Geometrical applications of differentiation

Solutions Main Menu

1 What are the x-coordinates of the two stationary points to the curve $y = 5 + 3x^3 - 2x^4$?

(A) $x = 0,\ x = \frac{2}{3}$

(B) $x = 0,\ x = \frac{3}{2}$

(C) $x = 0,\ x = \frac{8}{9}$

(D) $x = 0,\ x = \frac{9}{8}$

2 What are the x-coordinates of the two turning points to the curve $f(x) = x^3 - 12x^2 + 36x + 10$?

(A) $x = -2,\ x = -6$

(B) $x = 2,\ x = 6$

(C) $x = 0,\ x = 3$

(D) $x = 3,\ x = 4$

3 Consider the function $f(x) = 2x^3 - 6x + 1$ in the domain $-2 \le x \le 3$. What are the coordinates of its turning points and their nature?

(A) Maximum turning point at $(1,-3)$ and minimum turning point at $(-1,5)$

(B) Maximum turning point at $(-1,5)$ and minimum turning point at $(1,-3)$

(C) Maximum turning point at $(-2,-3)$ and minimum turning point at $(3,37)$

(D) Maximum turning point at $(3,37)$ and minimum turning point at $(-2,-3)$

4 Consider the function defined by $f(x) = x^3 - 6x^2 + 9x + 2$. What are the coordinates of its stationary points and their nature?

(A) Maximum stationary point at $(1,6)$ and minimum stationary point at $(3,2)$

(B) Maximum stationary point at $(3,2)$ and minimum stationary point at $(1,6)$

(C) Maximum stationary point at $(6,1)$ and minimum stationary point at $(2,3)$

(D) Maximum stationary point at $(2,3)$ and minimum stationary point at $(6,1)$

5 What values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

6 What is the equation of the tangent to the curve $y = x^2 - 5x$ at the point $(1,-4)$?

(A) $y = -3x - 1$

(B) $y = -3x - 7$

(C) $y = 3x + 7$

(D) $y = 3x - 7$

7 The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 2$ at the point $(-2,0)$. What is value of m and b?

(A) $m = 9$ and $b = -18$

(B) $m = 9$ and $b = 18$

(C) $m = 12$ and $b = -18$

(D) $m = 12$ and $b = 18$

8 What is the equation of the normal to the curve $y = x^2 - 4x$ at $(1,-3)$?

(A) $x + 2y - 7 = 0$

(B) $x - 2y - 7 = 0$

(C) $2x - y - 5 = 0$

(D) $2x + y + 5 = 0$

9 What is the equation of the normal to the curve $y = x^2 - x - 6$ at $(6,24)$?

(A) $x + 11y - 270 = 0$

(B) $x - 11y + 270 = 0$

(C) $11x - y - 42 = 0$

(D) $11x + y + 42 = 0$

10 The graph $y = f(x)$ passes through the point $(1,4)$ and $f'(x) = 3x^2 - 2$.
Which of the following expressions is $f(x)$?

(A) $x^3 - 2x$

(B) $2x - 1$

(C) $x^3 - 2x + 3$

(D) $x^3 - 2x + 5$

11 What points on the curve $y = x^3 - 4x^2 + 2x$ have a tangent parallel to the $2x + y = 3$?

(A) $(-\frac{2}{3}, -\frac{92}{27})$ and $(-2, -28)$

(B) $(-\frac{2}{3}, -\frac{92}{27})$ and $(2, -4)$

(C) $(\frac{2}{3}, -\frac{4}{27})$ and $(-2, -28)$

(D) $(\frac{2}{3}, -\frac{4}{27})$ and $(2, -4)$

12 Consider the curve given by $y = \frac{1}{2}x^4 - x^3$. What are the points of inflexion?

(A) only (0, 0)

(B) (0, 0) and $(-1, \frac{5}{8})$

(C) (0, 0) and $(1, -\frac{1}{2})$

(D) (0, 0) and $(\frac{3}{2}, -\frac{27}{32})$

13 A can of soup is the shape of a closed cylinder with a height h cm and a radius r cm. The volume of the can of soup is 400 cm^3. What is the radius of the can if the surface area of the metal used to make the can of soup is to be minimized?

(A) $\sqrt[3]{\frac{100}{\pi}}$

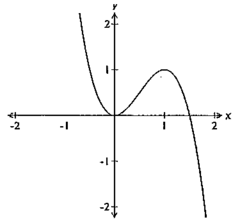
(B) $\sqrt[3]{\frac{200}{\pi}}$

(C) $\sqrt[3]{\frac{400}{\pi}}$

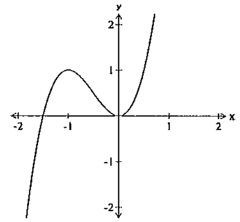
(D) $\sqrt[3]{\frac{800}{\pi}}$

14 Which of the following is the graph of $f(x) = 2x^3 - 3x^2$?

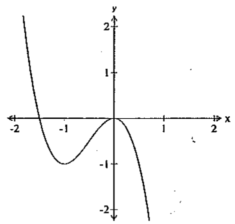
(A)



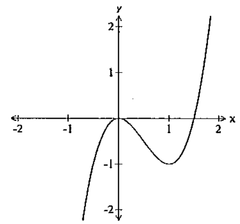
(B)



(C)



(D)



Objective Response Bank

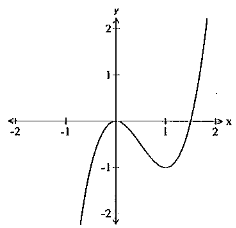
Year 12 Mathematics

Worked solutions

Geometrical applications of differentiation		Main Menu
	Solution	**Criteria**
1	Stationary points $\frac{dy}{dx}=0$ $9x^2-8x^3=0$ $x^2(9-8x)=0$ $x=0,\ x=\frac{9}{8}$	1 Mark: D
2	Turning points $f'(x)=0$ $3x^2-24x+36=0$ $3(x^2-8x+12)=0$ $3(x-2)(x-6)=0$ $x=2,\ x=6$	1 Mark: B
3	Turning points $f'(x)=0$ $6x^2-6=0$ $6(x+1)(x-1)=0$ $x=-1,\ x=1$ Turning points are $(-1,5)$ and $(1,-3)$. $f''(x)=12x$ At $(1,-3)$, $f''(1)=12>0$, Minimum turning point at $(1,-3)$ At $(-1,5)$, $f''(1)=-12<0$, Maximum turning point at $(-1,5)$	1 Mark: B
4	Stationary points $f'(x)=0$ $3x^2-12x+9=0$ $3(x-1)(x-3)=0$ $x=1,\ x=3$ Stationary points are $(1,6)$ and $(3,2)$. $f''(x)=6x-12$ At $(1,6)$, $f''(1)=-6>0$, Maximum stationary point At $(3,2)$, $f''(3)=6<0$, Minimum stationary point	1 Mark: A

5	Concave down when $f''(x)<0$ $f'(x)=6x^2+2x$ $f''(x)=12x+2$ $12x+2<0$ $x<-\frac{1}{6}$	1 Mark: A
6	$y=x^2-5x$ At the point $(1,-4)$ $\frac{dy}{dx}=2\times1-5=-3$ $\frac{dy}{dx}=2x-5$ Point slope formula $y-y_1=m(x-x_1)$ $y-(-4)=-3(x-1)$ $y=-3x-1$	1 Mark: A
7	$y=x^3-3x+2$ At the point $(-2,0)$ $\frac{dy}{dx}=3\times(-2)^2-3=9$ $\frac{dy}{dx}=3x^2-3$ Point slope formula $y-y_1=m(x-x_1)$ $y-0=9(x--2)$ $y=9x+18$ Hence $m=9$ and $b=18$	1 Mark: B
8	$y=x^2-4x$ $\frac{dy}{dx}=2x-4$ At $(1,-3)$ $\frac{dy}{dx}=2\times1-4=-2$ Normal is perpendicular to the gradient of the tangent $m_1m_2=-1,\quad m_1\times-2=-1,\quad m_1=\frac{1}{2}$ Equation of the normal at $(1,-3)$ $y-y_1=m(x-x_1)$ $y-(-3)=\frac{1}{2}(x-1)$ $2y+6=x-1$ $x-2y-7=0$	1 Mark: B

9	$y=x^2-x-6$ $\frac{dy}{dx}=2x-1$ At $(6,24)$ $\frac{dy}{dx}=2\times 6-1=11$ Normal is perpendicular to the gradient of the tangent $m_1m_2=-1$, $m_1\times 11=-1$, $m_1=-\frac{1}{11}$ Equation of the normal at $(6,24)$ $y-y_1=m(x-x_1)$ $y-24=-\frac{1}{11}(x-6)$ $11y-264=-x+6$ $x+11y-270=0$	1 Mark: A
10	$f'(x)=3x^2-2$ $f(x)=x^3-2x+c$ Point $(1,4)$ satisfies the function. $4=1^3-2\times 1+c$ or $c=5$ $\therefore f(x)=x^3-2x+5$	1 Mark: D
11	Gradient of line $2x+y=3$ is -2 $y=x^3-4x^2+2x$ $\frac{dy}{dx}=3x^2+8x+2$ Parallel lines have equal gradients $3x^2+8x+2=-2$ $3x^2+8x+4=0$ $(3x-2)(x-2)=0$ Therefore $x=\frac{2}{3}$ and $x=2$ Points are $(\frac{2}{3},-\frac{4}{27})$ and $(2,-4)$	1 Mark: D

12	Possible points of inflexion $\frac{d^2y}{dx^2}=0$ $6x^2-6x=0$ $6x(x-1)=0$ $x=0,\ x=1$ Check for change in concavity When $x=-0.1$ then $\frac{d^2y}{dx^2}=6\times -0.1\times(-0.1-1)>0$ When $x=0.1$ then $\frac{d^2y}{dx^2}=6\times 0.1\times(0.1-1)<0$ When $x=1.1$ then $\frac{d^2y}{dx^2}=6\times 1.1\times(1.1-1)>0$ Hence (0, 0) and $(1,-\frac{1}{2})$ are points of inflexion.	1 Mark: C
13	$V=\pi r^2h$ $400=\pi r^2\times h$ $h=\frac{400}{\pi r^2}$ $SA=2\pi r^2+2\pi rh$ $=2\pi r^2+2\pi r\times\frac{400}{\pi r^2}=2\pi r^2+\frac{800}{r}$ $\frac{dSA}{dr}=4\pi r-800r^{-2}$ Minimal SA occurs when $\frac{dSA}{dr}=0$ $4\pi r-800r^{-2}=0$ or $4r\left(\pi-\frac{200}{r^3}\right)=0$ Hence $r=0$ (no can) or $\pi-\frac{200}{r^3}=0$ $r=\sqrt[3]{\frac{200}{\pi}}$ $\frac{d^2SA}{dr^2}=4\pi+1600r^{-3}=4\pi+\frac{1600}{r^3}$ At $r=\sqrt[3]{\frac{200}{\pi}}$ $\frac{d^2SA}{dr^2}>0$ and is a minima	1 Mark: B
14		1 Mark: D