

Sydney Girls' High School



2010 MATHEMATICS YEAR 10 YEARLY EXAMINATION

Time Allowed: 75 minutes

TOPICS: Quadratic Equations, Probability, Consumer Arithmetic, Number Plane and Graphs, Surface Area and Volumes- 20%, Statistics, Similarity, Further Trigonometry, Functions & Logarithms (11.01-11.03 inclusive) Further Algebra (including Direct & Indirect Variation) – 80%.

INSTRUCTIONS:

- There are five (5) questions.
- Attempt ALL questions.
- Questions are of equal value.
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working.
- Marks may be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale.
- Board-approved calculators may be used.

Total: 100 marks

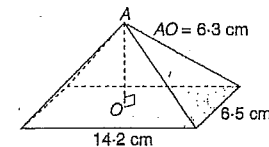
NAME:

CLASS TEACHER:

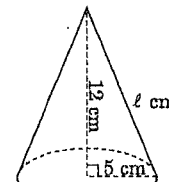
QUESTION 1 (20 marks)

Marks

- a) Andrew decided to buy a LCD marked \$3000. He pays 20% deposit and then the balance owing over 3 years, with simple interest charged at 15%p.a. on the balance.
- Find the deposit paid. (1)
 - Calculate the interest paid. (1)
 - What is his monthly repayment? (2)
- b) A box containing a light globe has a $\frac{1}{20}$ probability of holding a defective globe. If 180 boxes are checked, how many would be expected to be defective? (1)
- c) Solve
- $x^2 - 36 = 0$ (1)
 - $x^2 - 9x + 18 = 0$ (1)
 - $5x^2 - 4x - 3 = 0$ (2)
- d) A coin and a die are tossed. What is the probability of tossing a tail and an even number? (2)
- e) A parabola has the equation $y = x^2 - 5x + 6$. Find the equation of its axis of symmetry. (2)
- f) What is the equation of a circle with the origin as its centre and a radius of $\sqrt{5}$ units? (2)
- g) Calculate the volume of the pyramid shown below, correct to two decimal places. (2)



- h) A cone has dimensions as shown in the diagram below:



- Find the slant height l of the cone. (1)
- Hence, find the surface area of the cone. (2)

QUESTION 2 (20 marks)

Marks

a) Evaluate correct to three decimal places $\cos 142^\circ 30'$

(1)

b) An ordered stem and leaf table is given below for 30 scores

Stem	Leaf
4	1 8
5	0 3 4 7 9
6	2 4 5 6 6 7 8 8
7	1 3 3 3 5 7 9
8	0 4 4 5 8
9	1 3 7

- Find the:
- i) range
 - ii) mode
 - iii) median
 - iv) mean
 - v) standard deviation.

(1)

(1)

(1)

(1)

(1)

c) Given $f(x) = 2x - 5$ find i) $f(2)$
ii) $f(a+1)$

(1)

(2)

d) Given $R = \sqrt{\frac{ax}{b}}$ make x the subject.

(2)

e) By using a suitable substitution, factorise $(4y-1)^2 - 12(4y-1) + 20$

(2)

f) In the diagram shown,

i) Prove $\triangle ABC \parallel \triangle ADE$

(3)

ii) Find the values of x and y

(4)

QUESTION 3 (20 marks)

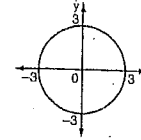
Marks

a) State whether each of the following represent a function or not a function

i) $(0,1) (1,2) (2,4) (3,8)$

(1)

ii)



(1)

iii) $y = x^2 + 3x$

(1)

b) Find θ to the nearest minute if $0^\circ < \theta < 180^\circ$ and $\tan \theta = -8.215$

(2)

c) Jasmine works for a real estate agency.

Her sales figures per month are given below.

Jasmine: 2 3 4 4 5 5 5 6 6 7 8 11

For Jasmine's sales:

i) Find the median

(1)

ii) Find the lower quartile (Q_1)

(1)

iii) Find the upper quartile (Q_3)

(1)

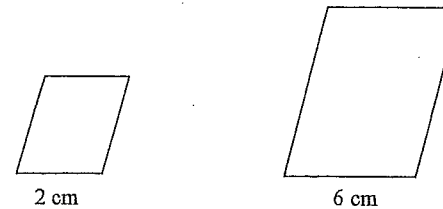
iv) Find the interquartile range

(1)

v) Display the data in a box-and-whisker plot.

(3)

d) The area of the smaller figure is 8cm^2 , find the area of the larger figure if the figures are similar.



(3)

e) The extension of a string is directly proportional to the mass added. The spring extends 28cm when 8g is added.

i) Find the constant of variation

(2)

ii) Write an equation to represent this relationship

(1)

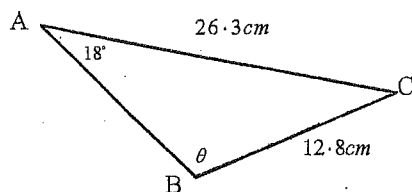
iii) How much will the spring extend when 40g is attached.

(2)

QUESTION 4 (20 marks)

Marks

a)



- If $90^\circ < \theta < 180^\circ$ in $\triangle ABC$ find
 i) θ (to the nearest minute) (3)
 ii) the area of the triangle. (3)

b) Solve simultaneously (4)

$$y = x^2 + 2x - 20$$

$$x + y = 8$$

c) i) Find the inverse function of $y = \frac{x}{3} + 2$ (2)

ii) Sketch $y = \frac{x}{3} + 2$ and its inverse on the same graph. (2)

d) Five students sat for a mathematics test and a science test. Their marks are given below:

Science	56	60	69	59	65
Mathematics	70	75	86	82	80

- i) Find the mean and standard deviation for each set of scores. (4)
 ii) Michael scored 65 in science and 80 in mathematics. In which subject did Michael perform better, compared with the class mean? Give reasons. (2)

QUESTION 5 (20 marks)

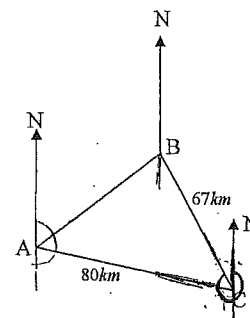
Marks

a) If $f(x) = x^2 + 2x$ find x if $f(x) = 0$ (2)

b) Given $y = 2^x$
 i) What are the possible x values? (1)
 ii) What are the possible y values? (1)

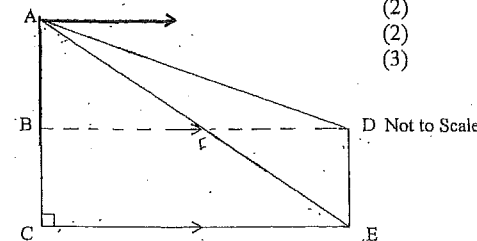
c) The surface area of two spheres are in the ratio 64 : 49.
 i) What is the ratio of their radii? (2)
 ii) Find the ratio of their volumes. (2)

d) The bearing of C from A is 130° , and the bearing of B from C is 330° .
 i) Show that $\angle BCA = 20^\circ$ (2)
 ii) Find AB to the nearest kilometre. (3)



e) AC and DE are two buildings. AC is 150 metres tall and CE is 100 metres. The angle of depression from A to D is 32° .

- Find:
 i) the distance AE (nearest metre) (2)
 ii) the distance AB (nearest metre) (2)
 iii) the size of $\angle DAE$ (3)



The End

Question 1

Deposit = $\frac{20}{100} \times 3000$
 = \$600

Interest = $\frac{15}{100} \times \$2400 \times 3$
 = \$1080

Monthly repayment
 = $\frac{2400 + 1080}{36}$
 = \$96.67

Defective = $\frac{1}{20} \times 180$
 = 9

i) $x^2 - 36 = 0$
 $(x-6)(x+6) = 0$
 $\therefore x = 6$ or $x = -6$

ii) $x^2 - 9x + 18 = 0$
 $(x-6)(x-3) = 0$
 $x = 6$ or $x = 3$

iii) $5x^2 - 4x - 3 = 0$
 $a = 5, b = -4, c = -3$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

c) iii) $x = \frac{4 \pm \sqrt{16 + 60}}{10}$

$x = \frac{4 \pm \sqrt{76}}{10}$

$x = \frac{4 \pm 2\sqrt{19}}{10}$

$x = \frac{2 \pm \sqrt{19}}{5}$

d) P(toil and even number)
 (T_2, T_4, T_6)
 $= \frac{3}{12}$
 $= \frac{1}{4}$

e) $y = x^2 - 5x + 6$
 $x = \frac{-b}{2a}$
 $x = \frac{5}{2}$

f) $x^2 + y^2 = (\sqrt{5})^2$
 $x^2 + y^2 = 5$

g) $V = \frac{1}{3} A \cdot H$
 $= \frac{1}{3} (14 \cdot 2)(6 \cdot 5)(6 \cdot 3)$
 $= 193.83 \text{ cm}^3$

h) i) $12^2 + 5^2 = l^2$
 $\sqrt{169} = l$
 $\therefore l = 13$

ii) $S.A = \pi r l + \pi r^2$
 $= \pi(5)(13) + \pi(5)^2$
 $= 90\pi \text{ cm}^2$

Question 2

a) $\cos 142^\circ 30' = -0.793$

b) i) Range = $97 - 41$
 = 56

ii) Mode = 73

iii) Median = $\frac{68 + 71}{2}$

= 69.5

iv) Mean = 70.4

v) S.D = 13.73

c) i) $f(2) = 2(2) - 5$
 $f(2) = -1$

ii) $f(a+1) = 2(a+1) - 5$
 $= 2a + 2 - 5$
 $= 2a - 3$

d) $R = \sqrt{\frac{ax}{b}}$

$R^2 = \frac{ax}{b}$

$\frac{bR^2}{a} = x$

$\therefore x = \frac{R^2 b}{a}$

e) let $m = (4y - 1)$
 $m^2 - 12m + 20$
 $(m - 10)(m - 2)$
 $(4y - 1 - 10)(4y - 1 - 2)$
 $(4y - 11)(4y - 3)$

f) i) Prove $\triangle ABC \sim \triangle ADE$

In $\triangle ABC$ and $\triangle ADE$
 $\angle A$ is common
 $\angle ADE = \angle ABC$ (corresponding \angle 's $DE \parallel BC$)
 $\angle AED = \angle ACB$ (corresponding \angle 's $DE \parallel BC$)
 hence $\triangle ABC \sim \triangle ADE$ (equiangular).

ii) Hence, corresponding sides in the same ratio

$\frac{DE}{BC} = \frac{AD}{AB}$

$\frac{12}{21} = \frac{4}{4+x}$

$12(4+x) = 4(21)$

$48 + 12x = 84$

$12x = 36$

$x = 3$

$\frac{DE}{BC} = \frac{AE}{AC}$

$\frac{12}{21} = \frac{y}{y+6}$

$12(y+6) = 21y$

$12y + 72 = 21y$

$72 = 9y$

$\therefore y = 8$

Question 3

- i) Function
- ii) Not a function
- iii) Function

$\tan \theta = -8.215$
 $\therefore \theta = 180^\circ - 83^\circ 4'$
 $\therefore \theta = 96^\circ 56'$

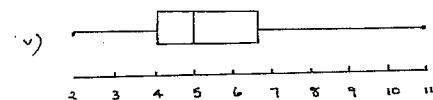
i) Median = 5

ii) $Q_1 = 4$

iii) $Q_3 = \frac{6+7}{2}$

= 6.5

iv) Interquartile range = $Q_3 - Q_1$
 $= 6.5 - 4$
 $= 2.5$



$\frac{A_L}{A_S} = \frac{6^2}{2^2}$

$A_L = \frac{36}{4} \times 8$

$A_L = 72 \text{ cm}^2$

i) $E \propto KM$

$28 = 8K$

$\therefore K = \frac{28}{8}$

$K = \frac{7}{2}$

ii) $E = KM$

$E = \frac{7}{2} M$

iii) $E = \frac{1}{2} M$ at $M = 40$

$E = \frac{1}{2} \times 40$

$E = 140 \text{ cm}$

Question 4

a) i) $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{12.8}{\sin 18^\circ} = \frac{26.3}{\sin \theta}$

$\sin \theta = \frac{26.3 \times \sin 18^\circ}{12.8}$

$\sin \theta = 0.6349$

$\theta = 39^\circ 25'$

$\therefore \theta = 180 - 39^\circ 25'$

$\theta = 140^\circ 35'$

(since $90^\circ < \theta < 180^\circ$)

ii) $\angle C = 180 - (18 + 140^\circ 35')$

$= 180 - 158^\circ 35'$

$= 21^\circ 25'$

$\therefore \text{Area} = \frac{1}{2} ab \sin C$

$= \frac{1}{2} (12.8)(26.3) \sin(21^\circ 25')$

$\text{Area} = 61.5 \text{ cm}^2$

b) $y = x^2 + 2x - 20$
 $x + y = 8$

$x + x^2 + 2x - 20 = 8$

$x^2 + 3x - 28 = 0$

$(x + 7)(x - 4)$

$x = -7$ or $x = 4$

$x + y = 8$

$-7 + y = 8$

$y = 15$

$x + y = 8$

$4 + y = 8$

$y = 4$

$\therefore P_1(-7, 15)$

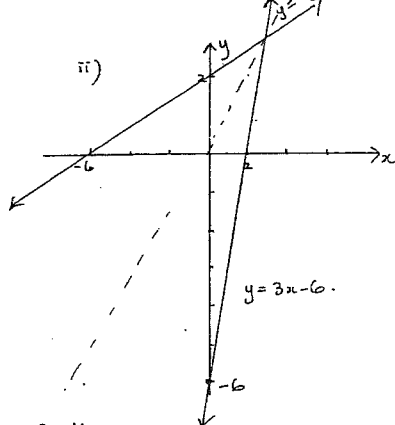
$P_2(4, 4)$

c) i) Inverse function of $y = \frac{x}{3} + 2$ is:

$x = \frac{y}{3} + 2$

$3x = y + 6$

$y = 3x - 6$



d) i)

	\bar{x}	σ
Science	61.8	4.62
Maths	78.6	5.57

ii) Science = $\frac{65 - 61.8}{4.62}$

$\hat{=} 0.7$

Maths = $\frac{80 - 78.6}{5.57}$

$\hat{=} 0.25$

His Science mark was better since he got a 0.7 score above

Question 5

a) $f(x) = x^2 + 2x$

$f(x) = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$ or $x = -2$

b) i) All real x

ii) $y > 0$

c) Surface Area 64:49

i) Ratio of radii

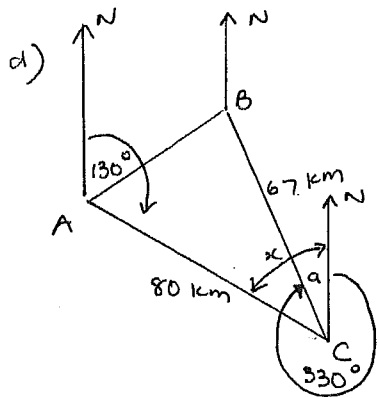
$\sqrt{64} : \sqrt{49}$

8 : 7

ii) Volumes

$8^3 : 7^3$

512 : 343



i)

$x + 130^\circ = 180^\circ$ (co-interior \angle 's,
 $x = 50^\circ$
 $a = 30^\circ$ ($360^\circ - 330^\circ$) parallel lines)

$\therefore \angle BCA = x - a$

d)

ii) $c^2 = a^2 + b^2 - 2ab \cos C$

$AB^2 = (67)^2 + (80)^2 - 2(67)(80) \cos 20^\circ$

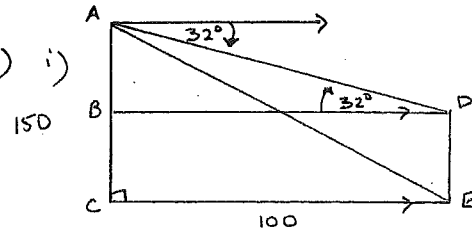
$AB^2 = 815.495$

$AB = \sqrt{815.495}$

$AB = 28.6$

$\therefore AB = 29 \text{ km}$ (nearest km)

e) i)



i) $AE^2 = 150^2 + 100^2$

$AE^2 = 32500$

$\therefore AE = 180 \text{ m}$

ii) $\tan 32^\circ = \frac{AB}{BD}$

$AB = \tan 32^\circ \times 100$

$AB = 62 \text{ m}$

iii) $DE = 150 - 62$

$= 88 \text{ m}$

$\angle ADE = 90^\circ + 32^\circ$

$= 122^\circ$

$\therefore \frac{\sin \angle DAE}{DE} = \frac{\sin \angle ADE}{AE}$

$\sin \angle DAE = \frac{88 \times \sin 122^\circ}{180}$

$= 0.4146$

$\therefore \angle DAE = 24^\circ 30'$