Student Number: ____________

St Catherine's School

Waverley

ASSESSMENT TASK 4

(Weighting 45%)

MATHEMATICS

YEAR 11

2011

General Instructions

- Reading time – 5 minutes
  Working time – 2 hours
- Complete each question in a separate booklet
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Marks for each question are indicated next to each part.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work

Total marks – 84

- Attempt Questions 1 - 7

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Total marks – 84

Attempt questions 1 – 7

All questions are of equal value

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

QUESTION 1 **(12 marks)** Use the Question 1 writing booklet

(a) Simplify $\frac{x+3}{2}+\frac{x+1}{3}$ /2

(b) Solve for x: $6-2x>14$ /2

(c) The line $2x+ky=7$ passes through the point (2,-1). Find the value of k. /2

(d) Find $\frac{d}{dx}\left(\frac{6}{x^2}\right)$ /2

(e) Given $k=1+\sqrt{2}$, find k^2-k in simplest form. /2

(f) Factorise fully: $4f^2-4fx-f+x$ /2

QUESTION 2 **(12 marks)** Use the Question 2 writing booklet

(a) Solve $|2x-5|<4$ /2

(b) If α and β are the roots of the equation $x^2+8x-5=0$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$. /2

(c) Given the equation of a circle is $x^2-4x+y^2-8y+16=0$ find the centre and radius /2

(d) A parabola has its vertex at $(-2,-1)$ and directrix $y=-3$. Find:

(i) the coordinates of the focus /2

(ii) the equation of the parabola 1

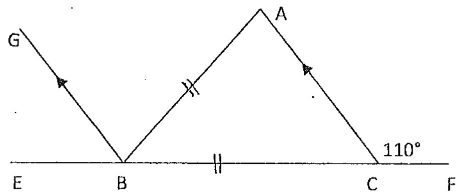
(e) A function is defined as $f(x)=\begin{cases}|x| : for\ x<0\\ x^2 : for\ 0\le x\le 2\\ 4 : for\ x>2\end{cases}$

(i) Evaluate $f(-1)+f(1)-f(3)$ /1

(ii) Sketch this function for $-2\le x\le 4$ /2

QUESTION 3 (12 marks) Use the Question 3 writing booklet

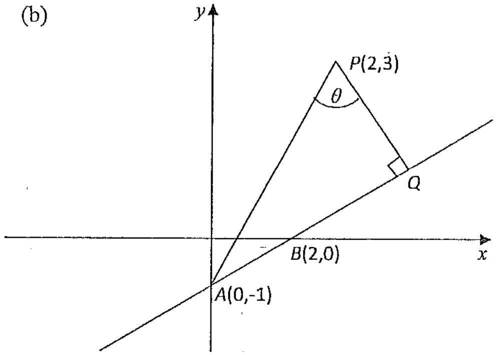
(a)



AB=BC and BG || CA.

Copy or trace this diagram into your writing booklet.
Prove BG bisects $\angle$EBA. /3

(b)



In the diagram above AQ $\perp$ PQ and $\angle$APQ = θ. A=(0,-1), B=(2,0) and P=(2,3).
ABQ is a straight line.

(i) Find the length of AP /1

(ii) Show the line AB has equation $x-2y-2=0$. /2

(iii) Use the perpendicular distance formula to find the length of PQ. /2

(iv) Show that $cos\theta = \frac{3}{5}$. /1

(v) What is the value of $sin\theta$? /1

(vi) Find the area of ΔAPQ /2

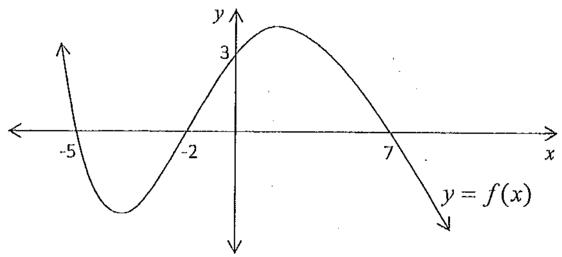
QUESTION 4 (12 marks) Use the Question 4 writing booklet

(a) Find the values of a, b and c if:

$2x^2+3x-6 \equiv a(x+1)^2 + b(x+1) + c$ for all values of x. /3

(b) Solve $2^{2x} = 8^{x-1}$ /2

(c) The graph of $y = f(x)$ is given below



(i) Find the value of $f(0)$ /1

(ii) For what value(s) of x is $f(x) < 0$? /2

(iii) Copy or trace the diagram into your answer booklet. Draw the graph of $y = f'(x)$ onto the same axes. /2

(d) Consider the function $y = \sqrt{9-x^2}$.

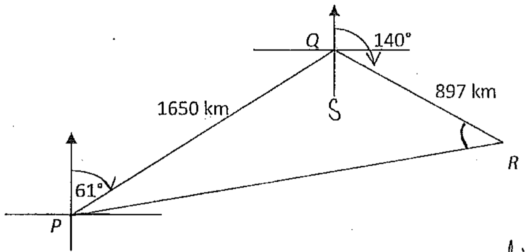
(i) State its domain /1

(ii) Show that this is an even function /1

QUESTION 5 (12 marks) Use the Question 5 writing booklet

(a) Find the value(s) of k for which the quadratic equation $x^2-(k-6)x+4=0$ has real roots. /2

(b) A plane travels 1650 km from P to Q on a bearing of 61°T. The plane then travels 897 km on a bearing of 140°T to a point R.



diagram

(i) Show that $\angle PQR$ is 101°, giving reasons. /1

(ii) Find the distance PR correct to one decimal place. /2

(iii) Find the size of $\angle PRQ$ to the nearest minute. /2

(iv) Find the bearing of P from R to the nearest degree. /1

(c) Solve $2\cos\theta=-\sqrt{3}$ for $0° \le \theta \le 360°$ /2

(d) Find the simplest expression for $\tan\theta\sqrt{1-\sin^2\theta}$ /2

QUESTION 6 (12 marks) Use the Question 6 writing booklet

(a) Differentiate with respect to x:

(i) $f(x)=4x^5-\frac{x}{3}+7$ /1

(ii) $y=(3x+2)^4$ /2

(iii) $y=x\sqrt{x}$ /2

(b) Evaluate $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2+2x-8}$ /2

(c) (i) If $y=\dfrac{2x-1}{x+1}$ show that $\dfrac{dy}{dx}=\dfrac{3}{(x+1)^2}$ /2

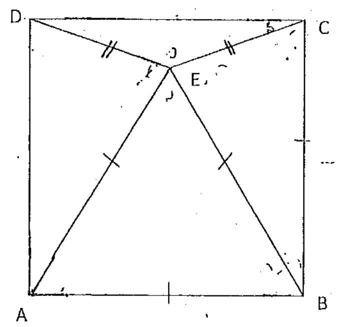
(ii) Find the gradient of the tangent to the curve $y=\dfrac{2x-1}{x+1}$ at the point where $x=1$. /1

(iii) Find the equation of the normal to the curve $y=\dfrac{2x-1}{x+1}$ at the point where $x=1$. /2

QUESTION 7 (12 marks) Use the Question 7 writing booklet

7. (a) One root of the equation $x^2 + qx + r = 0$ is three times the other.
Prove $3q^2 = 16r$. /3

(b) ABCD is a square. ΔABE is equilateral.



(i) Prove $\angle EBC = 30°$. /1

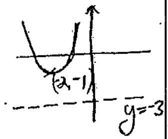
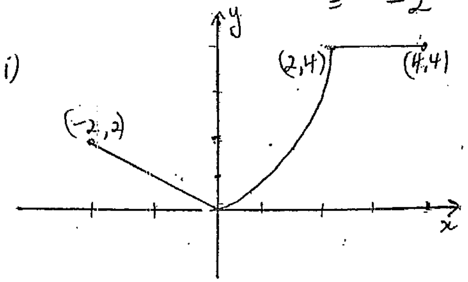
(ii) Prove ΔEBC and ΔEAD are congruent /3

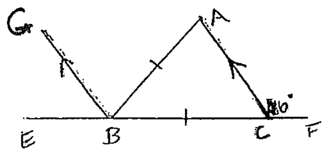
(iii) If the lengths of the sides of the square are k cm, prove that the area of ΔABE is $\frac{k^2\sqrt{3}}{4}$. /2

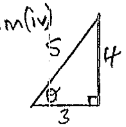
(iv) Hence, or otherwise show the area of ΔCDE is $\frac{k^2(2-\sqrt{3})}{4}$ /3

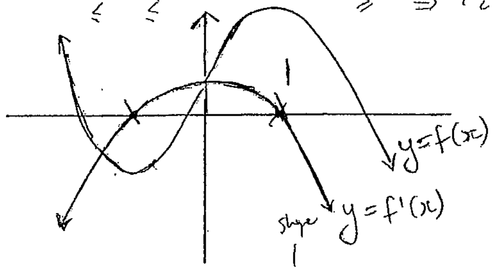
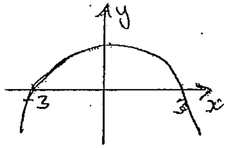
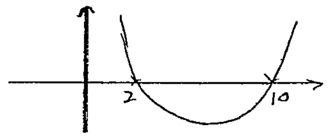
End of paper

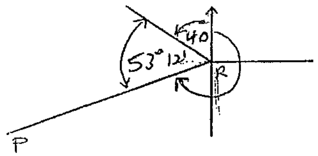
	Solutions	Marks	Comments: Criteria
1(a)	$\dfrac{x+3}{2} + \dfrac{x+1}{3}$ $= \dfrac{3(x+3)+2(x+1)}{6}$ $= \dfrac{3x+9+2x+2}{6}$ $= \dfrac{5x+11}{6}$	2	−1 if try to solve.
(b)	$6 - 2x > 14$ $-2x > 14-6$ $-2x > 8$ $x < -4$	2	
(c)	$2x + ky = 7$ $2(2) + k(-1) = 7$ $-k = 3$ $k = -3$	2	
(d)	$\dfrac{d}{dx}\left(\dfrac{6}{x^2}\right) = \dfrac{d}{dx}(6x^{-2})$ ½ $= -12x^{-3}$ ✓ $= \dfrac{-12}{x^3}$ $\cancel{½}$	2	
(e)	$k^2 - k = (1+\sqrt{2})^2 - (1+\sqrt{2})$ $= 1 + 2\sqrt{2} + 2 - 1 - \sqrt{2}$ $= 2 + \sqrt{2}$	2	no bracket −1 −1 if expand incorrectly
(f)	$4f^2 - 4fx - f + x$ $4f(f-x) - (f-x)$ $(4f-1)(f-x)$	2	

Qn	Solutions	Marks	Comments: Criteria
2	a) $\lvert 2x-5\rvert < 4$ $2x - 5 < 4$ OR $-(2x-5) < 4$ $2x < 9$; $2x-5 > -4$ $x < \frac{9}{2}$; $2x > 1$ $x > \frac{1}{2}$	2	
	b) $\alpha + \beta = -\dfrac{b}{a} = \dfrac{-8}{1}$; $\alpha\beta = \dfrac{c}{a} = \dfrac{-5}{1}$ $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\beta+\alpha}{\alpha\beta} = \dfrac{-8}{-5} = \dfrac{8}{5}$	2	
	c) $x^2 - 4x + y^2 - 8y + 16 = 0$ $x^2 - 4x + \left(\frac{-4}{2}\right)^2 + y^2 - 8y + \left(\frac{-8}{2}\right)^2 = -16 + \left(\frac{4}{2}\right)^2 + \left(\frac{-8}{2}\right)^2$ $(x-2)^2 + (y-4)^2 = -16 + 4 + 16$ $(x-2)^2 + (y-4)^2 = 4$ centre is $(2,4)$ radius is 2	2	
	d)(i) focal length $= -1 \to -3$ $= 2$ $\therefore$ focus $= (-2, 1)$ 	2	
	(ii) equation is $(x - -2)^2 = 2\times 4(y - -1)$ $(x+2)^2 = 8(y+1)$	1	
	e)(i) $f(-1) + f(1) - f(3) = \lvert -1\rvert + (1)^2 - 4$ $= -2$	1	
	(ii) $f(-2) = \lvert -2\rvert = 2$ $f(4) = 4$	2	$-\frac{1}{2}$ each mistake

Solutions	Marks	Comments: Criteria
3(a)  One of many solutions! $\angle ACF + \angle ACB = 180$ straight angle $\frac{1}{2}$ $\therefore \angle ACB = 70^\circ$ $AB = BC$ given $\triangle ABC$ is isosceles $\frac{1}{2}$ $\therefore \angle BAC = 70^\circ$ $\angle GBA = 70^\circ$ (alternate angles on $BG \parallel CA$) $\frac{1}{2}$ $\angle ABC = (180-70-70)$ $= 40$ (angle sum of isosceles triangle) $\frac{1}{2}$ $\angle EBG + \angle GBA + \angle ABC = 180^\circ$ (straight angle) $\frac{1}{2}$ $\angle EBG + 70 + 40 = 180$ $\angle EBG = 70$ $= \angle GBA$ $\frac{1}{2}$ $\therefore$ BG bisects $\angle EBA$	3	
(b)(i) $AP = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(2-0)^2+(3--1)^2}$ $= \sqrt{4+16}$ $= 2\sqrt{5}$	1	
(ii) $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$ $= \frac{0--1}{2-0}$ $= \frac{1}{2}$ $y-y_1 = m(x-x_1)$ using $(2,0)$ $y-0 = \frac{1}{2}(x-2)$ $2y = x-2$ $x-2y-2=0$	2	
(iii) $PQ = \left\vert\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\vert$ $a=1$ $b=-2$ $c=-2$ $x_1=2$ $y_1=3$ $= \left\vert\frac{2(1)-3(2)-2}{\sqrt{1^2+2^2}}\right\vert$ $= \frac{6}{\sqrt{5}}$	2	

Qn	Solutions	Marks	Comments: Criteria
3	iv) $\cos\theta = \frac{PQ}{AP}$ $= \frac{\frac{6}{\sqrt{5}}}{2\sqrt{5}}$ $= \frac{3}{5}$	1	
	v) from(iv)  $\sin\theta = \frac{4}{5}$	1	
	vi) Area $= \frac{1}{2}ab\sin\theta$ $= \frac{1}{2} AP \times PQ \times \sin\theta$ $= \frac{1}{2} \times 2\sqrt{5} \times \frac{6}{\sqrt{5}} \times \frac{4}{5}$ $= 4.8$ units^2	2	
4	(a) $2x^2+3x-6 \equiv a(x+1)^2+b(x+1)+c$ $= a(x^2+2x+1)+bx+b+c$ $= ax^2+2ax+a+bx+b+c$ $= ax^2+(2a+b)x+a+b+c$ ✓ $a=2$ ① $2a+b=3$ ② $a+b+c=-6$ ③ $\Rightarrow 2(2)+b=3$ $b=-1$ ✓ $\Rightarrow 2+(-1)+c=-6$ $c=-7$ $\therefore a=2$ $b=-1$ $c=-7$ ✓	3	small error correct $\frac{2\frac{1}{2}}{3}$
	(b) $2^{2x} = 8^{x-1}$ $2^{2x} = (2^3)^{x-1}$ ✓ $2^{2x} = 2^{3x-3}$ $\frac{1}{2}$ $\therefore 2x = 3x-3$ $\frac{1}{2}$ $x = 3$	2	

Solutions	Marks	Comments: Criteria
4(c)(i) $f(0) = 3$ $\frac{1}{2}$	1	
(ii) $-5 < x < -2$ ✓ and $x > 7$ ✓ $\frac{1}{2}$ $\leq \quad \leq \qquad \geq \Rightarrow 1\frac{1}{2}$	2	$x = -2, -5$
(iii) 	2	
(d)(i) Domain: $9-x^2 \geq 0$ $(3-x)(3+x) \geq 0$ $-3 \leq x \leq 3$ $-3 < x < 3$ $\frac{1}{2}$ other way round $\Rightarrow \frac{1}{2}$	1	
(ii) $f(x) = \sqrt{9-x^2}$ $f(-x) = \sqrt{9-(-x)^2}$ $\frac{1}{2}$ $\therefore f(x) = f(-x)$ $\frac{1}{2}$ $\therefore$ even function Symmetrical about y-axis with sketch $\Rightarrow 1$.	1	
5(a) $b^2 - 4ac \geq 0$ $(k-6)^2 - 4 \times 1 \times 4 \geq 0$ $k^2 - 12k + 36 - 16 \geq 0$ $k^2 - 12k + 20 \geq 0$ $(k-10)(k-2) \geq 0$ ✓ $\therefore\ k \leq 2$ $\frac{1}{2}$ OR $k \geq 10$ $\frac{1}{2}$	2	

Qn	Solutions	Marks	Comments: Criteria
5	(b)(i) $\angle PQR = 61 + (180-140)$ $= 101°$ (alternate angle plus straight angle)	1	
	(ii) $PR^2 = 1650^2 + 897^2 - 2(1650)(897)\cos 101°$ ✓ $PR = 2022.9$ km ✓	2	
	(iii) $\frac{\sin\theta}{1650} = \frac{\sin 101°}{2022.9}$ $\sin\theta = \frac{\sin 101° \times 1650}{2022.9}$ $\theta = 53°12'$ (nearest minute)	2	
	(iv) Bearing is $360 - (40 + 53°12')$ $= 266°48'$ T 	1	
	(c) $2\cos\theta = -\sqrt{3}$ incorrect quads $\Rightarrow \frac{1}{2}$. $\cos\theta = -\frac{\sqrt{3}}{2}$ $(\frac{1}{2})$ (2nd, 3rd quadrants) $\cos ? = \frac{\sqrt{3}}{2} \Rightarrow ? = 30°$ $\therefore \theta = 180-30,\ 180+30$ $= 150°$ ✓, $210°$ ✓	2	
	(d) $\tan\theta\sqrt{1-\sin^2\theta}$ $= \tan\theta\sqrt{(\cos^2\theta)}$ $(\frac{1}{2})$ $= \tan\theta \cos\theta$ $= \frac{\sin\theta}{\cos\theta}(\frac{1}{2})\cos\theta(\frac{1}{2})$ $= \sin\theta$ $(\frac{1}{2})$	2	

Solutions	Marks	Comments: Criteria

6a(i) $f(x) = 4x^5 - \frac{x}{3} + 7$

$f'(x) = 20x^4 - \frac{1}{3}$ — 1 ONLY OR 0. — 1

(ii) $y = (3x+2)^4$

$\frac{dy}{dx} = 4 \times 3(3x+2)^3$ — 1 FOR $4(3x+2)^3$

$= 12(3x+2)^3$ — 1 FOR $12(3x+2)^2$

(iii) $y = x\sqrt{x}$

$= x^1 \times x^{\frac{1}{2}}$

$= x^{3/2}$ $(\frac{1}{2})$ — 1 for $y' = 1 \times \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$ (1)

$= \frac{3\sqrt{x}}{2}$ $(\frac{1}{2})$ — 2

(b) $\lim_{x\to 2} \frac{x^3 - 8}{x^2 + 2x - 8} = \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x+4)(x-2)}$ — 1 — $\frac{1}{2}$ for some correct factorising

$= \lim_{x\to 2} \frac{x^2+2x+4}{x+4}$

$= \frac{2^2 + 2(2) + 4}{2+4}$

$= 2$ — 1 — 2

(c)(i) $y = \frac{2x-1}{x+1}$ $\quad u = 2x-1 \quad v = x+1 \quad \frac{du}{dx} = 2 \quad \frac{dv}{dx} = 1$ — 1

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{2(x+1) - 1(2x-1)}{(x+1)^2}$ — 1

$= \frac{3}{(x+1)^2}$ — (-1 MISTAKE) — 2

(ii) at $x=1$ $\frac{dy}{dx} = \frac{3}{(1+1)^2}$

$= \frac{3}{4}$ — (1 ONLY) — 1

(iii) when $x=1$, $y = \frac{2(1)-1}{1+1} = \frac{1}{2}$ $(\frac{1}{2})$ $\quad m_\perp = \frac{-4}{3}$ $(\frac{1}{2})$ — 2

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{-4}{3}(x-1)$ ← $(\frac{1}{2})$ — OR $y = -\frac{4}{3}x + \frac{11}{6}$

$3y - \frac{3}{2} = -4x + 4 \implies 4x + 3y - \frac{11}{2} = 0$ ↙ $(\frac{1}{2})$

Qn	Solutions	Marks	Comments: Criteria

7 a) Let the roots be α and β and $\beta = 3\alpha$ $\therefore \alpha + \beta = \alpha + 3\alpha = \frac{-q}{1}$

ie $4\alpha = -q$ ①

also $\alpha\beta = \alpha \times 3\alpha = \frac{r}{1}$

$3\alpha^2 = r$ ②

from ① $\alpha = \frac{-q}{4}$ sub into ②

$3\left(-\frac{q}{4}\right)^2 = r$

$\therefore 3q^2 = 16r$ — 3

Criteria: $x = \alpha, 3\alpha$ $\frac{1}{2}$; $4\alpha = -q$ $\frac{1}{2}$; $3\alpha^2 = r$ $\frac{1}{2}$; $\alpha = -\frac{q}{4}$ $\frac{1}{2}$; $3(-\frac{q}{4})^2 = r$ $\frac{1}{2}$; $3q^2 = 16r$ $\frac{1}{2}$

b)(i) $\triangle ABE$ is equilateral

$\therefore \angle ABE = 60°$

$\angle EBC = 90 - \angle ABE$ (angle of square)

$= 30°$ — 1

Criteria: $\angle ABE = 60°$ ($\angle$ in equilateral Δ) $\frac{1}{2}$; $\angle EBC = 90 - \angle ABE$ ($\angle$ square) $= 30°$ $\frac{1}{2}$

(ii) In $\triangle EBC$ and $\triangle EAD$ — $-\frac{1}{2}$ PER WRONG REASON. FACT 1 wrong = 2 MARKS.

$EB = EA$ (given)

$BC = AD$ (sides of square)

$\angle DAE = 90 - 60$

$= 30$

$= \angle EBC$

$\therefore \triangle EBC \equiv \triangle EAD$ (SAS) — 3

(iii) area $= \frac{1}{2}ab\sin C$

$AB = AE$ (given)

$\therefore$ area $= \frac{1}{2} \times k \times k \times \sin 60$ — 1

$= \frac{k^2}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{k^2\sqrt{3}}{4}$ — 2

iv) area $\triangle CDE$ = area$_{ABCD}$ − area$_{AEB}$ − 2 area$_{DAE}$

$= k^2 - \frac{k^2\sqrt{3}}{4} - 2\left(\frac{1}{2} \times k \times k \times \sin 30°\right)$

$= k^2 - \frac{k^2\sqrt{3}}{4} - \frac{k^2}{2}$ — 3

$= \frac{k^2(2 - \sqrt{3})}{4}$