Student Number___________

MORIAH COLLEGE

Year 12 – Task 2 - Pre-Trial

MATHEMATICS 2011

Time Allowed: **3 hours**

Examiners: O. Golan, G. Busuttil

OUTCOMES ADDRESSED: P3,P5,H2,H4,H5,H6,H7,H8

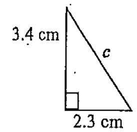
General Instructions

- Reading time: 5 minutes
- Working time: 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in each question
- There are 10 questions in this examination paper.
- Questions are printed on both sides of the paper.
- **All questions are of equal value. Total marks: 120**

Marks

Question 1 – (Start a new page)

(a) Evaluate c, the hypotenuse of the following right-angle triangle, correct to three significant figures: 2



(b) Graph the solution of: $|x+3|<9$ 2

(c) Simplify: $\dfrac{3x^2-27}{2x+6}$ 2

(d) Solve: $\dfrac{3x-1}{2}-\dfrac{x+2}{5}=3$ 2

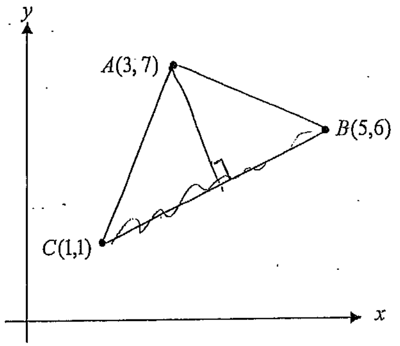
(e) Find a and b such that: $(3+\sqrt{2})^2=a+b\sqrt{2}$ 2

(f) The surface area, A, of a sphere is given by: $A=4\pi R^2$ where R is the radius of the sphere. If a sphere has a surface area of 120cm^2, find the radius correct to two decimal places. 2

Marks

Question 2 – (Start a new page)

(a)



NOT TO SCALE

The triangle above has vertices A, B and C whose coordinates are: $(3,7)$, $(5,6)$ and $(1,1)$ respectively.

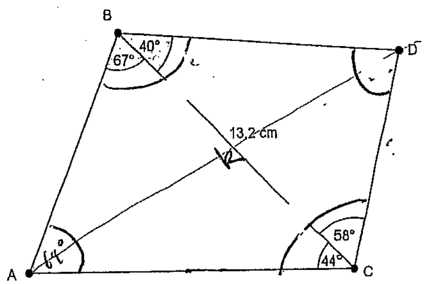
(i) Show that the equation of the line BC is $5x-4y=1$ **2**

(ii) Calculate the length of the segment BC in exact form **2**

(iii) Find the perpendicular distance of the point A from the line BC **2**

(iv) Find the area of the triangle ABC **2**

(b) In the quadrilateral $ABDC$
$BC=13.2\,cm$ $\angle ABC=67^\circ$; $\angle BCA=44^\circ$; $\angle DBC=40^\circ$; $\angle BCD=58^\circ$



(i) Using the sine rule, find the length of sides AC and DC correct to two decimal places **2**

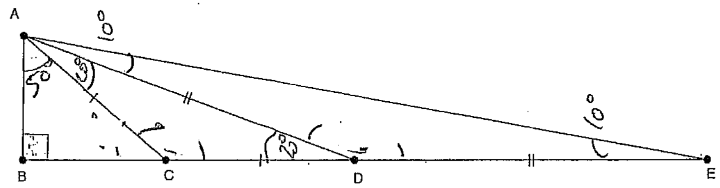
(ii) Using the cosine rule, find the distance between A and D correct to two decimal places **2**

Marks

Question 3 – (Start a new page)

(a) Calculate the angle between the line $3y-5x=17$ and the positive direction of the x-axis, correct to the nearest degree. **2**

(b)



2

NOT TO SCALE

Triangle ABC is right angled at B
$\angle BAC=50^\circ$, $AC=CD$, $AD=DE$
Find the size of $\angle AEB$. Give reasons.

(c) Find the centre and radius of the circle with equation: $x^2+y^2=4x-2y$ **3**

(d) Solve the inequality: $(x-2)(x+3)<6$ **3**

(e) Show that if: $\ln(a+1)-\ln(a-1)=\frac{1}{2}$ then $a=\frac{\sqrt{e}+1}{\sqrt{e}-1}$ **2**

Marks

Question 4 – (Start a new page)

(a) The roots of the quadratic equation: $2x^2 + 4x + 1 = 0$ are α and β. Without solving this equation, find:

(i) $\alpha\beta$ — 1

(ii) $\alpha + \beta$ — 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ — 2

(b) The recurring decimal $0.7\dot{5}$ can be written as $(0.7 + S)$ where S is the limiting sum of a geometric progression.

(i) State the first term and the common ratio of this geometric progression — 2

(ii) Find the limiting sum of this geometric progression and, hence, write $0.7\dot{5}$ as a simplified fraction — 2

(c) A parabola has a vertex at (0, 3) and a focus at (-2,3)

(i) Draw a sketch of the parabola, clearly indicating this information. — 1

(ii) Find the equation of this parabola. — 2

(iii) Add the directrix of this parabola to your sketch and state its equation — 1

Marks

Question 5 – (Start a new page)

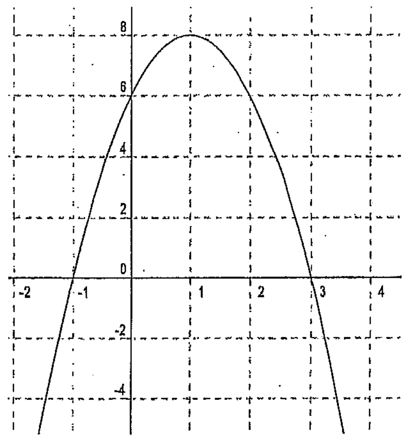
(a) Nathan decides to start running every morning. He runs 1.5 km on the first day and then every day after that he runs 0.5 km further.

(i) How far did he run on the seventeenth day? — 1

(ii) On what day did he run exactly 16 km? — 2

(iii) Each day after his run, Nathan records the total number of kilometers he has run so far. After how many days will his accumulated distance be 161 km? — 2

(b)



The parabola above is drawn to scale.

(i) State the coordinates of the points where the parabola crosses the x axis and the y axis. — 2

(ii) Hence, or otherwise, find the equation of the parabola in the form $y = ax^2 + bx + c$ — 2

(c) By using the substitution $K = x^2 + 2x$, solve the equation: — 3

$$(x^2 + 2x)(x^2 + 2x - 1) = 6$$

Marks

Question 6 – (Start a new page)

(a) Given the curve $y = \frac{1}{x-2}$ and the line $y = -x + c$

(i) Show that the x-coordinate at the point of the point of intersection between the curve and the line must satisfy: $x^2 - (c+2)x + 2c + 1 = 0$ **2**

(ii) Find the two values of c for which the line touches the curve at one point. **2**

(b) Evaluate:

(i) $\lim_{x \to 4} \frac{x^3 - 64}{2x - 8}$ **2**

(ii) $\lim_{h \to 0} \frac{3(x+h)^2 + 2(x+h) - (3x^2 + 2x)}{h}$ **3**

(c) Find all the values of x in the interval $0^\circ \le x \le 360^\circ$ for which $2\cos x = -\sqrt{3}$ **3**

Marks

Question 7 – (Start a new page)

(a) Differentiate:

(i) $y = \frac{2}{x} + 3x^2 - 1$ **2**

(ii) $y = \ln(e^x + 3)$ **2**

(b) Find:

(i) $\int \left(\frac{x^2 + 2x - 4}{x}\right) dx$ **2**

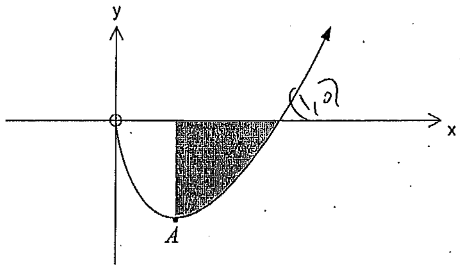
(ii) $\int_0^1 \frac{dx}{\sqrt{3x+1}}$ **3**

(c) Find the equation of the normal to the curve $y = x^4 + x - 1$ at the point where $x=1$ **3**

Marks

Question 8 – (Start a new page)

(a)



The curve above is the graph of $y = x\ln x$

(i) Show that the coordinates of A, the stationary point of the curve, are $(\frac{1}{e}, -\frac{1}{e})$ **3**

(ii) Verify that the graph crosses the x-axis at $(1,0)$ **1**

(iii) Consider the function: $y = x^2(2\ln x - 1)$. Show that $\frac{dy}{dx} = 4x\ln x$ **2**

(iv) From A, a perpendicular to the x axis is dropped. Using (iii) or otherwise, show that the shaded area is $\frac{1-3e^{-2}}{4}$ squared units. **3**

(b) The function $y = ax^4 + bx^3$ has a point of inflexion at $(2,16)$. Find the value of a and b **3**

Marks

Question 9 – (Start a new page)

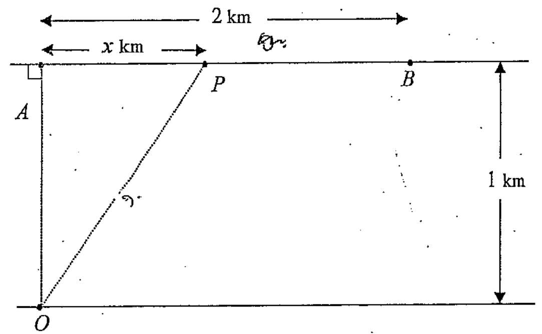
(a) (i) Given that all cubics of the type: $y = ax^3 + bx^2 + cx + d$ have one single point of inflexion, show that the of x coordinate of the point of inflexion of a cubic graph, x_I, is given by the formula: $x_I = -\frac{b}{3a}$ **2**

(ii) What conclusion can you draw about the point of inflexion of the cubic: $y = mx^3 + nx$ **1**

(b) Consider the function given by $y = \frac{8}{2+x^2}$

Using Simpson's rule with 5 function values, estimate the area under the curve from 0 to 4. **3**

(c)



The diagram shows a straight section of a river with parallel riverbanks 1 km wide. Ben is at point O on the bank. He needs to reach point B on the opposite bank. The point A is directly opposite him on the other side of the river and the distance between A and B is 2 kilometres.

Ben can swim at 6km/h and jog at 10km/h. He wants to swim in a straight line to the other side of the river, to a point P (between A and B), and then jog the rest of the way to B. Let the distance from A to P be x.

(i). Show that the time T, in hours that Ben takes to reach B is given by: **2**

$$T = \frac{\sqrt{x^2+1}}{6} + \frac{2-x}{10}$$

(ii) Show that if Ben wishes to minimize the time taken to complete the journey from O to B, then he should swim to a point P, 0.75km from A. **3**

(iii) Find the minimum time it takes Ben to complete his journey, to the nearest minute. **1**

Marks

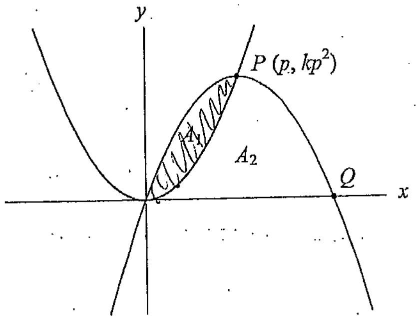
Question 10 – (Start a new page)

(a) The terms $t_1, t_2, t_3, \ldots t_n$ form a geometric progression **2**

(i) Show that the terms: $\ln t_1, \ln t_2, \ln t_3, \ldots \ln t_n$ form an arithmetic progression **2**

(ii) Show that: $t_1 \times t_2 \times, t_3 \times \ldots t_n = (t_1 \times t_n)^{\frac{n}{2}}$

(b)



The parabola above has equation: $y = kx^2$ ($k > 0$) and the point P (p, kp^2) lies on it.
A second parabola has a vertex at P and passes through the origin, as shown.

(i) Show that the equation of the second parabola is $y = -kx^2 + 2kpx$ **2**

(ii) Show that the coordinates of the point Q, the positive x intercept of the parabola $y = -kx^2 + 2kpx$, are: $(2p, 0)$ **1**

(iii) Show that the area bounded by the parabola $y = -kx^2 + 2kpx$ and the x-axis is: $\frac{4}{3}kp^3$ square units. **2**

(iv) The parabola $y = kx^2$ divides the area bounded by the parabola $y = -kx^2 + 2kpx$ and the x-axis into two areas: A_1 and A_2, as shown. **3**

Prove that: $\frac{A_1}{A_2} = \frac{1}{3}$

END

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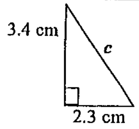
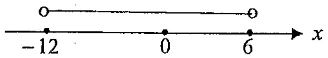
MATHEMATICS
2011

SOLUTIONS

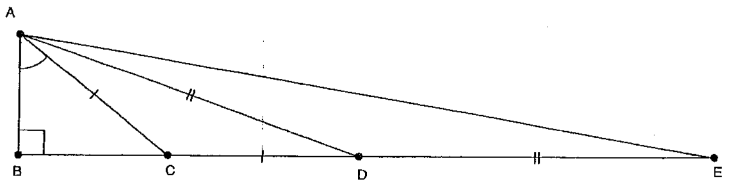
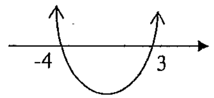
Question 1

(a)	 $c=\sqrt{2.3^2+3.4^2}=4.10cm$	2
(b)	$\lvert x+3\rvert<9$ $-9<x+3<9$ $-12<x<6$  	2
(c)	$\dfrac{3x^2-27}{2x+6}=\dfrac{3(x+3)(x-3)}{2(x+3)}=\dfrac{3(x-3)}{2}$	2
(d)	$\dfrac{3x-1}{2}-\dfrac{x+2}{5}=3 \quad /\times 10$ $5(3x-1)-2(x+2)=30$ $15x-5-2x-4=30$ $13x-9=30$ $13x=39$ $x=3$	2
(e)	$(3+\sqrt{2})^2=a+b\sqrt{2}$ $9+6\sqrt{2}+2=a+b\sqrt{2}$ $11+6\sqrt{2}=a+b\sqrt{2}$ $a=11 \quad b=6$	2
(f)	$A=4\pi R^2$ $120=4\pi R^2$ $\dfrac{30}{\pi}=R^2 \qquad R=\sqrt{\dfrac{30}{\pi}}=3.09cm$	2

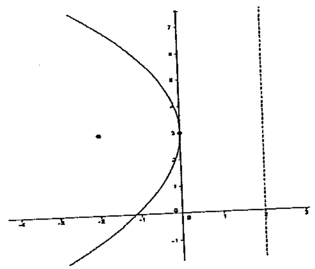
Question 2

(a)		In the triangle ABC above the vertices are at (1,1), (5,6) and (3,7) respectively	
	(i)	$m_{BC}=\dfrac{6-1}{5-1}=\dfrac{5}{4} \qquad y-1=\dfrac{5}{4}(x-1) \qquad 4(y-1)=5(x-1) \qquad 4y-4=5x-5$ $5x-4y=1$	2
	(ii)	$BC=\sqrt{(5-1)^2+(6-1)^2}=\sqrt{16+25}=\sqrt{41}units$	2
	(iii)	$d=\left\lvert\dfrac{5(3)-4(7)-1}{\sqrt{5^2+4^2}}\right\rvert \qquad d=\dfrac{14}{\sqrt{41}}units$	2
	(iv)	Area $ABC=\dfrac{14}{\sqrt{41}}\times\sqrt{41}\times\dfrac{1}{2}=7units^2$	2
(b)	(i)	$AC=\dfrac{13.2\sin 67}{\sin 69}=13.02cm \qquad DC=\dfrac{13.2\sin 40}{\sin 82}=8.57cm$	2
	(ii)	$AD^2=13.2^2+8.57^2-2(13.2)(8.57)\cos 102$ $AD=17.01\text{cm}$	2

Question 3

(a)	Rearrange $3y-5x=17$ $\quad 3y=5x+17$ $y=\frac{5}{3}x+\frac{17}{3}$ $\quad m=\frac{5}{3}$ $\quad \tan\theta=\frac{5}{3}$ $\quad \theta=59^\circ$	2
(b)	 NOT TO SCALE Triangle ABC is right angled at B $\angle ACB=40^\circ$ (complementary) $\angle ADC=20^\circ$ (external angle, base angles equal) $\angle AEB=10^\circ$ (external angle, base angles equal)	2
(c)	$x^2+y^2=4x-2y$ $x^2-4x+y^2+2y=0$ $(x-2)^2-4+(y+1)^2-1=0$ $(x-2)^2+(y+1)^2=5$ Centre at (2,-1) $R=\sqrt{5}$	3
(c)	$(x-2)(x+3)<6$ $x^2+x-6<6$ $x^2+x-12<$ $(x-3)(x+4)<0$ $-4<x<3$ 	3
(d)	$\ln(a+1)-\ln(a-1)=\frac{1}{2}$ $\Rightarrow$ $\ln(\frac{a+1}{a-1})=\frac{1}{2}$ $\Rightarrow$ $\frac{a+1}{a-1}=e^{\frac{1}{2}}=\sqrt{e}$ $a+1=\sqrt{e}(a-1)$ $\Rightarrow$ $a+1=a\sqrt{e}-\sqrt{e}$ $\Rightarrow$ $1+\sqrt{e}=a\sqrt{e}-a$ $1+\sqrt{e}=a(\sqrt{e}-1)$ $\Rightarrow$ $\frac{1+\sqrt{e}}{\sqrt{e}-1}=a$ as required	2

Question 4

(a)	$2x^2+4x+1=0$ are α and β. $a=2$ $\quad b=4$ $\quad c=1$ (i) $\alpha\beta=\frac{c}{a}=\frac{1}{2}$ (ii) $\alpha+\beta=-\frac{b}{a}=-2$ (iii) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-2}{\frac{1}{2}}=-4$	1 1 2
(b)	The recurring decimal $0.7\dot{5}$ can be written as (0.7 +S) where S is the limiting sum of a geometric progression. (i) $0.7\dot{5}=0.7+0.05+0.005+0.0005+\ldots$ $a=0.05$ $\quad r=0.1$ (ii) $S=\frac{0.05}{1-0.1}=\frac{1}{18}$ $0.7\dot{5}=0.7+\frac{1}{18}=\frac{34}{45}$	2 2
(c)	A parabola has a vertex at (0, 3) and a focus at (-2,3) (i) Draw a sketch of the parabola, clearly indicating this information.  (ii) Parabola of type: $-4ax=y^2$ From diagram, focal length a=2 $-8x=(y-3)^2$ (iii) Directrix x=2	1 2 1

Question 5

(a) Arithmetic profession in which: $a = 1.5$ km and $d = 0.5$

(i) $T_{17} = 1.5 + 16(0.5) = 9.5km$

(ii) $16 = 1.5 + (n-1)(0.5)$ **1**

$32 = n + 2$ **2**

$n = 29$ *days*

(iii) $161 = \frac{n}{2}[3 + (n-1)(0.5)]$ **2**

$322 = n[3 + (n-1)(0.5)]$

$644 = n[6 + (n-1)]$

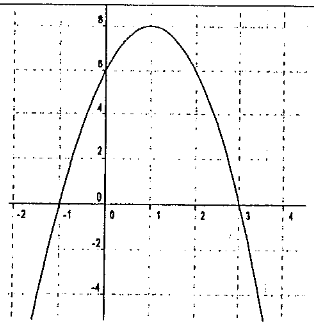
$644 = n[5 + n]$

$0 = n^2 + 5n - 644$

$0 = (n-23)(n+28)$

n=23 days (rule out negative solution)

(b)



(i) x-intercepts: (-1,0), (3,0) **2**

y-intercept: (0,6)

(ii). General equation of parabola with x-intercepts: (-1,0), (3,0) : $y = a(x+1)(x-3)$ **2**

Substitute $x = 0, y = 6$ to find a

$6 = a(0+1)(0-3) \qquad a = -2$

$y = -2(x+1)(x-3)$

$y = -2x^2 + 4x + 6$

(c) $(x^2 + 2x)(x^2 + 2x - 1) = 6$ Let: $K = x^2 + 2x$ **3**

$K(K-1) = 6$

$K^2 - K - 6 = 0$

$(K-3)(K+2) = 0$

K=3 or K=-2

$3 = x^2 + 2x$	$-2 = x^2 + 2x$
$0 = x^2 + 2x - 3$	$0 = x^2 + 2x + 2$
$0 = (x+3)(x-2)$	$0 = (x+1)^2 + 1$
x = -3 or x = 2	No solution

Question 6

(a) To find intersection between line and curve solve:

(i) $\begin{cases} y = -x + c \\ y = \dfrac{1}{x-2} \end{cases} \Rightarrow -x + c = \dfrac{1}{x-2}$ and the line must satisfy: **2**

$\Rightarrow (x-2)(-x+c) = 1$

$\Rightarrow -x^2 + cx + 2x - 2c = 1$

$\Rightarrow x^2 - (c+2)x + 2c + 1 = 0$

(ii) Set $\Delta = b^2 - 4ac = 0 \quad \Rightarrow (c+2)^2 - 4(2c+1) = 0$ **2**

$\Rightarrow c^2 - 4c = 0$

$\Rightarrow c(c-4) = 0$

$\Rightarrow c = 0 \quad or \quad c = 4$

(b) Evaluate:

(i) $\lim_{x \to 4} \frac{x^3 - 4^3}{2x - 8} = \lim_{x \to 4} \frac{(x-4)(x^2 + 4x + 16)}{2(x-4)} = \lim_{x \to 4} \frac{x^2 + 4x + 16}{2} = 24$ **2**

(ii) $\lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 2x + 2h - 3x^2 - 2x}{h}$ **3**

$\lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 3x^2 - 2x}{h}$

$\lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h}$

$\lim_{h \to 0} \frac{h(6x + 3h + 2)}{h}$

$\lim_{h \to 0} (6x + 3h + 2) = 6x + 2$

(c) $2\cos x = -\sqrt{3}$ **3**

$\cos x = -\frac{\sqrt{3}}{2}$

Consider: $\cos\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = 30^\circ$

Cosine is negative in the second and third quadrants:

$\Rightarrow \quad \theta = 150^\circ, 210^\circ$

Question 7

(a)

(i) $y = 2x^{-1} + 3x^2 - 1 \quad \Rightarrow \quad y' = -2x^{-2} + 6x$ **[2]**

(ii) $y = \ln(e^x + 3) \quad \Rightarrow \quad y' = \dfrac{e^x}{e^x + 3}$ **[2]**

(b) Find:

(i) $\int (\frac{x^2 + 2x - 4}{x}) dx \quad \Rightarrow \quad \int (x + 2 - \frac{4}{x}) dx = \frac{x^2}{2} + 2x - 4\ln x + C$ **[2]**

(ii) $\int_0^1 \frac{dx}{\sqrt{3x+1}} \quad \Rightarrow \quad \int_0^1 (3x+1)^{-\frac{1}{2}} dx$

$\Rightarrow \quad \left[2\frac{(3x+1)^{\frac{1}{2}}}{3} \right]_0^1 = \frac{2}{3}\left[\sqrt{3x+1}\right]_0^1 = \frac{2}{3}\left[\sqrt{4} - \sqrt{1}\right] = \frac{2}{3}$ **[3]**

(c)

$y = x^4 + x - 1$

At $x=1, y=1$ The point is: $(1,1)$

$y' = 4x^3 + 1$

At $x=1$, $y' = 5 \quad \Rightarrow \quad m_{normal} = -\frac{1}{5}$

$y - 1 = -\frac{1}{5}(x-1)$

$5y - 5 = -(x-1)$

$x + 5y = 6$ (or any other presentation) **[3]**

Question 8

(a)

i. $y = x\ln x$ Let: $u = x$, $u' = 1$; $v = \ln x$, $v' = \frac{1}{x}$

$y' = 1(\ln x) + x(\frac{1}{x}) = \ln x + 1$ To find stationary point, set $y' = 0$

$0 = \ln x + 1 \quad -1 = \ln x \quad \Rightarrow \quad -1 = \log_e x \quad \Rightarrow \quad e^{-1} = x$

Substitute $x = e^{-1}$ in $y = x\ln x$: $\quad y = e^{-1}\ln(e^{-1}) = -e^{-1} = -\frac{1}{e}$

So A, the stationary point of the curve, is at $(\frac{1}{e}, -\frac{1}{e})$ as required **[3]**

ii. At $x = 1 \quad y = 1\ln(1) = 0 \quad (1,0)$ as required **[1]**

iii. $y = x^2(2\ln x - 1)$ Let: $u = x^2$, $u' = 2x$; $v = 2\ln x - 1$, $v' = \frac{2}{x}$

$\frac{dy}{dx} = 2x(2\ln x - 1) + x^2(\frac{2}{x})$

$\frac{dy}{dx} = 4x\ln x - 2x + 2x$

$\frac{dy}{dx} = 4x\ln x$ **[2]**

iv. $A = \left| \int_{\frac{1}{e}}^{1} x\ln x dx \right| = \frac{1}{4}\int_{1}^{\frac{1}{e}} 4x\ln x dx = \frac{1}{4}\left[x^2(2\ln x - 1)\right]_1^{\frac{1}{e}}$

$A = \frac{1}{4}\left[\frac{1}{e^2}(2\ln e^{-1} - 1) - 1^2(2\ln 1 - 1)\right]$

$A = \frac{1}{4}\left[\frac{1}{e^2}(-2-1) + 1\right] = \frac{1}{4}\left[-\frac{3}{e^2} + 1\right] = \frac{1}{4}\left[1 - 3e^{-2}\right]$

$A = \frac{1 - 3e^{-2}}{4}$ square units. **[3]**

(b) The point satisfies $y = ax^4 + bx^3$ so, substitute: $16 = a(2)^4 + b(2)^3 \quad \Rightarrow \quad 16 = 16a + 8b$

$y' = 4ax^3 + 3bx^2 \quad \Rightarrow \quad y'' = 12ax^2 + 6bx$

At point of inflexion $x=2$ and $y'' = 0 \Rightarrow \quad 0 = 48a + 12b \quad \Rightarrow \quad 0 = 16a + 4b$

Solving simultaneously: $\begin{cases} 16 = 16a + 8b \\ 0 = 16a + 4b \end{cases} \Rightarrow \quad b = 4, \quad a = -1$ **[3]**

Question 9

(a) (i) $y = ax^3 + bx^2 + cx + d \qquad y' = 3ax^2 + 2bx + cx \qquad y'' = 6ax + 2b$

For point of inflexion set $y'' = 0$

$0 = 6ax + 2b \qquad -6ax = 2b$ **2**

$x_i = -\dfrac{b}{3a}$ as required **1**

(ii) $x_i = -\dfrac{0}{3m} = 0$ Therefore, the point of inflexion is at the origin.

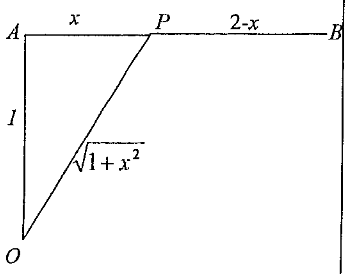
(b) **3**

x	0	1	2	3	4
y	4	$\frac{8}{3}$	$\frac{8}{6}$	$\frac{8}{11}$	$\frac{8}{18}$
	$\times 1$	$\times 4$	$\times 2$	$\times 4$	$\times 1$

$A \approx \dfrac{1}{3}\left[4 + \dfrac{32}{3} + \dfrac{16}{6} + \dfrac{32}{11} + \dfrac{8}{18}\right] = \dfrac{2048}{297}u^2$

(c) Using Pythagoras, $OP = \sqrt{1+x^2}$; $PB = 2-x$

Using distance-speed-time formula $T = \dfrac{D}{S}$



(i) $T = \dfrac{\sqrt{1+x^2}}{6} + \dfrac{2-x}{10}$ **2**

(ii) $T = \dfrac{(1+x^2)^{0.5}}{6} - \dfrac{1}{10}x + \dfrac{1}{5}$

$T' = \dfrac{2x(1+x^2)^{-0.5}}{6\times 2} - \dfrac{1}{10} \qquad T' = \dfrac{x}{6\sqrt{1+x^2}} - \dfrac{1}{10}$ **3**

For stationary point, set $T' = 0 \qquad 0 = \dfrac{x}{6\sqrt{1+x^2}} - \dfrac{1}{10}$

$\dfrac{x}{6\sqrt{1+x^2}} = \dfrac{1}{10} \quad \Rightarrow \quad 10x = 6\sqrt{1+x^2} \quad \Rightarrow \quad 5x = 3\sqrt{1+x^2}$ **1**

$\Rightarrow \quad 25x^2 = 9(1+x^2) \quad \Rightarrow \quad 25x^2 - 9x^2 = 9$

$\Rightarrow \quad 16x^2 = 9 \quad \Rightarrow \quad x^2 = \dfrac{9}{16} \Rightarrow \quad x = \dfrac{3}{4}$

Nature of stationary point:

x	0	0.75	1
T'	-0.1	0	0.017
	↘	→	↗

MIN

(iii) To find minimum time, put $x = 0.75$ in the time formula:

$T = \dfrac{\sqrt{1+0.75^2}}{6} + \dfrac{2-0.75}{10} = \dfrac{1}{3}$ hour, or 20 minutes.

Question 10

(a) (i) Let $t_1, t_2, t_3, \ldots t_n$ be a GP with first term t_1 and common ratio r.

So, $\dfrac{t_n}{t_{n-1}} = r$ (where r is a constant) Now, in the progression: $\ln t_1, \ln t_2, \ln t_3, \ldots \ln t_n$,

If the difference between two consecutive terms is d, then:

$d = \ln t_n - \ln t_{n-1} = \ln(\dfrac{t_n}{t_{n-1}}) = \ln(r)$, i.e. the difference is constant, so the sequence is an AP. **2**

(ii) $t_1 \times t_2 \times t_3 \times \ldots \times t_n = t_1 \times t_1 r \times t_1 r^2 \times \ldots \times t_1 r^{n-1}$

$t_1 \times t_2 \times t_3 \times \ldots \times t_n = (t_1)^n \times r \times r^2 \times \ldots \times r^{n-1}$

$t_1 \times t_2 \times t_3 \times \ldots \times t_n = (t_1)^n \times r^{(0+1+2+\ldots n-1)}$

The index of r is the sum of an AP with a=0 and d=1 where there are n terms.

$0 + 1 + 2 + \ldots (n-1) = \dfrac{n(n-1)}{2}$

$t_1 \times t_2 \times t_3 \times \ldots \times t_n = (t_1)^n \times r^{\frac{n(n-1)}{2})}$ **2**

$t_1 \times t_2 \times t_3 \times \ldots \times t_n = (t_1)^{\frac{2n}{2}} \times r^{\frac{n(n-1)}{2})}$

$t_1 \times t_2 \times t_3 \times \ldots \times t_n = \left[(t_1)^2 \times r^{(n-1)}\right]^{\frac{n}{2}}$

$t_1 \times t_2 \times t_3 \times \ldots \times t_n = \left[(t_1) \times (t_1) r^{(n-1)}\right]^{\frac{n}{2}}$

$t_1 \times t_2 \times t_3 \times \ldots \times t_n = \left[(t_1) \times (t_n)\right]^{\frac{n}{2}}$ as required

(b) (i) General equation of parabola with vertex at (p, kp^2) is: $y = a(x-p)^2 + kp^2$ for some a. **2**

To find the value of a, put (0,0): $\quad 0 = a(0-p)^2 + kp^2$

$0 = ap^2 + kp^2$

$0 = p^2(a+k) \quad \Rightarrow \quad a + k = 0 \quad \Rightarrow \quad a = -k$

Equation of parabola is: $y = -k(x-p)^2 + kp^2 \quad \Rightarrow \quad y = -kx^2 + 2kpx$ as required

(ii) For x intercept put: $0 = -kx^2 + 2kpx \Rightarrow \quad 0 = -kx(x+2p)$ so, solutions are: **1**

$x = 0$ (the origin) or : $x = -2p$ as required.

(iii) $A_1 + A_2 = \displaystyle\int_0^{2p}(-kx^2 + 2kpx)dx = \left[\dfrac{-kx^3}{3} + kpx^2\right]_0^{2p} = \left(\dfrac{-8kp^3}{3} + 4kp^3\right) - 0 = \dfrac{4kp^3}{3}$ **2**

as required.

(iv) $A_1 = \displaystyle\int_0^{p}(-kx^2 + 2kpx - kx^2)dx = \left[\dfrac{-2kx^3}{3} + kpx^2\right]_0^{p} = \left(\dfrac{-2kp^3}{3} + kp^3\right) = \dfrac{kp^3}{3}$ **3**

Subtracting from our finding in (iii), we get $A_2 = kp^3 \qquad \dfrac{A_1}{A_2} = \dfrac{\frac{1}{3}kp^3}{kp^3} = \dfrac{1}{3}$

as required