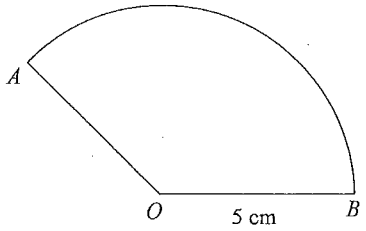
Trigonometric functions

Solutions Main Menu

32 AOB is a sector of a circle, centre O and radius 5 cm. The sector has an area of 10π.

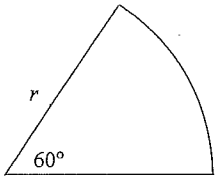


Not to scale

What is the arc length of the sector?

(A) 2π
(B) 4π
(C) 6π
(D) 10π

33 The sector below has an area of 10π square units.

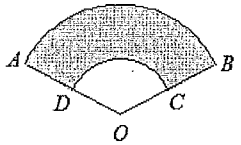


Not to scale

What is the value of r?

(A) $\sqrt{60}$
(B) $\sqrt{60}\pi$
(C) $\sqrt{\frac{\pi}{3}}$
(D) $\sqrt{\frac{1}{3}}$

34 A car windscreen wiper traces out the area $ABCD$ where AB and CD are arcs of circles with a centre O and radii 40 cm and 20 cm respectively. Angle AOB measures 120°.



Not to scale

What is the area of $ABCD$?

(A) 419 cm^2
(B) 1257 cm^2
(C) 1676 cm^2
(D) 2095 cm^2

35 What is the greatest value of the function $y = 4 - 2\cos x$?

(A) 2
(B) 4
(C) 6
(D) 8

36 What is the solution to the equation $\cos 2x = \frac{1}{2}$ in the domain $-\pi \le x \le \pi$?

(A) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-5\pi}{6}, \frac{-\pi}{6}$
(B) $x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{-11\pi}{12}, \frac{-\pi}{12}$
(C) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
(D) $x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

37 What is the solution to the equation $(\sin x + 2)(2\sin x + 1) = 0$ in the domain $0 \le x \le 2\pi$?

(A) $x = \frac{\pi}{3}, \frac{2\pi}{3}$
(B) $x = \frac{4\pi}{3}, \frac{5\pi}{3}$
(C) $x = \frac{\pi}{6}, \frac{5\pi}{6}$
(D) $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

38 What is the solution to the equation $\frac{\cos^3\theta}{\sin\theta}+\sin\theta\cos\theta=1$ in the domain $0\le x\le 2\pi$?

(A) $\theta=\frac{\pi}{4},\frac{3\pi}{4}$

(B) $\theta=\frac{\pi}{4},\frac{5\pi}{4}$

(C) $\theta=\frac{5\pi}{4},\frac{7\pi}{4}$

(D) $\theta=\frac{3\pi}{4},\frac{5\pi}{4}$

39 What is the derivative of $\sin x+x^2$ with respect to x?

(A) $-\cos x+2x$

(B) $\cos x+2x$

(C) $-\cos x+\frac{x^3}{3}$

(D) $\cos x+\frac{x^3}{3}$

40 What is the derivative of $\cos(x^2+1)$ with respect to x?

(A) $\cos(x^2-1)$

(B) $-2x\cos(x^2-1)$

(C) $\sin(x^2-1)$

(D) $-2x\sin(x^2-1)$

41 What is the derivative of $x\cos x$ with respect to x?

(A) $-\sin x$

(B) $-x\sin x$

(C) $x\sin x-\cos x$

(D) $-x\sin x+\cos x$

42 What is the derivative of $\frac{\tan x}{x}$ with respect to x?

(A) $\frac{-\sec^2 x}{x^2}$

(B) $\frac{\sec^2 x}{x^2}$

(C) $\frac{x\sec^2 x-\tan x}{x^2}$

(D) $\frac{\tan x-x\sec^2 x}{x^2}$

43 What is correct expression for the integral $\int\cos\frac{x}{3}dx$?

(A) $-3\sin\frac{x}{3}+c$

(B) $-3\cos\frac{x}{3}+c$

(C) $3\sin\frac{x}{3}+c$

(D) $3\cos\frac{x}{3}+c$

44 What is correct expression for the integral $\int\sin 4x dx$?

(A) $-4\cos 4x+c$

(B) $\frac{-\cos 4x}{4}+c$

(C) $4\cos 4x+c$

(D) $\frac{\cos 4x}{4}+c$

45 What is correct expression for the integral $\int\sec^2 3x dx$?

(A) $\frac{\tan x}{3}+c$

(B) $\frac{\tan 3x}{3}+c$

(C) $\tan x+c$

(D) $\tan 3x+c$

46 What is correct expression for the integral $\int(\sin x \cos x)\,dx$?

(A) $-\frac{1}{2}\cos^2 x + c$

(B) $-\frac{1}{2}\sin^2 x + c$

(C) $\frac{1}{2}\cos^2 x + c$

(D) $\frac{1}{2}\sin^2 x + c$

47 What is the value of the integral $\int_0^{\frac{\pi}{2}}(1-\cos 2x - \sin 2x)dx$?

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}-1$

(C) $\frac{\pi}{2}-1$

(D) 1

48 What is the value of $\int_0^{\frac{\pi}{4}}(\sec^2 x - x)dx$?

(A) $1-\frac{\pi^2}{32}$

(B) $1-\frac{\pi^2}{16}$

(C) $1-\frac{\pi}{8}$

(D) $1-\frac{\pi}{4}$

49 What is the value of $\int_0^1 \cos(\pi x - \frac{\pi}{3})dx$?

(A) $\frac{\sqrt{3}}{\pi}$

(B) $\frac{2}{\pi}$

(C) $\sqrt{3}$

(D) 2

Trigonometric functions		Main Menu
	Solution	**Criteria**
32	Arc length is given by $l = r\theta$ $10\pi = \frac{1}{2} \times r^2 \times \theta$ $r^2\theta = 20\pi$ $r\theta = 4\pi$ (radius is 5)	1 Mark: B
33	$10\pi = \frac{1}{2} \times r^2 \times \frac{\pi}{3}$ $r^2 = \frac{60\pi}{\pi}$ $r = \sqrt{60}$	1 Mark: A
34	Area of sector $ABO = \frac{1}{2} \times 40^2 \times \frac{2\pi}{3}$ $= \frac{1600\pi}{3}$ Area of sector $CDO = \frac{1}{2} \times 20^2 \times \frac{2\pi}{3}$ $= \frac{400\pi}{3}$ Area of $ABCD = \frac{1600\pi}{3} - \frac{400\pi}{3}$ $= 400\pi$ ≈ 1257 cm^2	1 Mark: B
35	Range of values for $y = \cos x$ is $-1 \le y \le 1$ Greatest value for $4 - 2\cos x$ occurs when $\cos x = -1$ Therefore the greatest value is 6 ($4 - 2\cos x = 4 - 2 \times -1 = 6$)	1 Mark: C
36	$\cos 2x = \frac{1}{2}$ $2x = \frac{\pi}{3}$ or $x = \frac{\pi}{6}$ In domain $-\pi \le x \le \pi$ the solution is $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-5\pi}{6}, \frac{-\pi}{6}$	1 Mark: A
37	$2\sin x + 1 = 0$ $\quad\quad$ $\sin x + 2 = 0$ $\sin x = -\frac{1}{2}$ or $x = \frac{\pi}{6}$ $\quad\quad$ $\sin x = -2$ $\quad\quad$ No solution In domain $-\pi \le x \le \pi$ the solution is $x = \frac{7\pi}{6}, \frac{11\pi}{6}$	1 Mark: D

38	$\frac{\cos^3\theta}{\sin\theta} + \sin\theta\cos\theta = \frac{\cos\theta}{\sin\theta}\left(\cos^2\theta + \sin^2\theta\right)$ $= \cot\theta$ $\cot\theta = 1$ $\theta = \frac{\pi}{4}$ or $\theta = \frac{5\pi}{4}$	1 Mark: B
39	$\frac{d}{dx}(\sin x + x^2) = \cos x + 2x$	1 Mark: B
40	$\frac{d}{dx}\cos(x^2 - 1) = -2x\sin(x^2 - 1)$	1 Mark: D
41	$\frac{d}{dx}x\cos x = -x\sin x + \cos x$	1 Mark: D
42	$\frac{d}{dx}\left(\frac{\tan x}{x}\right) = \frac{x\sec^2 x - \tan x \times 1}{x^2}$ $= \frac{x\sec^2 x - \tan x}{x^2}$	1 Mark: C
43	$\int \cos\frac{x}{3}dx = 3\sin\frac{x}{3} + c$	1 Mark: C
44	$\int \sin 4x dx = -\frac{\cos 4x}{4} + c$	1 Mark: B
45	$\int \sec^2 3x dx = \frac{\tan 3x}{3} + c$	1 Mark: B
46	$\int \sin x \cos x dx = -\frac{1}{2}\cos^2 x + c$	1 Mark: A
47	$\int_0^{\frac{\pi}{2}} (1 - \cos 2x - \sin 2x)dx$ $= \left[x - \frac{1}{2}\sin 2x + \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$ $= \left(\frac{\pi}{2} - \frac{1}{2}\sin 2 \times \frac{\pi}{2} + \frac{1}{2}\cos 2 \times \frac{\pi}{2}\right) - \left(0 - \frac{1}{2}\sin 2 \times 0 + \frac{1}{2}\cos 2 \times 0\right)$ $= \left(\frac{\pi}{2} - \frac{1}{2}\right) - \left(\frac{1}{2}\right)$ $= \frac{\pi}{2} - 1$	1 Mark: C

48	$\int_0^{\frac{\pi}{4}}(\sec^2 x - x)dx = \left[\tan x - \frac{x^2}{2}\right]_0^{\frac{\pi}{4}}$ $= \left(\tan\frac{\pi}{4} - \frac{\left(\frac{\pi}{4}\right)^2}{2}\right) - \left(\tan 0 - \frac{0^2}{2}\right)$ $= 1 - \frac{\pi^2}{32}$	1 Mark: A
49	$\int_0^1 \cos(\pi x - \frac{\pi}{3})dx = \left[\frac{\sin(\pi x - \frac{\pi}{3})}{\pi}\right]_0^1$ $= \frac{\sin(x - \frac{\pi}{3})}{\pi} - \frac{\sin(-\frac{\pi}{3})}{\pi}$ $= \frac{\sin\frac{\pi}{3} + \sin\frac{\pi}{3}}{\pi}$ $= \frac{2\frac{\sqrt{3}}{2}}{\pi} = \frac{\sqrt{3}}{\pi}$	1 Mark: A