RANDWICK GIRLS HIGH SCHOOL

YEAR 11 ASSESSMENT TASK 1
Mathematics – Extension 1

Time: 1½ hours **Examiner: D. Posener**

INSTRUCTIONS:

- All questions may be attempted
- Marks may be deducted for careless or badly arranged work
- Show all necessary working
- The mark value of each question is shown
- Approved calculators may be used
- Begin each question on a new page

NAME: _____

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/10
6	/12
TOTAL	/70 /67

Question 1: (12 marks)

(a) Evaluate, correct to 2 significant figures $\frac{1}{91.6-27.02}$ **1**

(b) Factorise $2a^3 - 16$ **2**

(c) Factorise $x^2 - y^2 + 2y - 1$ **2**

(d) Simplify $1 - \frac{m-n}{m+n}$ **2**

(e) Simplify $\frac{4}{x^2-4} - \frac{2}{x^2-2x}$ **2**

(f) Solve $(x-1)^3 = -8$ **1**

(g) Solve $\frac{1}{3^n} < 0.001$ **2**

Question 2: (12 marks)

(a) Simplify $2\sqrt{63} - \sqrt{343}$ **1**

(b) Express $\frac{5\sqrt{2}-4}{3\sqrt{2}-4}$ in its lowest terms, with rational denominator **2**

(c) Solve $|2x+1| \leq 7$ **2**

(d) (i) $\lim_{x\to -1} \frac{x^2-1}{x+1}$ **2**

(ii) $\lim_{x\to \infty} \frac{3-x^2+2x}{5-3x+4x^2}$

(e) State the domain and range of $f(x) = \frac{-1}{\sqrt{4-x^2}}$ **2**

(f) Make neat sketches of the following regions: **3**

(i) $4 < x^2 + y^2 \leq 9$

(ii) $y \geq 4x^{-1}$

Question 3: **(12 marks)**

(a) Simplify $\frac{9^x-1}{3^x-1}$ **2**

(b) Given $f(x) = 3x - 4$, find $g(x)$ if $f(g(x)) = x$ **2**

(c) A function is defined by the rule:

$$f(x) = \begin{cases} 1-x^2 & \text{for } -2 \le x \le 0 \\ 2^x & \text{for } 0 < x < 2 \\ \frac{1}{2x} & \text{for } 2 \le x \le 4 \end{cases}$$

(i) Find the value of $f(-2) + f(1) + f(3)$ **1**

(ii) Sketch the function for $-2 \le x \le 4$, indicating the main features. **3**

(d) Sketch

(i) $y = \left|\frac{1}{x}\right|$ **2**

(ii) $y = log_{10}(x-1)$ **2**

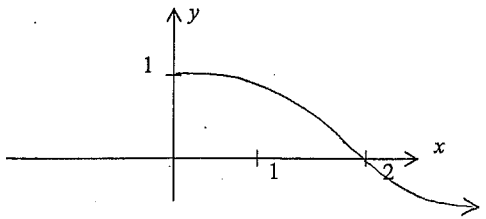
Question 4: **(12 marks)**

(a) Solve $\frac{4x-3}{2x+1} \le 3$ **3**

(b) Solve $\frac{3}{1+x} \le \frac{1}{1-x}$ **4**

(c) Find the centre and radius of the circle whose equation is $x^2 + y^2 + 6y = 0$ **3**

(d) Part of the graph of the function $y = f(x)$ is shown below **2**



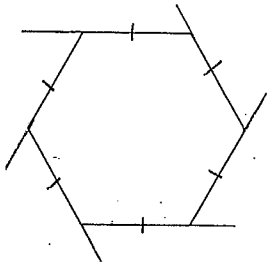
Draw two neat copies of this graph and label them (a) and (b)

Complete the graphs of $y = f(x)$ on each sketch so that:

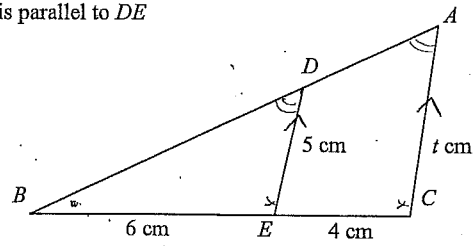
(i) In (a) $y = f(x)$ is an even function
(ii) in (b) $y = f(x)$ is an odd function

Question 5: **(10 marks)**

(a) The diagram below shows the molecular structure of benzene, which is in the shape of a regular hexagon. Find the exterior angle of such a figure. **2**



(b) AC is parallel to DE



(i) Prove $\Delta ABC \,|||\, \Delta DBE$ **3**

(ii) Find the value of t. **2**

Question 6: **(12 marks)**

(a) Sketch $y = \sqrt{2-x}$ **2**

(b) Sketch $y = 2 + \frac{1}{x-1}$ **2**

(c) Solve simultaneously $x + y - 2 = 0$ and $xy = 1$ **2**

(d) Sketch $y = \frac{1-x^2}{1+x^2}$ **2**

(e) Sketch $y = \frac{x+4}{x(x+8)}$ **4**

RGHS - Yr11 - Ext1 Task1 - Solutions

Question 1

a) $\frac{91.6 - 27.02}{1000} = 0.0646$

$= 0.06$ (to 2 sig. fig.) ✗

b) $2(a^3 - 8)$

$2(a-2)(a^2+2a+4)$ ✓

c) $x^2 - (y-1)^2$

$= (x-(y-1))(x+(y-1)) = (x-y+1)(x+y-1)$ ✓

d) $1 - \frac{m-n}{m+n}$

$\frac{m+n-(m-n)}{m+n} = \frac{m+n-m+n}{m+n}$

$= \frac{2n}{m+n}$ ✓

e) $\frac{4}{(x-2)(x+2)} - \frac{2}{x(x-2)}$

$= \frac{4x - 2(x+2)}{x(x+2)(x-2)}$

$= \frac{4x - 2x - 4}{x(x+2)(x-2)}$

$= \frac{2x-4}{x(x+2)(x-2)} = \frac{2(x-2)}{x(x+2)(x-2)}$

$= \frac{2}{x(x+2)}$ ✓

f) $(x-1)^3 = -8$

$x - 1 = -2$

$x = -1$ ✓

g) $\frac{1}{3^n} < 0.001$ $\quad 3^n > 1000$ $\quad \frac{1}{3^n} < 0.001$ $\quad \log 3^n > \log 1000$

$3^{-n} < 0.001$ $\quad \frac{1}{3^n} < \frac{1}{1000}$ $\quad n \log 3 > \log 1000$

$\frac{\log 0.001}{\log 3}$ $\quad 3^n > 1000$ $\quad n > \frac{\log 1000}{\log 3}$

$= -6.288\ldots$ ✗ $\quad n > 6.29$

$\therefore n > -6$

Question 2.

a) $2\sqrt{63} - \sqrt{343}$

$6\sqrt{7} - 7\sqrt{7}$ ✓

$= -\sqrt{7}$

b) $\frac{5\sqrt{2}-4}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4}$

$\frac{30 + 20\sqrt{2} - 12\sqrt{2} - 16}{18 - 16}$

$= \frac{14 + 8\sqrt{2}}{2}$ ✓

$= 7 + 4\sqrt{2}$ ✓

c) $|2x+1| \leq 7$

$2x+1 \leq 7 \qquad 2x+1 \geq -7$

$2x \leq 6 \qquad 2x \geq -8$

$x \leq 3 \qquad x \geq -4$ ✓

$\therefore -4 \leq x \leq 3$

d) (i) $\lim_{x \to -1} \frac{x^2-1}{x+1}$

$x \to -1 \quad \frac{(x-1)(x+1)}{(x+1)}$

$x \to -1 \quad x - 1$

$-1 - 1$ ✓

$= -2$ ✓

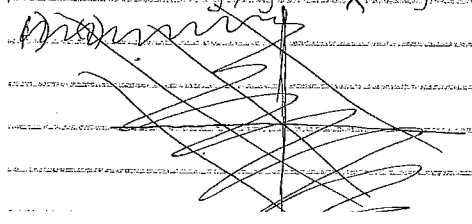
(ii) $\lim_{x \to \infty} \frac{3 - x^2 + 2x}{5 - 3x + 4x^2}$

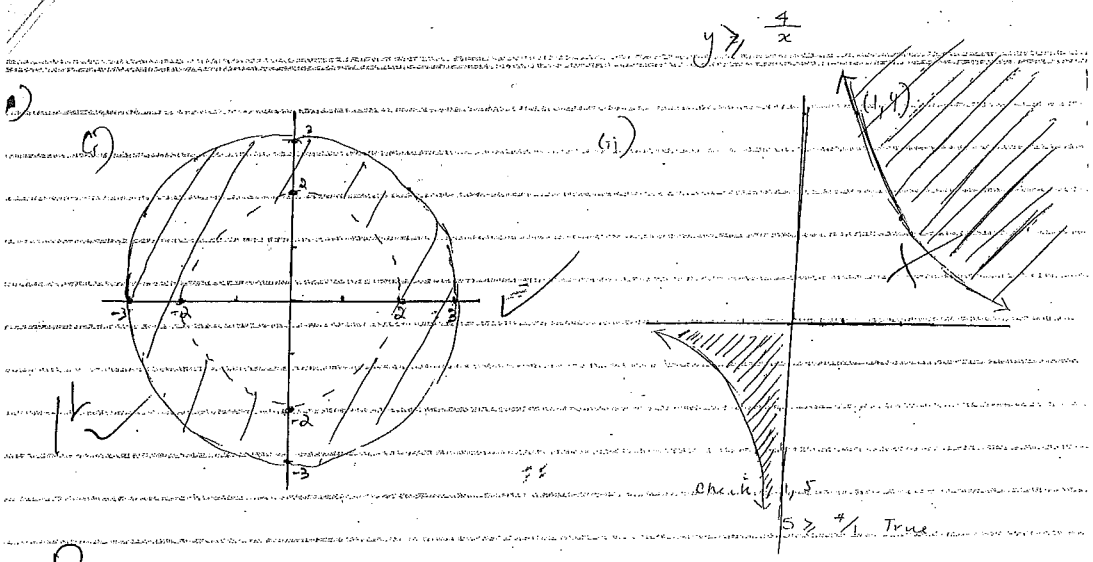
$\frac{\frac{3}{x^2} - \frac{x^2}{x^2} + \frac{2x}{x^2}}{\frac{5}{x^2} - \frac{3x}{x^2} + \frac{4x^2}{x^2}} = \frac{0 - 1 + 0}{0 - 0 + 4}$

$= -1/4$ ✓

e) D: $-2 \leq x \leq 2$ ✓

R: all real y, $y \neq 0$. ✗ $y < 0$





Question 3

a) $\frac{9^x - 1}{3^x - 1} = \frac{(3^x)^2 - 1}{3^x - 1} = \frac{(3^x + 1)(3^x - 1)}{(3^x - 1)}$

$\frac{3^{2x} - 1}{3^x - 1} = \frac{3^x - 1}{3^x - 1} = 3^x + 1$

$= -3^x + 1$

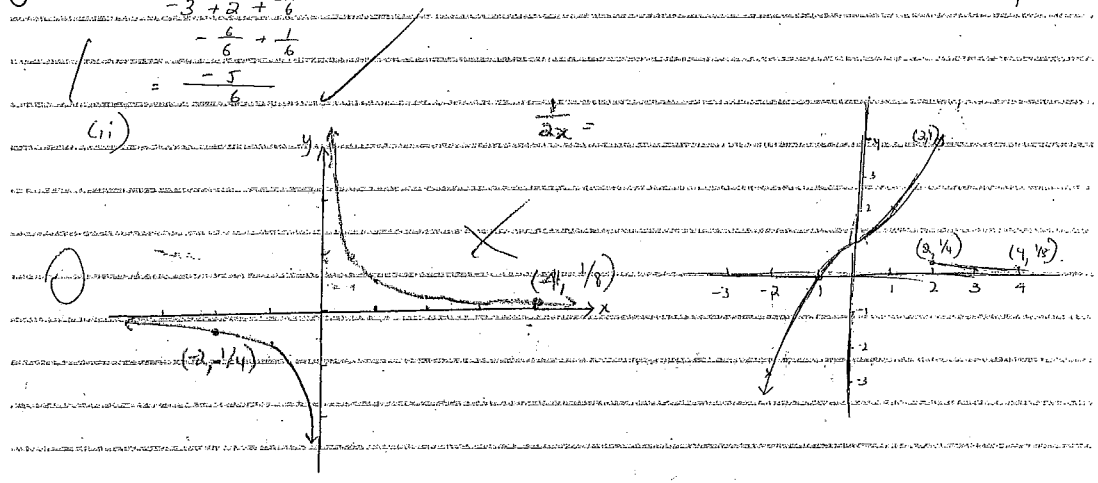
b) (i) $(1 - 4) + 2' + \frac{1}{6}$

$-3 + 2 + \frac{1}{6}$

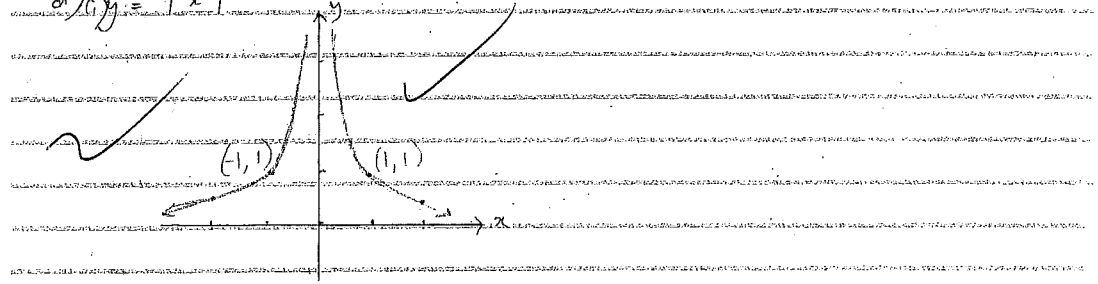
$-\frac{6}{6} + \frac{1}{6}$

$= \frac{-5}{6}$

(ii)

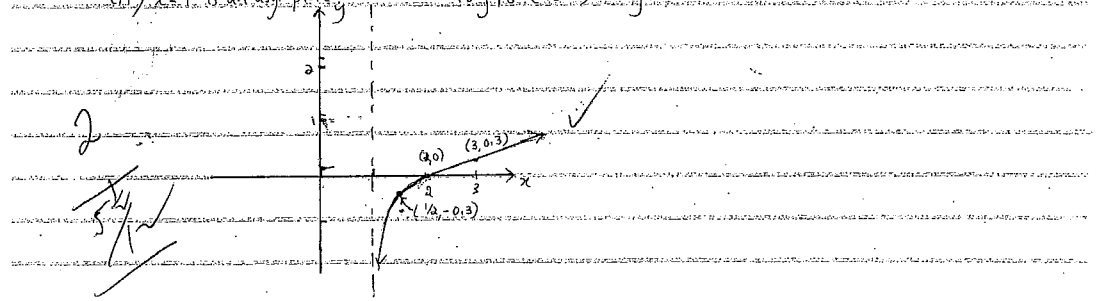


c) (i) $y = \left|\frac{1}{x}\right|$



(ii) $x = 1$ is an asymptote

$\log_{10}(x - 1) = y$



$f(x) = 3x - 4$

$f(g(x)) = 3g(x) - 4 = x$

$3g(x) = x + 4$

$g(x) = \frac{x + 4}{3}$

Question 4

a) $(4x-3)(2x+1) \leq 3(2x+1)^2$ $\quad x \neq -\frac{1}{2}$.

$(4x-3)(2x+1) - 3(2x+1)^2 \leq 0$

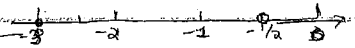
$(2x+1)(4x-3-3(2x+1)) \leq 0$

$(2x+1)(4x-3-6x-3) \leq 0$

$(2x+1)(-2x-6) \leq 0$

$-\frac{1}{2}$ $\quad -3$

Test 0. $\quad -3 \leq 3$ TRUE



$x \leq -3$ or $x \geq -\frac{1}{2}$ ✓

b) $\frac{3}{1+x} \leq \frac{1}{1-x}$

Test 0. $\quad 3 \leq 1$ FALSE

Test 2 $\quad 1 \leq -1$

$3(1+x) \leq \frac{(1+x)^2}{1-x}$ $\quad x \neq \pm 1$

$3(1-x^2)(1-x)^2 \leq (1+x)^2(1-x)$

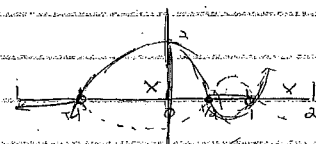
$3(1+x)(1-x)^2 - (1+x)^2(1-x) \leq 0$

$(1+x)(1-x)[3(1-x) - (1+x)] \leq 0$

$(1+x)(1-x)(3-3x-1-x) \leq 0$

$(1+x)(1-x)(2-4x) \leq 0$

Test 0. $\quad 1 \times 1 \times 2 = 2$



$\therefore \quad x \leq -1$ and $\frac{1}{2} \leq x \leq 1$ ✓

c) $x^2 + y^2 + 6y = 0$

$x^2 + y^2 + 6y + 9 = 9$

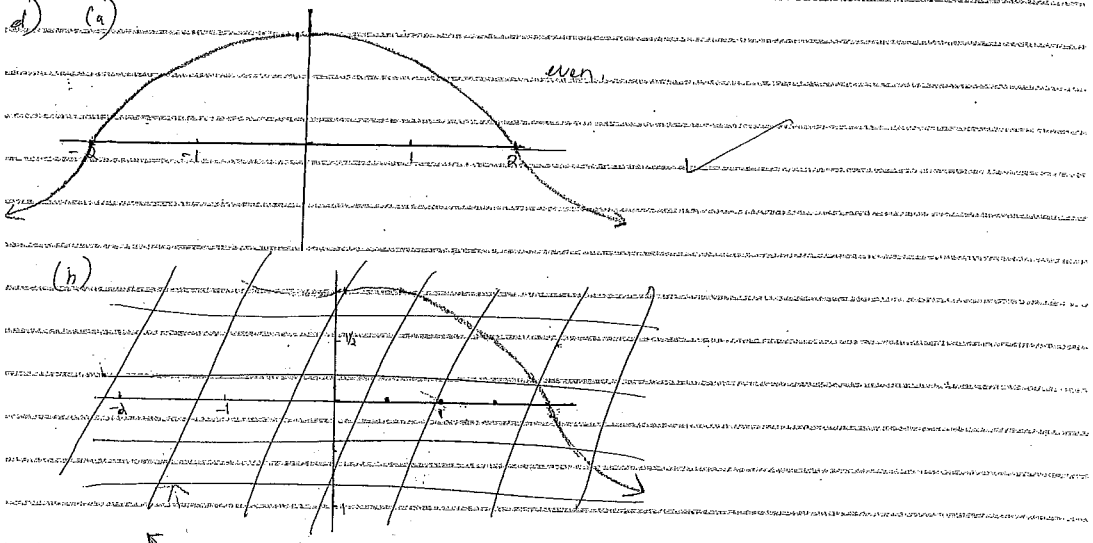
$x^2 + (y+3)^2 = 9$

Centre $= (0, -3)$ ✓

Radius $= 3$ ✓

Question 4 Contd

d) (a)



(b)

Question 5

a) $\frac{360}{n} = \frac{360}{6}$

ext $\angle = 60°$ ✓

b) (i) Consider $\triangle ABC$ and $\triangle DBE$

1) $\angle B$ is common ✓

2) $\angle BED = \angle BCA$ (corresponding $\angle$'s, $ED \parallel AC$) ✓

3) $\angle BDE = \angle BAC$ (corresponding $\angle$'s, $ED \parallel AC$)

Since there are three pairs of equal angles

$\triangle ABC \,|||\, \triangle DBE$ (equiangular) ✓

(ii) $\frac{AC}{DE} = \frac{BC}{BE}$ (proportional sides of sim $\triangle$'s)

$\frac{t}{5} = \frac{10}{6}$ ✓

$t = \frac{10 \times 5}{6}$

$= \frac{50}{6}$

$= 8\frac{1}{3}$ cm ✓

or 8.33cm (to 2 d.p.).

✓✓

Question 6 positive.

a) $y = \sqrt{2-x}$ (2, 2)

$x \leq 2$

$y \geq 0$.

$y^2 = 2 - x$

$x = 2 - y^2$

let $y = 0$ let $x = 0$

$x = 2$ $y^2 = 2$

$y = \sqrt{2}$.

b) $y = 2 + \frac{1}{x-1}$ $x \neq 1$

let $y = 0$ let $x = 0$

$\frac{1}{x-1} = -2$ $2 - 1$

$1 = -2(x-1)$ $y = 1$

$= -2x + 2$

$2x = 2$

$x = 1$ ✗

$\lim_{x \to \infty} \frac{2}{x} + \frac{\frac{1}{x}}{\frac{x}{x} - \frac{1}{x}}$

$0 + \frac{0}{1 - 0}$

$= 0$.

$y \to 0$ asymptote.

Make x subject

$y = 2 + \frac{1}{x-1}$

$f(x) = 2 + \frac{1}{x-1}$

$f(-x) = 2 + \frac{1}{-x-1}$ NOT EVEN.

$-f(-x) = -2 - \frac{1}{-x-1}$

$= -2 + \frac{1}{x-1}$ NOT ODD.

NEITHER.

$x - y = 1$ ——— ①

c) $x + y - 2 = 0$ ——— ②

$y = 1/x$

Sub $1/x$ into ②

$x + 1/x - 2 = 0$

$x^2 + 1 - 2x = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

Sub $x = 1$ into ①

$1 - y = 1$

$y = 1$

$\therefore x = 1$ ✓

$y = 1$ ✓

d) $y = \frac{1 - x^2}{1 + x^2}$

let $y = 0$ let $x = 0$

$x = 0$ $\frac{1-0}{1+0}$

$= 1$

$y = 1$

$f(-x) = \frac{1 - (-x)^2}{1 + (-x)^2}$

$= \frac{1 - x^2}{1 + x^2}$

$f(x) = f(-x)$

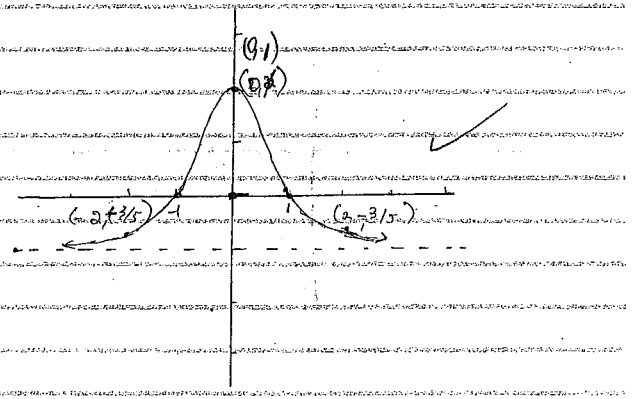
= even

$\lim_{x \to \infty} \frac{\frac{1}{x^2} - \frac{x^2}{x^2}}{\frac{1}{x^2} + \frac{x^2}{x^2}}$

$= \frac{0 - 1}{0 + 1}$

$= -1$

$y \to -1$



e) $y = \frac{x+4}{x(x+8)} = \frac{x+4}{x^2+8x}$ $x \neq 0$

let $y = 0$ let $x = 0$

$x = 0$ $y = 0$

$\lim_{x \to \infty} \frac{\frac{x}{x^2} + \frac{4}{x^2}}{\frac{x^2}{x^2} + \frac{8x}{x^2}}$

$\frac{0+0}{1+0}$

$y \to 0$

$f(-x) = \frac{-x+4}{-x(-x+8)}$

$\therefore$ not even

$-f(-x) = -\left(\frac{-x+4}{-x(-x+8)}\right)$

$\therefore$ not odd

0