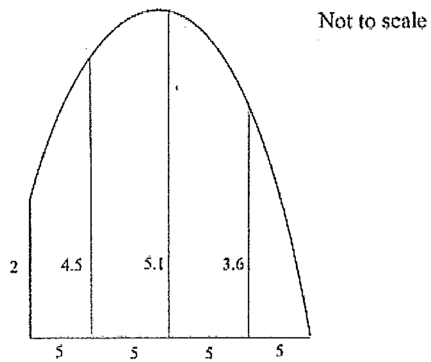
Integration

Solutions Main Menu

15 The diagram below shows a native garden. All measurements are in metres.



What is an approximate value for the area of the native garden using the trapezoidal Rule with 4 intervals?

(A) 31 m^2
(B) 62 m^2
(C) 71 m^2
(D) 74 m^2

16 The table below shows the values of a function $f(x)=\sqrt{25-x^2}$ for six values of x.

x	0	1	2	3	4	5
$f(x)$	5.00	4.90	4.58	4.00	3.00	0.00

What value is an estimate for $\int_0^5 \sqrt{25-x^2}\,dx$ using trapezoidal rule with these six values?

(A) 10.74
(B) 12.65
(C) 18.98
(D) 37.96

17 An area is bounded by the curve $y=\frac{2}{3}\sqrt{9-x^2}$, the coordinate axes and the line $x=2$.

What is an approximation for this area using the trapezoidal rule and three function values

(A) 1.82
(B) 2.69
(C) 3.63
(D) 7.26

18 The table below shows the values of a function $f(x)$ for five values of x.

x	2	2.5	3	3.5	4
$f(x)$	4	1	−2	3	8

What value is an estimate for $\int_2^4 f(x)dx$ using Simpson's rule with these five values?

(A) 4
(B) 6
(C) 8
(D) 12

19 The table below shows the values of a function $f(x)$ for five values of x.

x	0	2	4	6	8
$f(x)$	10	42	26	10	42

What value is an estimate for $\int_0^8 f(x)dx$ using Simpson's rule with these five values?

(A) 104
(B) 208
(C) 312
(D) 624

20 What value is an estimate for $\int_3^5 (x+1)^{-2}dx$ using Simpson's rule with 4 strips?

(A) 0.043 (B) 0.063

(C) 0.083 (D) 0.250

21 What is the value of $\int_{-1}^{2} x^2+1dx$?

(A) 4 (B) 5

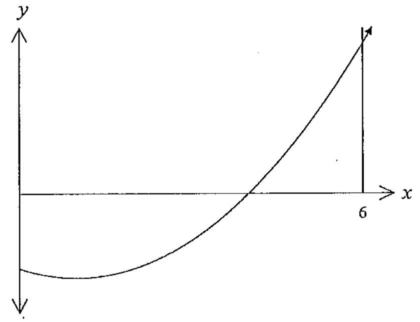
(C) 6 (D) 7

22 What is the area enclosed between the curves $y=x^2+1$ and $y=3x+1$?

(A) $\frac{3}{2}$ square units (B) $\frac{9}{2}$ square units

(C) $\frac{27}{2}$ square units (D) $\frac{45}{2}$ square units

23 The diagram below shows the graph of $y=x^2-2x-8$.



What is the correct expression for the area bounded by the x-axis and the curve $y=x^2-2x-8$ between $0\le x\le 6$?

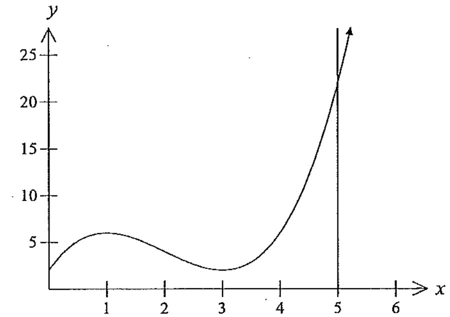
(A) $A=\int_0^5 x^2-2x-8dx+\left|\int_5^6 x^2-2x-8dx\right|$

(B) $A=\int_0^4 x^2-2x-8dx+\left|\int_4^6 x^2-2x-8dx\right|$

(C) $A=\left|\int_0^5 x^2-2x-8dx\right|+\int_5^6 x^2-2x-8dx$

(D) $A=\left|\int_0^4 x^2-2x-8dx\right|+\int_4^6 x^2-2x-8dx$

24 The diagram below shows the graph of $y=x^3-6x^2+9x+2$ and the line $x=5$.



What is the value of the area bounded by the x-axis and the curve $y=x^3-6x^2+9x+2$ between $0\le x\le 5$?

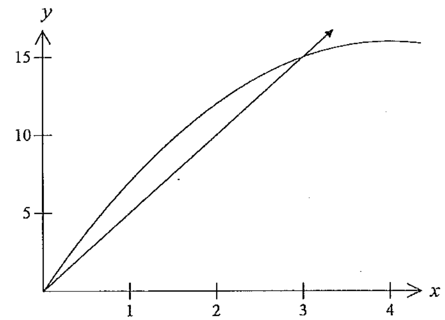
(A) 22.00 square units

(B) 25.25 square units

(C) 27.00 square units

(D) 28.75 square units

25 The diagram below shows the graph of $y=5x$ and $y=8x-x^2$.



What is the area between the curves $y=5x$ and $y=8x-x^2$?

(A) 4.5 units^2

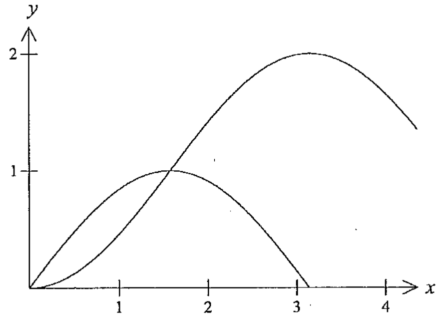
(B) 5.5 units^2

(C) 9.0 units^2

(D) 13.5 units^2

26 The diagram below shows the graph of $y = \sin x$ and $y = 1 - \cos x$.

These graphs intersect at $(0,0)$ and $(\frac{\pi}{2}, 1)$.



What is the value of the area between $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \le x \le \pi$?

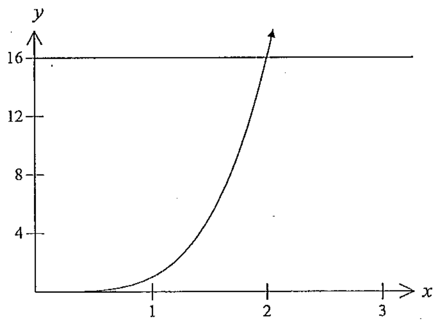
(A) 2

(B) $2 + \pi$

(C) $2 - \pi$

(D) π

27 A region in the diagram is bounded by the curve $y = x^4$, the y-axis and the line $y = 16$.



Which of the following expressions is correct for the volume of the solid of revolution when this region is rotated about the y-axis?

(A) $V = \pi \int_0^2 x^8 \, dx$

(B) $V = \pi \int_0^{16} x^8 \, dx$

(C) $V = \pi \int_0^2 y^{\frac{1}{2}} \, dy$

(D) $V = \pi \int_0^{16} y^{\frac{1}{2}} \, dy$

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28 The semi-circle $y = \sqrt{9 - x^2}$ is rotated about the x-axis. Which of the following expressions is correct for the volume of the solid of revolution?

(A) $V = \pi \int_0^3 (9 - x^2) \, dx$

(B) $V = 2\pi \int_0^3 (9 - x^2) \, dx$

(C) $V = \pi \int_0^3 (9 - y^2) \, dy$

(D) $V = 2\pi \int_0^3 (9 - y^2) \, dy$

29 The region in the first quadrant bounded by the curve $x^2 + 4y^2 = 16$ and the coordinate axes is rotated about the y-axis. What is the volume of the solid formed by this rotation?

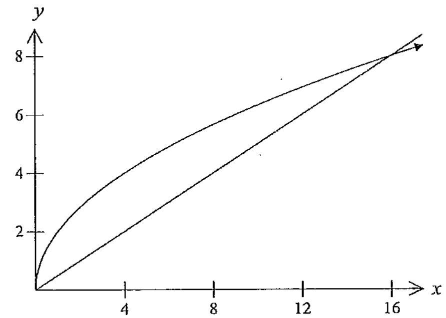
(A) 4π units^3

(B) 8π units^3

(C) $\frac{64\pi}{3}$ units^3

(D) $\frac{128\pi}{3}$ units^3

30 The diagram below shows the graph of $y = 2\sqrt{x}$ and $y = \frac{x}{4}$.



Which of the following is the correct expression for the volume of the solid of revolution when the area between the curve $y = 2\sqrt{x}$ and $y = \frac{x}{4}$ is rotated around the x-axis?

(A) $V = \int_0^8 (4y - \frac{y^2}{2}) \, dy$

(B) $V = \int_0^{16} (2\sqrt{x} - \frac{x}{4}) \, dx$

(C) $V = \pi \int_0^8 (16y^2 - \frac{y^4}{4}) \, dy$

(D) $V = \pi \int_0^{16} (4x - \frac{x^2}{16}) \, dx$

31 The area under the curve $y=\frac{5}{\sqrt{x}}$, for $1 \le x \le e^3$, is rotated about the x axis. What is the exact volume of the solid of revolution?

(A) 15π units^3

(B) 25π units^3

(C) 28π units^3

(D) 75π units^3

Integration		Main Menu
	Solution	**Criteria**
15	$A=\frac{h}{2}(d_f+2d_m+d_l)+\frac{h}{2}(d_f+2d_m+d_l)$ $=\frac{5}{2}(2+2\times 4.5+5.1)+\frac{5}{2}(5.1+2\times 3.6+0)$ $=71\text{ m}^2$	1 Mark: C
16	$\int_0^5\sqrt{25-x^2}\,dx=\frac{h}{2}[y_0+y_5+2\times(y_1+y_2+y_3+y_4)]$ $=\frac{1}{2}[5+0+2\times(4.90+4.58+4+3)]$ $=18.98$	1 Mark: C
17	$\int_0^2\frac{2}{3}\sqrt{9-x^2}\,dx=\frac{h}{2}[y_0+y_2+2\times(y_1)]$ $=\frac{1}{2}\left[2+\frac{2\sqrt{8}}{3}+2\times(\frac{2\sqrt{5}}{3})\right]$ $=3.630974...\approx 3.63$	1 Mark: C
18	$\int_2^4 f(x)dx=\frac{h}{3}[y_0+y_4+4(y_1+y_3)+2y_2]$ $=\frac{\frac{1}{2}}{3}[4+8+4\times(1+3)+2\times -2]$ $=4$	1 Mark: A
19	$\int_0^8 f(x)dx=\frac{h}{3}[y_0+y_4+4(y_1+y_3)+2y_2]$ $=\frac{2}{3}[10+42+4\times(42+10)+2\times 26]$ $=208$	1 Mark: B
20	$\int_3^5(x+1)^{-2}dx=\frac{h}{3}[y_0+y_4+4(y_1+y_3)+2y_2]$ $=\frac{0.5}{3}[0.0625+0.0278+4\times(0.0494+0.0331)+2\times 0.04]$ $=0.0833833...$ ≈ 0.083	1 Mark: C
21	$\int_{-1}^2 x^2+1\,dx=\left[\frac{x^3}{3}+x\right]_{-1}^2$ $=\left[\left(\frac{2^3}{3}+2\right)-\left(\frac{-1^3}{3}+-1\right)\right]$ $=6$	1 Mark: C

22	$x^2+1=3x+1$ $x^2-3x=0$ $x(x-3)=0$ Point of intersection occurs when $x=0$ and $x=3$ $A=\int_0^3(3x+1)-(x^2+1)dx$ $=\int_0^3(3x-x^2)dx$ $=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3$ $=\left[\left(\frac{3\times 3^2}{2}-\frac{3^3}{3}\right)-\left(\frac{3\times 0^2}{2}-\frac{0^3}{3}\right)\right]$ $=\frac{9}{2}$ square units	1 Mark: B
23	Point P is an x-intercept. $x^2-2x-8=0$ $(x-4)(x+2)=0$ Therefore $x=4$ or $x=-2$ The x value of point P is positive (diagram). Coordinates of P is $(4,0)$ $A=\left\|\int_0^4 x^2-2x-8dx\right\|+\int_4^6 x^2-2x-8dx$	1 Mark: D
24	$A=\int_0^5(x^3-6x^2+9x+2)dx$ $=\left[\frac{x^4}{4}-\frac{6x^3}{3}+\frac{9x^2}{2}+2x\right]_0^5$ $=\left[\left(\frac{5^4}{4}-\frac{6\times 5^3}{3}+\frac{9\times 5^2}{2}+2\times 5\right)-0\right]$ $=28.75$ square units	1 Mark: D
25	$A=\int_0^3\left((8x-x^2)-5x\right)dx$ $=\left[-\frac{x^3}{3}+\frac{3x^2}{2}\right]_0^3$ $=\left[\left(-\frac{3^3}{3}+\frac{3\times 3^2}{2}\right)-0\right]$ $=4.5\text{ units}^2$	1 Mark: A

26	$A=\int_0^{\frac{\pi}{2}}\left(\sin x-(1-\cos x)\right)dx+\int_{\frac{\pi}{2}}^{\pi}\left(1-\cos x-\sin x\right)dx$ $=\left[-\cos x-x+\sin x\right]_0^{\frac{\pi}{2}}+\left[x-\sin x+\cos x\right]_{\frac{\pi}{2}}^{\pi}$ $=\left(0-\frac{\pi}{2}+1-(-1)\right)+\left(\pi-0-1-\left(\frac{\pi}{2}-1\right)\right)$ $=2-\frac{\pi}{2}+\frac{\pi}{2}=2$	1 Mark: A
27	Now $y=x^4$ or $y^{\frac{1}{2}}=x^2$ $V=\pi\int_0^{16}x^2dy$ $=\pi\int_0^{16}y^{\frac{1}{2}}dy$	1 Mark: D
28	Now $y=\sqrt{9-x^2}$ or $y^2=9-x^2$ $V=\pi\int_{-3}^{3}y^2dx$ $=2\pi\int_0^{3}(9-x^2)dx$	1 Mark: B
29	Substitute $x=0$ to find where the curve cuts the y-axis. $x=0$ then $0^2+4y^2=16$ $y^2=4$ $y=\pm2$ Solid is in the first quadrant only hence $0<y<2$ Also $x^2+4y^2=16$ or $x^2=16-4y^2$ $V=\pi\int_a^b x^2dy$ $=\pi\int_0^2(16-4y^2)dy$ $=\pi\left[16y-\frac{4}{3}y^3\right]_0^2$ $=\pi\left[16\times2-\frac{4}{3}\times2^3\right]$ $=\frac{64\pi}{3}$ units^3	1 Mark: C
30	$V=\pi\int_0^{16}y^2dx$ $=\pi\int_0^{16}\left((2\sqrt{x})^2-(\frac{x}{4})^2\right)dx$ $=\pi\int_0^{16}(4x-\frac{x^2}{16})dx$	1 Mark: D

31	$V=\pi\int_a^b y^2dx$ $=\pi\int_0^{e^3}(\frac{5}{\sqrt{x}})^2dx$ $=\pi\int_0^{e^3}\frac{25}{x}dx$ $=\pi\left[25\ln x\right]_0^{e^3}$ $=25\pi\left[\ln e^3-\ln 1\right]$ $=75\pi$ units^3	1 Mark: D