St Catherine's School

Waverley

Student Name: ______________________

Teacher Name: ______________________

# Year 11 Mathematics

## Preliminary Task #2

## 3rd May 2011

| | |
|---|---|
| ***Time allowed:*** | ***90 minutes + 5 minutes reading time*** |
| ***Total marks:*** | ***80 marks*** |
| ***Weighting:*** | ***25%*** |

INSTRUCTIONS

- There are 4 questions of different values.
- Marks for each part of a question are indicated.
- Questions 1 & 2 should be attempted in one booklet.
- Questions 3 & 4 should be attempted in a separate booklet
- Start each question on a new page.
- All necessary working should be shown.
- Approved scientific calculators and drawing templates may be used.
- Marks may be deducted for careless or badly arranged work.



QUESTION 1 Start a new booklet 30 marks

(a) The mass of 1 atom of oxygen is $2.7\times10^{-23}$ grams. What is the mass of $6.23\times10^{28}$ atoms of oxygen? Give your answer in scientific notation correct to 2 significant figures 2

(b) Simplify $\dfrac{x^3-y^3}{x^2-y^2}$ 2

(c) Solve $2x^2-4x-1=0$, leaving answers in exact surd form. 2

(d) Solve the following for $x$ and $y$ simultaneously

$$y=x-8 \quad\text{and}\quad 2x-y=18$$ 3

(e) Solve (i) $-2\le 4-2x$ 2

(ii) $|5x-1|\le 9$ 3

(iii) $\dfrac{1}{2}(x+2)+\dfrac{1}{3}(2x-4)=9$ 3

(f) Find the value of $x$ and $y$ if $\dfrac{\sqrt{3}-2}{2+\sqrt{3}}=x+\sqrt{y}$ 3

(g) Given that $p=\left(\sqrt{5}-\sqrt{3}\right)^2+\sqrt{60}$, find $p$ in its simplest form 2

(h) Express $\dfrac{x+1}{x^2-x}-\dfrac{x-1}{x^2+x}$ as a fraction in simplest form 3

(i) Solve (i) $3^{2x+1}=9$ 2

(ii) $|2x+1|=3x-4$ 3

QUESTION 2 Start a new page 20 marks

(a) State the natural domain and range of each of the following functions:

(i) $x^2 + y^2 = 25$ 2

(ii) $y = 3^x$ 2

(iii) $y = 3x^2 - 2$ 2

(b) Neatly sketch each of the following on separate axes. Clearly label all essential features, including any intercepts and asymptotes.

(i) $y = |x| + 2$ 2

(ii) $y = x^3 - 2$ 2

(iii) $y = \frac{1}{x-4}$ 2

(c) Sketch the graph of $y = (x-1)^2$ *for* $-1 \le x \le 2$. State the range of the function. 3

(d) (i) Sketch the curve $y = 4 - 3x - x^2$ onto a number plane 2

(ii) What is the maximum value of $y = 4 - 3x - x^2$? 2

(iii) Hence or otherwise, solve the inequation $4 - 3x - x^2 \le 0$ 1



QUESTION 3 Start a new booklet 12 marks

(a) Show whether the function $g(x) = \frac{x^3}{x^4 - 4}$ is odd, even or neither 2

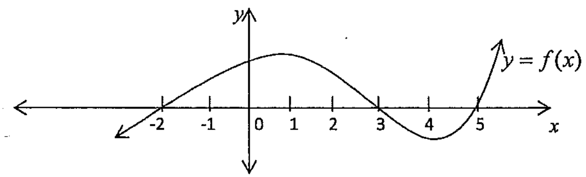
(b) A piecewise function, $f(x)$, is defined as follows:

$$f(x) = \begin{cases} ax & \text{for } x \le -1 \\ 4 & \text{for } -1 < x \le 2 \\ 3 - x & \text{for } x > 2 \end{cases}$$

If $f(-2) = f(0) + f(4)$, find the value of $a$. 3

(c) If $g(x) = 3x^2 - 5x + 4$ solve $g(x) = 6$. 2

(d)




For the graph of $y = f(x)$ above, state the value(s) of $x$ for which $f(x)$ is increasing 2

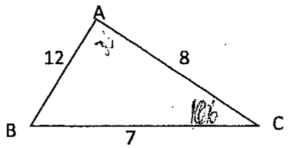
(e) Shade the region bounded by the following graphs: $y \le x + 1$ and $y \ge x^2 + 1$ 3

QUESTION 4 Start a new page 18 marks

(a) Find the exact value of $2\operatorname{cosec} 45° - \sec 60°$ 2

(b) Find the value of $b$ if $\cot (3b+5)° = \tan (2b-20)°$ 2

(c)

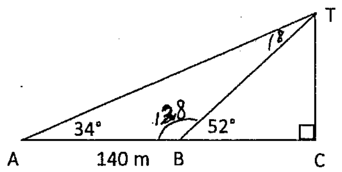



In triangle $ABC$ $a = 7,\ b = 8\ and\ c = 12$.

(i) Find the size of $\angle ABC$ to the nearest degree 2

(ii) Hence, or otherwise, find the area of the triangle $ABC$ 1

(d)




In the diagram A and B are two points in the same horizontal plane as the base of the television tower CT.

(i) If AB =140 metres, $\angle TAB = 34°$ and $\angle TBC = 52°$,

show that $TB = \dfrac{140 \sin 34°}{\sin 18°}$ 2

(ii) Calculate the height of the tower to the nearest metre 2





(e) Two ships sail in a straight line from a port B. The first ship sails 12 km in the direction 050°T and the second ship sails 20 km in the direction 110°T at the same time.

(i) Draw a picture of the information described 1

(ii) Show that the ships are 17 kilometres apart (to the nearest km) 3

(iii) What is the bearing of the first ship as seen from the second ship? 3

END OF TEST

YR 11 2 UNIT TASK 2 2011 [redacted]

a) $(2.7\times10^{-23})\times(6.23\times10^{28})$

$= 1682100$

$= 1.7\times10^{6}$ 2

b) $\dfrac{x^3-y^3}{x^2-y^2} = \dfrac{(x-y)(x^2+xy+y^2)}{(x-y)(x+y)}$

$= \dfrac{x^2+xy+y^2}{x+y}$ 2

c) $2x^2-4x-1=0$

$a=2 \quad b=-4 \quad c=-1$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-4)\pm\sqrt{(-4)^2-4\times2\times-1}}{2\times2}$

$= \dfrac{4\pm\sqrt{16+8}}{4}$

$= \dfrac{4\pm\sqrt{24}}{4}$

$= \dfrac{4\pm2\sqrt6}{4}$

$= \dfrac{2(2\pm\sqrt6)}{4}$

$= \dfrac{2\pm\sqrt6}{2}$ 2

d) $y=x-8$ (1) $\quad 2x-y=18$ (2)

sub (1) into (2)

$2x-(x-8)=18$

$2x-x+8=18$

$x=10$

sub $x=10$ into (1)

$y=10-8 \quad \therefore y=2$ 3

e)(i) $-2\le4-2x$

$-6\le-2x$

$\dfrac{-6}{-2}\ge x$

$\therefore x\le3$ 2

(ii) $|5x-1|\le9$

$5x-1\le9$ OR $-(5x-1)\le9$

$5x\le10 \qquad 5x-1\ge-9$

$x\le2 \qquad 5x\ge-8$

$x\ge-\dfrac{8}{5}$ 3

ie $-\dfrac{8}{5}\le x\le2$

(iii) $\dfrac{1}{2}(x+2)+\dfrac{1}{3}(2x-4)=9$

$\dfrac{3(x+2)+2(2x-4)}{6}=9$

$3x+6+4x-8=54$

$7x-2=54$

$7x=56$

$x=8$ 3

(f) $\dfrac{\sqrt3-2}{2+\sqrt3} = \dfrac{\sqrt3-2}{2+\sqrt3}\times\dfrac{2-\sqrt3}{2-\sqrt3}$

$= \dfrac{2\sqrt3-4-3+2\sqrt3}{4-3}$

$= -7+4\sqrt3$

$= -7+\sqrt{48}$

$\therefore x=-7 \quad y=48$ 3

(g) $p=(\sqrt5-\sqrt3)^2+\sqrt{60}$

$=5-2\sqrt{15}+3+2\sqrt{15}$

$=8$ 2

(h) $\dfrac{x+1}{x^2-x}-\dfrac{x-1}{x^2+x}$

$= \dfrac{x+1}{x(x-1)}-\dfrac{x-1}{x(x+1)}$

$= \dfrac{(x+1)(x+1)-(x-1)(x-1)}{x(x-1)(x+1)}$

$= \dfrac{x^2+2x+1-(x^2-2x+1)}{x(x-1)(x+1)}$

$= \dfrac{4}{(x-1)(x+1)}$ 3

(i)(i) $3^{2x+1}=9$

$3^{2x+1}=3^2$

$2x+1=2$

$2x=1$

$x=\dfrac{1}{2}$ 2

(ii) $|2x+1|=3x-4$

$2x+1=3x-4$ OR $-(2x+1)=3x-4$

$-x=-5 \qquad -2x-1=3x-4$

$x=5 \qquad -5x=-3$

$x=\dfrac{3}{5}$

st $x=5$: $|2\times5+1|=3\times5-4$, $11=11$ ✓

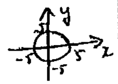
test $x=\dfrac{3}{5}$: $|2\times\frac{3}{5}+1|=3\times\frac{3}{5}-4$ ✗

∴ only $x=5$ is a solution 3

QUESTION 2

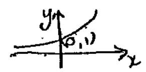
a) (i) D: $-5\le x\le5$

R: $-5\le y\le5$ 2



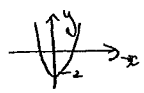
(ii) D: all real $x$

R: $y>0$ 2

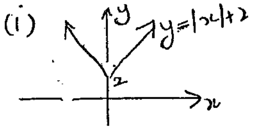


(iii) D: all real $x$

R: $y\ge-2$ 2

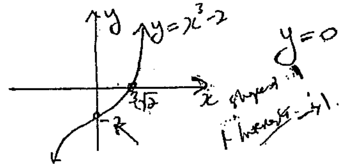


c) (i) $x=0 \quad y=2$ 2



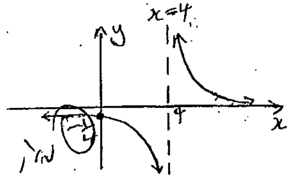

(ii) $y=0 \Rightarrow x^3=2$

$x=\sqrt[3]{2}$ 2




(iii) $x=0 \Rightarrow y=\dfrac{1}{0-4}$

$y=-\dfrac{1}{4}$ 2



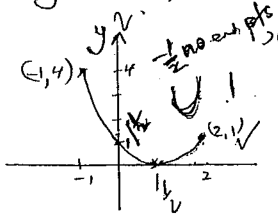

d) $y=(x-1)^2$

$x=-1 \quad y=(-1-1)^2=4$

$x=2 \quad y=(2-1)^2=1$

$x=0 \quad y=1$

$y=0: \quad 0=(x-1)^2, \; x=1$ 3



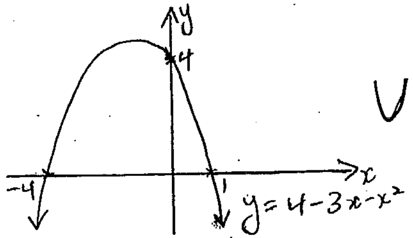
(d) (i) $y=4-3x-x^2$

$y=(4+x)(1-x)$

when $y=0$ $x=-4,1$

when $x=0$ $y=4$

concave down 2




(ii) axis of symmetry

$x=\dfrac{-4+1}{2}$

$=-\dfrac{3}{2}$

sub $x=-\dfrac{3}{2}$ into $y=4-3x-x^2$

$y=4-3(-\frac{3}{2})-(-\frac{3}{2})^2$

$=4+\dfrac{9}{2}-\dfrac{9}{4}$

$=6.25$

∴ max value is 6.25 2

(iii) $4-3x-x^2\le0$

when $x\le-4$ and $x\ge1$ 1

$g(x) = \frac{x^3}{x^4-4}$ (2)

$g(-x) = \frac{(-x)^3}{(-x)^4-4}$

$= \frac{-x^3}{x^4-4}$

$g(x) = -\frac{x^3}{(x^4-4)}$ 1

$g(-x) = -g(x)$ ∴ odd function 1 2

$f(-2) = -2a$
$f(0) = 4$
$f(4) = 3-4 = -1$

1 FOR ANY CORRECT (3)
1 FOR CORRECT EXPRESSION

$-2a = 4 + -1$ 1

$-2a = 3$

$a = -\frac{3}{2}$ 1

3

$g(x) = 3x^2 - 5x + 4$ (2)

$6 = 3x^2 - 5x + 4$

$0 = 3x^2 - 5x - 2$

$0 = (3x+1)(x-2)$ 1

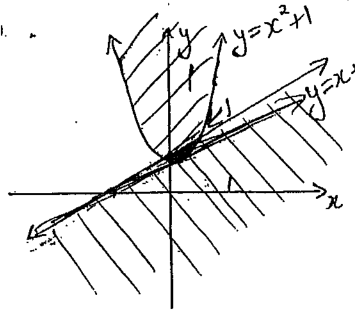
$3x+1 = 0$ OR $x-2=0$

$x = -\frac{1}{3}$  $x = 2$ 1

2

(2) increasing for $x < 1$ and $x > 4$ 2

$1\frac{1}{2}$ FOR ≤




1 FOR GRAPHS
1 FOR ONE CORRECT REGION
(3)

$y \geq x^2+1$
test (0, 0)
$0 \geq 0^2+1$
$0 \geq 1$ false

$y \leq x+1$
test (0, 0)
$0 \leq 0+1$ true

3

Question 4

4(a) $2\operatorname{cosec}45° - \sec 60°$

$= 2 \times \sqrt{2} - 2$

$= 2\sqrt{2} - 2$ 2

(b) $\cot(3b+5) = \tan(2b-20)$

$\tan(90-(3b+5)) = \tan(2b-20)$

$85 - 3b = 2b - 20$

$105 = 5b$

$b = 21$ 2

(c)(i) $\cos B = \frac{12^2+7^2-8^2}{2\times12\times7}$

$= 0.7678...$

$\angle ABC = 40°$ (nearest degree) 2

(ii) area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times12\times7\times\sin40°$

$= 27$ units$^2$ 1

(d)(i) $\angle ATB = 52 - 34$
$= 18$

using sine rule

$\frac{TB}{\sin34°} = \frac{140}{\sin18}$

$TB = \frac{140\sin34°}{\sin18}$ 2

(ii) In △TBC

$\sin52 = \frac{TC}{TB}$

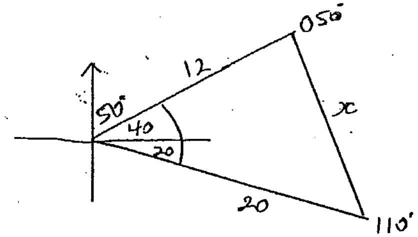
$TC = TB \times \sin52$

$= \frac{140\sin34}{\sin18} \times \sin52$

$= 200$ m (nearest metre) 2

(e)

(i)



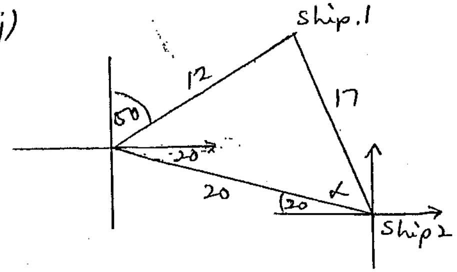

1

(ii) $x^2 = 12^2 + 20^2 - 2\times12\times20\times\cos60$

$= 304$

$x = 17$ km (nearest km) 3

(iii)




$\frac{\sin\alpha}{12} = \frac{\sin60}{17}$

$\sin\alpha = \frac{12\sin60}{17}$

$= 37°$ (nearest degree)

∴ bearing of ship A from ship B
is $270 + 20 + 37$
ie $327°T$. 3