{\ 2})$ $=\pi((2\sqrt{x})^2-(2x^3)^2)$ $=\pi(4x-4x^6)=4\pi(x-x^6)$ $V=\lim_{\delta x\to 0}\sum_{x=0}^{1}4\pi(x-x^6)\delta x$ $=\int_0^1 4\pi(x-x^6)dx$ $=4\pi\int_0^1(x-x^6)dx$ $=4\pi\left[\frac{x^2}{2}-\frac{x^7}{7}\right]_0^1$ $=4\pi\left[(\frac{1}{2}-\frac{1}{7})-0\right]=\frac{10\pi}{7}$	1 Mark: D
66	$\sqrt{30-x^2}=x^2$ $30-x^2=x^4$ $x^4+x^2-30=0$ $(x^2+6)(x^2-5)=0$ $x=\pm\sqrt{5}$ Cylindrical shells radius is x and height y $V=\lim_{\delta x\to 0}\sum_{x=0}^{\sqrt{5}}2\pi xy\delta x$ $=\int_0^{\sqrt{5}}2\pi x\left(\sqrt{30-x^2}-x^2\right)dx$	1 Mark: D
67	Cylindrical shells radius is x and height y $V=\lim_{\delta x\to 0}\sum_{x=0}^{2}2\pi xy\delta x$ $=2\pi\int_0^2 xydx$ $=2\pi\int_0^2 x(4x^2-x^4)dx$ $=2\pi\int_0^2 4x^3-x^5)dx$ $=2\pi\left[x^4-\frac{x^6}{6}\right]_0^2$ $=2\pi\left[16-\frac{32}{6}\right]$ $=\frac{32\pi}{3}$	1 Mark: D

68	Cylindrical shells radius is y and height $a-x=a-\frac{y^2}{4a}$ $V=\lim_{\delta y\to 0}\sum_{y=0}^{2a}2\pi y(a-\frac{y^2}{4a})\delta y$ $=2\pi\int_0^{2a}y(a-\frac{y^2}{4a})dy$ $=2\pi\int_0^{2a}(ya-\frac{y^3}{4a})dy$ $=2\pi\left[\frac{ay^2}{2}-\frac{y^4}{16a}\right]_0^{2a}$ $=2\pi\left[(\frac{4a^3}{2}-\frac{16a^4}{16a})-0\right]$ $=2\pi a^3$	1 Mark: D
69	Cylindrical shells radius is x and height $\sin x$ $V=\lim_{\delta x\to 0}\sum_{x=0}^{\frac{\pi}{2}}2\pi x\sin x\delta x$ $=2\pi\int_0^{\frac{\pi}{2}}x\sin xdx$ $=2\pi\left(\left[x\cos x\right]_0^{\frac{\pi}{2}}\right)+\int_0^{\frac{\pi}{2}}\cos xdx$ $=2\pi\left[\sin x\right]_0^{\frac{\pi}{2}}$ $=2\pi[1-0]$ $=2\pi$	1 Mark: D