# Logarithmic and exponential functions   <u>Solutions</u>   <u>Main Menu</u>

**50**  What is the solution to the equation $4^x = 32$?

(A)  0.4

(B)  2.5

(C)  3

(D)  8

**51**  What is the solution to the equation $\log_e(x+2) - \log_e x = \log_e 4$?

(A)  $\dfrac{2}{5}$

(B)  $\dfrac{2}{3}$

(C)  $\dfrac{3}{2}$

(D)  $\dfrac{5}{2}$

**52**  Which of the following is an expression for $\dfrac{dy}{dx}$ if $y = \ln(x^2 - 4)$?

(A)  $\dfrac{dy}{dx} = 2x$

(B)  $\dfrac{dy}{dx} = \dfrac{1}{x^2 - 4}$

(C)  $\dfrac{dy}{dx} = \dfrac{2}{x^2 - 4}$

(D)  $\dfrac{dy}{dx} = \dfrac{2x}{x^2 - 4}$

**53**  What are the solutions to the equation $e^{6x} - 7e^{3x} + 6 = 0$?

(A)  $x = 1$ and $x = 6$

(B)  $x = 0$ and $x = \dfrac{\ln 6}{2}$

(C)  $x = 0$ and $x = \dfrac{\ln 6}{3}$

(D)  $x = 1$ and $x = \dfrac{\ln 6}{2}$

**54**  What is the derivative of $(1 + \log_e x)^4$?

(A)  $4(1 + \log_e x)^3$

(B)  $\dfrac{(1 + \log_e x)^5}{5}$

(C)  $\dfrac{4(1 + \log_e x)^3}{x}$

(D)  $\dfrac{(1 + \log_e x)^5}{5x}$

**55**  What is the derivative of $\log_2 x$?

(A)  $\dfrac{1}{x}$

(B)  $\dfrac{1}{2x}$

(C)  $\ln 2x$

(D)  $\dfrac{1}{x \ln 2}$

**56**  What is the derivative of $x^2 e^x$?

(A)  $2xe^x$

(B)  $e^x(2 + x)$

(C)  $\dfrac{x^3}{3}e^x$

(D)  $xe^x(2 + x)$

**57**  What is the derivative of $2e^x \cos x$ with respect to $x$?

(A)  $-2e^x \sin x$

(B)  $-2e^x \cos x$

(C)  $2e^x(\sin x - \cos x)$

(D)  $2e^x(\cos x - \sin x)$

**58** What is the derivative of $\dfrac{e^x}{x^2}$?

(A) $\dfrac{e^x}{2x}$

(B) $\dfrac{3e^x}{x^3}$

(C) $\dfrac{e^x(x-2)}{x^3}$

(D) $\dfrac{e^x(x^2-2x)}{x^2}$

**59** What is the derivative of $x^2 \log_e x$ with respect to $x$?

(A) $2e^{\sin^2 x} \sin x$

(B) $2e^{\sin^2 x} \sin x \cos x$

(C) $2e^{\cos^2 x} \sin x$

(D) $2e^{\cos^2 x} \sin x \cos x$

**60** What is the derivative of $\log_e(\cos x)$ with respect to $x$?

(A) $-\dfrac{1}{\sin x}$

(B) $-\tan x$

(C) $\dfrac{1}{\cos x}$

(D) $\tan x$

**61** What is the derivative of $\log_e\left(\dfrac{x+1}{x-1}\right)$ with respect to $x$?

(A) $\dfrac{x+1}{x-1}$

(B) $\dfrac{-2}{x-1}$

(C) $\dfrac{-2}{x^2-1}$

(D) $\dfrac{2}{x^2-1}$

**62** What is the value of $\displaystyle\int_{-4}^{1} \dfrac{2}{x+5}dx$?

(A) $2\ln 5$

(B) $2\ln 6$

(C) $2\ln 7$

(D) $\ln 12$

**63** What is the value of $\displaystyle\int_{2}^{6} \dfrac{1}{x+2}dx$?

(A) $\ln 2$

(B) $\ln 4$

(C) $\ln 6$

(D) $\ln 8$

**64** What is the value of $\displaystyle\int_{0}^{1} (e^{2x}+1)dx$?

(A) $\dfrac{1}{2}e^2$

(B) $\dfrac{1}{2}(e^2+1)$

(C) $e^2$

(D) $e^2+1$

# Applications of calculus to the physical world <u>Solutions</u>    <u>Main Menu</u>

65  Ten kilograms of chlorine is placed in water and begins to dissolve. After $t$ hours the
amount $A$ kg of undissolved chlorine is given by $A = 10e^{-kt}$. What is the value of $k$ given
that $A = 3.6$ and $t = 5$?

(A)  −0.717

(B)  −0.204

(C)  0.204

(D)  0.717

66  The population of a colony of bugs is increasing continuously at a rate proportional to
the existing population. The present population is 20 000 and the population 3 months
ago was 8000. What is the value of $k$?

(A)  −0.916

(B)  −0.305

(C)  0.305

(D)  0.916

67  The population of a town is falling at a constant rate, so that after 25 years the
population will have halved, and $\dfrac{dP}{dt} = -kP$, where $P$ is the population of the town and $t$
is the time in years. What is the value of $k$?

(A)  $\dfrac{\ln 0.5}{-25}$

(B)  $\dfrac{\ln 0.5}{25}$

(C)  $\dfrac{\ln 2}{-25}$

(D)  $\dfrac{\ln 2}{25}$

68  It is assumed that the number $N(t)$ of ants in a certain nest at time $t \geq 0$ is given by
$N(t) = \dfrac{A}{1 + e^{-t}}$ where $A$ is a constant and $t$ is measured in months. At time $t = 0$, $N(t)$ is
estimated at $2 \times 10^5$ ants. What is the value of $A$?

(A)  $2 \times 10^5$

(B)  $2 \times 10^{-5}$

(C)  $4 \times 10^5$

(D)  $4 \times 10^{-5}$

| | **Logarithmic and exponential functions** | |
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| | **Solution** | **Criteria** |
| 50 | $4^x = 32$ <br> $(2^2)^x = 2^5$ <br> $2x = 5x$ <br> $x = 2.5$ | 1 Mark: B |
| 51 | $\log_e(\frac{x+2}{x}) = \log_e 4$ <br> $(\frac{x+2}{x}) = 4$ <br> $x + 2 = 4x$ <br> $3x = 2$ <br> $x = \frac{2}{3}$ | 1 Mark: B |
| 52 | $y = \ln(x^2 - 4)$ <br> $\frac{dy}{dx} = \frac{1}{x^2 - 4} \times 2x = \frac{2x}{x^2 - 4}$ | 1 Mark: D |
| 53 | $(e^{3x} - 1)(e^{3x} - 6) = 0$ <br> $e^{3x} = 1, 6$ <br> $3x = 0$ or $3x = \ln 6$ <br> $x = 0$ or $x = \frac{\ln 6}{3}$ | 1 Mark: C |
| 54 | $\frac{d}{dx}(1 + \log_e x)^4 = 4(1 + \log_e x)^3 \times \frac{1}{x}$ <br> $= \frac{4(1 + \log_e x)^3}{x}$ | 1 Mark: C |
| 55 | $\frac{d}{dx}\log_2 x = \frac{d}{dx}\left(\frac{\ln x}{\ln 2}\right)$ <br> $= \frac{1}{\ln 2} \times \frac{1}{x}$ <br> $= \frac{1}{x \ln 2}$ | 1 Mark: D |
| 56 | $\frac{d}{dx}(x^2 e^x) = 2xe^x + x^2 e^x$ <br> $= xe^x(2 + x)$ | 1 Mark: D |

| | | |
|---|---|---|
| 57 | $\frac{d}{dx}2e^x \cos x = 2e^x(-\sin x) + \cos x\, 2e^x$ <br> $= 2e^x(\cos x - \sin x)$ | 1 Mark: D |
| 58 | $\frac{d}{dx}\left(\frac{e^x}{x^2}\right) = \frac{x^2 e^x - e^x 2x}{(x^2)^2}$ <br> $= \frac{e^x(x - 2)}{x^3}$ | 1 Mark: C |
| 59 | $\frac{d}{dx}e^{\sin^2 x} = e^{\sin^2 x} \times \frac{d}{dx}(\sin^2 x)$ <br> $= e^{\sin^2 x} \times 2\sin x \cos x$ <br> $= 2e^{\sin^2 x}\sin x \cos x$ | 1 Mark: B |
| 60 | $\frac{d}{dx}\log_e(\cos x) = \frac{1}{\cos x} \times (-\sin x)$ <br> $= -\tan x$ | 1 Mark: B |
| 61 | $\frac{d}{dx}\log_e\left(\frac{x+1}{x-1}\right) = \frac{(x-1)}{(x+1)} \times \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$ <br> $= \frac{x - 1 - x - 1}{(x+1)(x-1)}$ <br> $= \frac{-2}{x^2 - 1}$ | 1 Mark: C |
| 62 | $\int_{-4}^{1}\frac{2}{x+5}dx = 2\left[\ln(x+5)\right]_{-4}^{1}$ <br> $= 2(\ln 6 - \ln 1)$ <br> $= 2\ln 6$ | 1 Mark: B |
| 63 | $\int_{2}^{6}\frac{1}{x+2}dx = \left[\ln(x+2)\right]_{2}^{6}$ <br> $= (\ln 8) - (\ln 2)$ <br> $= \ln 2$ | 1 Mark: A |
| 64 | $\int_{0}^{1}(e^{2x} + 1)dx = \left[\frac{1}{2}e^{2x} + x\right]_{0}^{1}$ <br> $= (\frac{1}{2}e^2 + 1) - (\frac{1}{2})$ <br> $= \frac{1}{2}(e^2 + 1)$ | 1 Mark: B |