Sydney Girls' High School

2011
MATHEMATICS EXTENSION 1

YEAR 11
HALF-YEARLY EXAMINATION

Time Allowed: 60 minutes + 3 minutes (Reading)

TOPICS: Harder Mathematics of Basic Algebra and Equations, Special Quadrilaterals, Functions and Trigonometry.

Directions to Candidates

- There are four (4) questions.
- Attempt ALL questions.
- Questions are of equal value.
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working. Marks will be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale.
- Board-approved calculators may be used.

Total: 60 marks

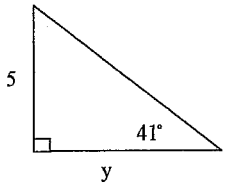
QUESTION 1 (15 marks) Marks

a) Evaluate $\frac{12\sin 33^\circ}{\sin 31^\circ}$ (correct to 4 significant figures) 2

b) If $f(x)=3x^2-5x+4$ and $g(x)=2x+10$ find $f(1)-g(-1)$. 3

c) Find the exact value of $\cos 225^\circ$. 2

d) Find correct to 1 decimal place the value of y. 2



e) State the domain and range of the function $f(x)=\frac{1}{\sqrt{x+3}}$. 2

f) Solve for x $\frac{5}{x-3} \geq 2$. 4

QUESTION 2 (15 marks)

Marks

a) Sketch the following graphs on separate number planes showing all relevant features.

i) $y = x^2 + 5x - 6$ 2

ii) $y = -\sqrt{4 - x^2}$ 2

iii) $y = 2^{-x}$ 2

iv) $y = |x - 2|$ 2

v) $y = \dfrac{3}{x+1}$ 2

b)

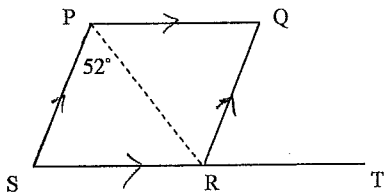


Figure not to scale

In the diagram PQRS is a rhombus where $\angle SPR = 52^\circ$ and SR is produced to T.

i) Find the value of $\angle SPQ$. Give reasons. 2

ii) Find the value of $\angle QRT$. Give reasons. 3

QUESTION 3 (15 marks)

Marks

a) Expand and simplify $(1+2x^2)^3$. 3

b) Find the exact value of $\sin\theta$ if $\tan\theta = \dfrac{3}{7}$ and $\cos\theta < 0$. 3

c) Show whether the following function is odd, even or neither. 2

$$f(x) = \frac{x}{x^2 - 1}$$

d) Solve the equation $2\cos x = \sqrt{3}$, where $0 \le x \le 360^\circ$ 3

e) Sketch the curve $y = \sec x$, for $0 \le x \le 360^\circ$. 2

f) Prove that $\dfrac{1}{\sec\theta - \tan\theta} - \dfrac{1}{\sec\theta + \tan\theta} = 2\tan\theta$ 2

QUESTION 4 (15 marks) Marks

a) i) Factorise $2^n + 2^{n+1}$ 1

ii) Hence, write $\frac{2^{100} + 2^{101}}{3}$ as a power of 2. 2

b) On a number plane shade in the region given by the two conditions

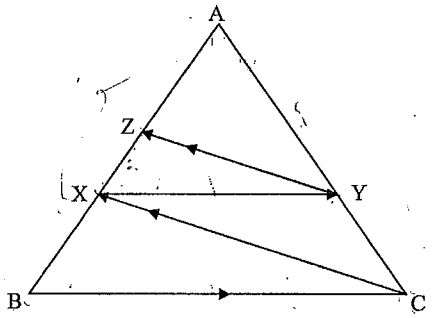
$x^2 + y^2 \leq 4$ and $x + y > 1$. 3

c) If $(3-\sqrt{2})^4 = P + Q\sqrt{2}$ where P and Q are integers.

Find the values of P and Q. 3

d) In the diagram below $XY \parallel BC$ *and* $ZY \parallel XC$.

$AY = 8cm$, $YC = 4cm$ *and* $XB = 3cm$.

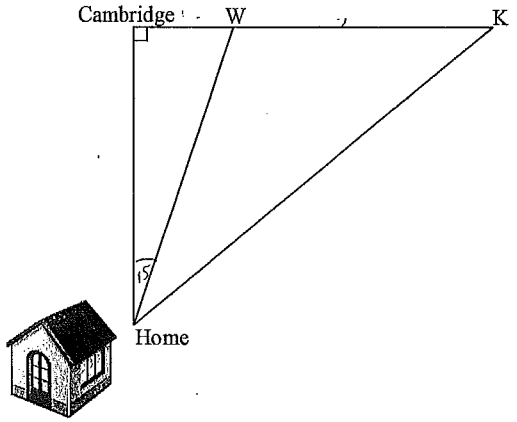


i) Copy this diagram onto your examination paper labelling information given.

ii) Find the value of ZX. 3

QUESTION 4(continued) Marks

e) Both William and Kate are due east of Cambridge.
Cambridge is 9 km due north of their home.
Kate is 15 km from their home and William is on a bearing of 015° from their home.



i) Copy this diagram onto your examination paper labelling information given.

ii) How far apart are William and Kate? 3

THE END

Year 11 - 2011 - Half-yealy examination - Solutions (60 mar

Tota

Question 1. (15 marks)

a) 12.69 (2)

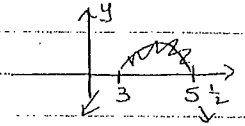
b) $f(1) = 3(1)^2 - 5(1) + 4$
$= 2$
$g(-1) = (2)(-1) + 10$
$= 8$
$\therefore f(1) - g(-1) = 2 - 8$
$= -6$ (3)

c) Exact value of $\cos 225^\circ$
$= -\cos 45^\circ$
$= \frac{-1}{\sqrt{2}}$ or $\frac{-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$ (2)

d) $\tan 41^\circ = \frac{5}{y}$
$\therefore y = \frac{5}{\tan 41^\circ}$
$y = 5.8$ (2)

e) Domain: $x + 3 > 0$
$x > -3$
Range: $y > 0$ (2)

f) $x \neq 3$
$\frac{5}{x-3} - 2 \geq 0$
$\frac{5 - 2(x-3)}{(x-3)} \geq 0$
$\frac{11 - 2x}{(x-3)} \geq 0$
$\frac{(x-3)^2(11-2x)}{(x-3)} \geq 0$
$(x-3)(11-2x) \geq 0$

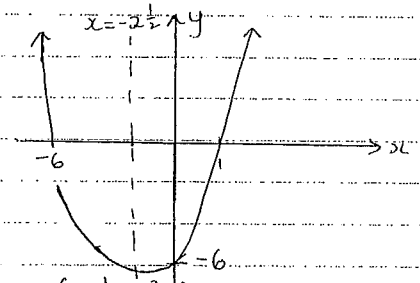


(4)

$\therefore$ Solution
$3 < x \leq 5\frac{1}{2}$

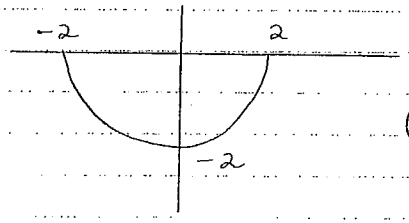
Question 2 (15 marks)

a) i) $y = x^2 + 5x - 6$
$y = (x+6)(x-1)$



(2)

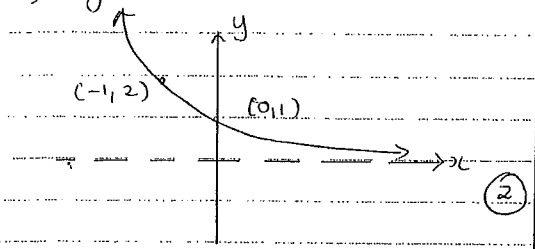
ii) $y = -\sqrt{4 - x^2}$



(2)

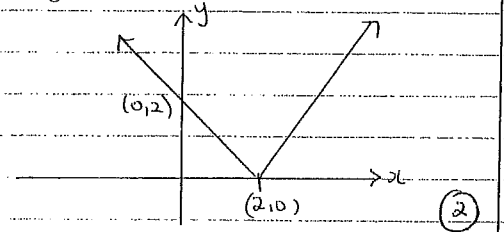
Question 2 (con't)

iii) $y = 2^{-x}$



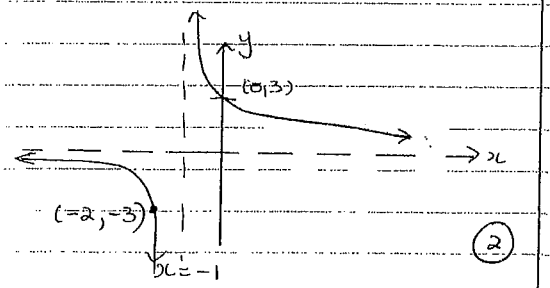
(2)

iv) $y = |x - 2|$



(2)

v) $y = \frac{3}{x+1}$



(2)

b) i) $\angle SPQ = 2(52^\circ)$
$= 104^\circ$ (2)
(diagonals of rhombus bisect $\angle$'s)

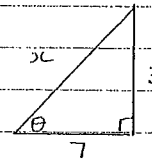
ii) $\angle SRQ = 104^\circ$ (opposite $\angle$'s of rhombus are equal)
$\therefore \angle QRT = 180 - 104$
$= 76^\circ$ (straight $\angle$) (3)

Question 3 (15 marks)

a)
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(1 + 2x^2)^3 = 1 + 3(1)^2(2x^2) + 3(1)(2x^2)^2 + (2x^2)^3$
$= 1 + 6x^2 + 12x^4 + 8x^6$ (

b) $\tan\theta = \frac{3}{7}$

S	A
T	C



$x^2 = 7^2 + 3^2$
$x = \sqrt{58}$
$\therefore \sin\theta = -\frac{3}{\sqrt{58}}$ (3)

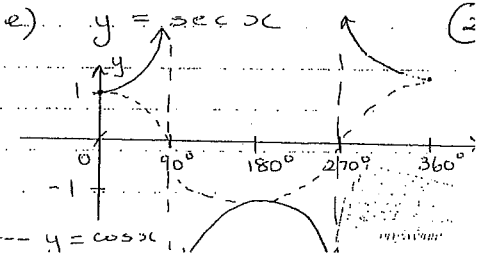
c) $f(x) = \frac{x}{x^2 - 1}$
$f(-x) = \frac{-x}{x^2 - 1}$ (
$\therefore$ since $f(x) = -f(x)$
$\therefore$ function is odd.

d) $2\cos x = \sqrt{3}$
$\cos x = \frac{\sqrt{3}}{2}$

S	A
T	C

(
$x = 30^\circ$ or 330°

e) $y = \sec x$ (2



Question 3 (cont'd)

f) Prove $\frac{1}{\sec\theta - \tan\theta} - \frac{1}{\sec\theta + \tan\theta} = 2\tan\theta$

L.H.S $= \frac{1}{\sec\theta - \tan\theta} - \frac{1}{\sec\theta + \tan\theta}$

$= \frac{\sec\theta + \tan\theta - (\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$

$= \frac{\sec\theta + \tan\theta - \sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$

$= 2\tan\theta$ (2)

$=$ R.H.S.

Question 4 (15 marks)

a) i) $2^n + 2^{n+1} = 2^n(1+2)$

$= 2^n \times 3$ (1)

ii) $\frac{2^{100} + 2^{101}}{3} = \frac{2^{100}(1+2)}{3}$

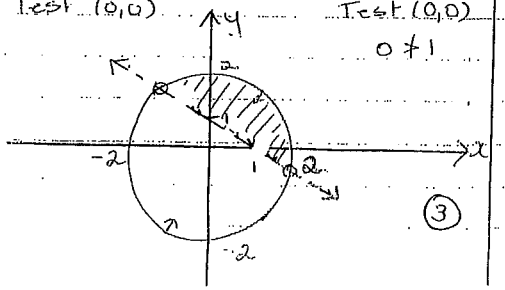
$= \frac{2^{100} \times 3}{3}$

$= 2^{100}$ (2)

b) $x^2 + y^2 \leq 4$ and $x + y > 1$

Test (0,0) $\quad$ Test (0,0)

$0 \not> 1$



(3)

c) $(3-\sqrt{2})^4 = P + Q\sqrt{2}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

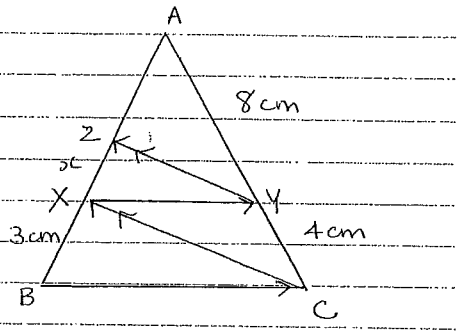
$(3-\sqrt{2})^4 = 3^4 + 4(3)^3(-\sqrt{2}) + 6(3)^2(-\sqrt{2})^2 + 4(3)(-\sqrt{2})^3 + (-\sqrt{2})^4$

$= 81 - 108\sqrt{2} + 108 - 24\sqrt{2} + 4$

$= 193 - 132\sqrt{2}$

$\therefore P = 193$ and $Q = -132$ (3)

d)



$\frac{AY}{YC} = \frac{AX}{XB}$

$\frac{8}{4} = \frac{AX}{3}$

$\therefore AX = 6\text{cm}$

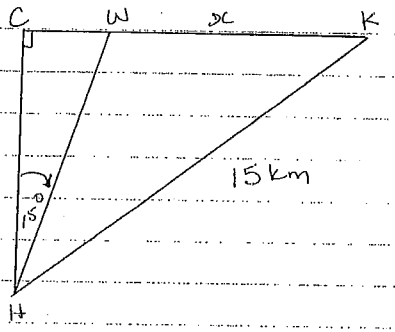
Now, $\frac{AY}{YC} = \frac{2}{1} = \frac{AZ}{ZX}$ (prop. intercept theorem $ZX \parallel XC$)

$\therefore 6\text{cm} \div 3 = ZX$

$\therefore XZ = 2\text{cm}$. (3)

Question 4

e) i)



ii) $CK^2 = 15^2 - 9^2$

$CK^2 = 144$

$\therefore CK = \sqrt{144}$

$\therefore CK = 12\text{ km}$

$\tan 15^\circ = \frac{CW}{9}$

$9 \times \tan 15^\circ = CW$

$\therefore CW = 2.4\text{ km}$

$\therefore x \doteq 12 - 2.4$

$\doteq 9.6$

$\therefore$ Kate and Wills are 9.6 km apart. (3)