St George Girls High School

Year 11

Common Test – 1

2011

Mathematics Extension 1

Marks: 60

Instructions

1. Working time – 75 minutes
2. All questions should be attempted.
3. Show all working.
4. **Start each question on a new page.**
5. Marks will be deducted for careless work or poorly presented solutions.
6. On the cover sheet of the answer booklet clearly show:

   a) your name
   b) your mathematics class and teacher

Question 1: (10 Marks) – Start A New Page Marks

a) If $(3x^m)^3 \times (3x)^{m-6} = ax^2$ 2

Find the value of a and m

b) Solve simultaneously 4

$$x + y + z = 6$$

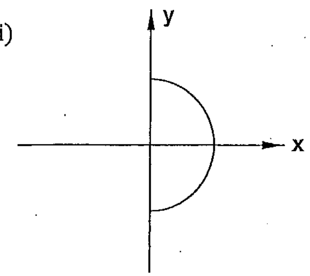
$$2x - y + z = 1$$

$$x + y - 2z = -9$$

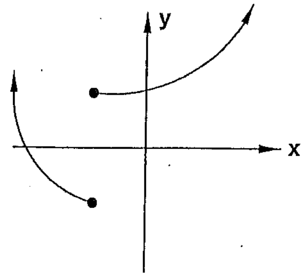
c) Find the inverse function $y = f^{-1}(x)$ if $f(x) = \frac{2x-1}{x+3}$ 2

d) Which of the following curves represent a function $y = f(x)$ 2

(i)



(ii)



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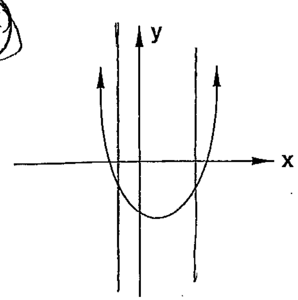
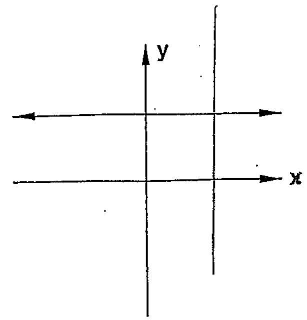


(iv)



Question 2: (10 Marks) – Start A New Page **Marks**

a) Suppose $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 3, 5, 7, 9\}$

with universal set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

List the members in 3

(i) $A \cup B$

(ii) $A \cap B$

(iii) $\bar{A} \cap \bar{B}$

b) If $n(A) = 15$, $n(B) = 24$ and $n(A \cap B) = 4$

Find $n(A \cup B)$ 1

c) If $\log_a 2 = A$, $\log_a 3 = B$, $\log_a 5 = C$

Find an expression in terms of A, B and C

(i) $\log_a 36$ 2

(ii) $\log_a 0.3$ 2

(iii) $\log_a \frac{5}{\sqrt[5]{a}}$ 2

Question 3: (10 Marks) – Start A New Page **Marks**

a) Solve for x (correct to 2 decimal places where necessary)

(i) $8^{x+1} = 2.4^{x-1}$ 2

(ii) $5^x = 9$ 2

b) Given the function $f(x) = 3^x$ and $g(x) = 2x + 1$ 2

Find $f(g(2))$

c) What would be a possible restriction on the domain of $y = x^2 + 9x + 14$ so that its inverse relation is a function? 2

d) (i) What is the natural domain of the function $y = \sqrt{x - 4}$? 1

(ii) What is the range of the above function? 1

Question 4: (10 Marks) – Start A New Page Marks

a) Rationalise the denominator 1

$$\frac{1}{\sqrt[3]{3}}$$

b) (i) Factorise $10 + 3x - x^2$ 1

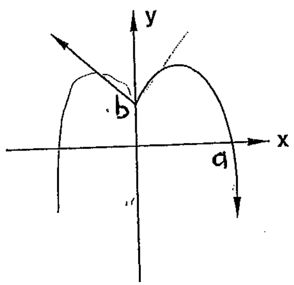
(ii) Find the intercepts and vertex of the parabola $y = 10 + 3x - x^2$ 2

(iii) Sketch the parabola showing all important features. 2

c) Find the number of terms in the series 2

$$128 + 162 + 196 + \cdots + 740$$

d) Below is the graph of $y = f(x)$ 2



On the number planes provided sketch the graph of:

(i) $y = -f(x)$

(ii) $y = f(-x)$

Question 5: (10 Marks) – Start A New Page Marks

a) Simplify 1

$$3^{n-2} \times 9^{n+1}$$

b) By completing the square find the centre and radius of the circle 3

$$x^2 + y^2 + 4x + 6y = 12$$

c) In an arithmetic progression $T_4 + T_9 = 109$, $T_6 + T_{15} = 181$. Find the first term, the common difference and the sum to 15 terms. 3

d) A square number is divisible by 7 if and only if the square root of that number is divisible by 7. 3

Without proving the above, prove that $\sqrt{7}$ is irrational.

Question 6: (10 Marks) – Start A New Page **Marks**

a) The sum of the first n terms of a series is given by $5n - 3n^2$. Find the sum of the first $(n-1)$ terms and hence find an expression for the nth term. 2

b) Evaluate 2

$$16 - 8 + 4 - 2 + 1 - \frac{1}{2} + \cdots$$

c) If a, b, c are terms in an arithmetic sequence show that $2^a, 2^b, 2^c$ are terms in a geometric sequence. 3

d) The sum of the first ten terms of the series 3

$$\log_2\left(\frac{1}{x}\right) + \log_2\left(\frac{1}{x^2}\right) + \log_2\left(\frac{1}{x^3}\right) + \cdots$$

is -110. Find the value of x

End of Paper

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Question 1

a) $(3x^m)^3 \times (3x)^{m-6} = ax^2$

LHS $3^3 . x^{3m} \times 3^{m-6} . x^{m-6}$

$= 3^{3+(m-6)} \times x^{3m+m-6}$

$= 3^{m-3} \times x^{4m-6}$

Now $3^{m-3} \times x^{4m-6} = ax^2$

Then $a = 3^{m-3}$ and $4m-6=2$

$4m=8$

$\therefore \underline{m=2}$

Substitute $m=2$ into $a=3^{m-3}$

$a=3^{2-3}$

$a=3^{-1}$

$\underline{a=\frac{1}{3}}$

b) $x+y+z=6$ ①

$2x-y+z=1$ ②

$x+y-2z=-9$ ③

①+② we obtain

$3x+2z=7$ ④

②+③ we get

$3x-z=-8$ ⑤

we have $3x+2z=7$ ④

$3x-z=-8$ ⑤

④−⑤

$3z=15$

$z=5$

Subst $z=5$ into ④ we obtain

$3x+2(5)=7$

$3x+10=7$

$3x=-3$

$x=-1$

Subst $x=-1$ and $z=5$ into ① we get

$-1+y+5=6$

$y+4=6$

$y=2$

Solns are

$x=-1$, $y=2$, $z=5$

c) $f(x) = \dfrac{2x-1}{x+3}$

$$y = \frac{2x-1}{x+3}$$

Inverse,

$$x = \frac{2y-1}{y+3}$$

$$x(y+3) = 2y-1$$

$$xy + 3x = 2y - 1$$

$$3x + 1 = 2y - xy$$

$$3x + 1 = y(2-x)$$

$$\therefore y = \frac{3x+1}{2-x}$$

so $f^{-1}(x) = \dfrac{3x+1}{2-x}$

d) (iii) and (iv)

Question 2

a) $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 3, 5, 7, 9\}$

$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

(i) $A \cup B = \{1, 2, 3, 5, 7, 9, 11\}$

(ii) $A \wedge B = \{3, 5, 7\}$

(iii) $\bar{A} \wedge \bar{B}$

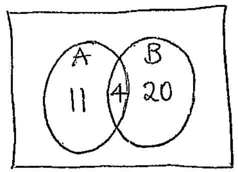
$\bar{A} = \{1, 4, 6, 8, 9, 10\}$

$\bar{B} = \{2, 4, 6, 8, 10, 11\}$

So $\bar{A} \wedge \bar{B} = \{4, 6, 8, 10\}$

b) $n(A) = 15$ $\quad n(A \wedge B) = 4$ $\quad$ Find $n(A \cup B)$

$n(B) = 24$



$\therefore n(A \cup B) = 24 + 11$

$= 35$

c) $\log_a 2 = A$, $\log_a 3 = B$, $\log_a 5 = C$

(i) $\log_a 36 = \log_a (4\times 9)$

$= \log_a 4 + \log_a 9$

$= \log_a 2^2 + \log_a 3^2$

$= 2\log_a 2 + 2\log_a 3$

$= 2A + 2B$

(ii) $\log_a 0{\cdot}3 = \log_a\left(\frac{3}{10}\right)$

$= \log_a 3 - \log_a 10$

$= \log_a 3 - (\log_a (5\times 2))$

$= \log_a 3 - [\log_a 5 + \log_a 2]$

$= B - [C + A]$

$= B - C - A$

(iii) $\log_a \frac{5}{\sqrt[5]{a}} = \log_a 5 - \log_a a^{\frac{1}{5}}$

$= C - \frac{1}{5}\log_a a$

$= C - \frac{1}{5}$

Question 3

a) (i) $8^{x+1} = 2.4^{x-1}$

$2^{3(x+1)} = 2^1 . 2^{2(x-1)}$

$2^{3(x+1)} = 2^{1+2(x-1)}$

$2^{3x+3} = 2^{1+2x-2}$

$2^{3x+3} = 2^{2x-1}$

$\therefore 3x+3 = 2x-1$

$\therefore x = -4$

(ii) $5^x = 9$

$\log_{10} 5^x = \log_{10} 9$

$x\log_{10} 5 = \log_{10} 9$

$\therefore x = \frac{\log_{10} 9}{\log_{10} 5}$

$x = 1{\cdot}37$ (to 2 decp

b) $f(x) = 3^x$ and $g(x) = 2x+1$

Find $f(g(2))$

Now $g(2) = 2(2)+1$

$g(2) = 5$

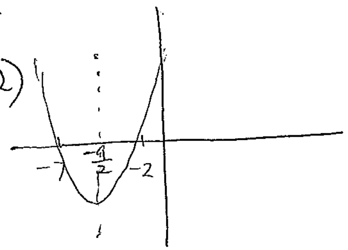
$\therefore f(g(2)) = 3^5$

$= 243$

c) $y = x^2 + 9x + 14$

$y = (x+7)(x+2)$

OR $x = \frac{-9}{2}$



Either restrict domain for

$x \geq -\frac{9}{2}$

OR $x \leq -\frac{9}{2}$

d) (i) $y=\sqrt{x-4}$

Domain: $x-4 \geq 0$

$\underline{x \geq 4}$

Range: $\underline{y \geq 0}$

<u>Question 4</u>

a) $\frac{1}{\sqrt[3]{3}} \times \frac{\sqrt[3]{9}}{\sqrt[3]{9}} = \frac{\sqrt[3]{9}}{\sqrt[3]{3\times 9}} = \frac{\sqrt[3]{9}}{\sqrt[3]{27}} = \frac{\sqrt[3]{9}}{3}$

b) (i) $10+3x-x^2$

$=(5-x)(x+2)$

(ii) $y=(5-x)(x+2)$

x intercepts, $x=-2$, $x=5$

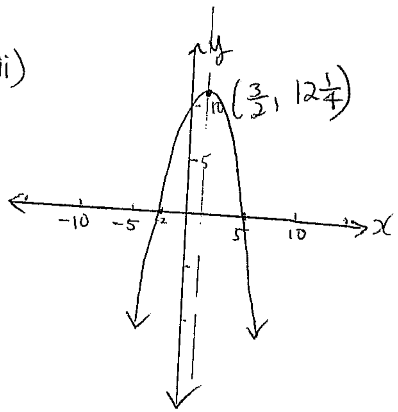
y-intercept $y=10$

Axis of symmetry $x=+\frac{3}{2}$

Vertex $y=10+3\left(\frac{3}{2}\right)-\left(\frac{3}{2}\right)^2$

$y=10+\frac{9}{2}-\frac{9}{4}$

(iii)



c) $a=128$

$d=T_3-T_2=T_2-T_1$

$d=34$

and $L=740$

$T_n=740$

so $a+(n-1)d=740$

$128+(n-1).34=740$

$128+34n-34=740$

$94+34n=740$

$34n=646$

$n=19$

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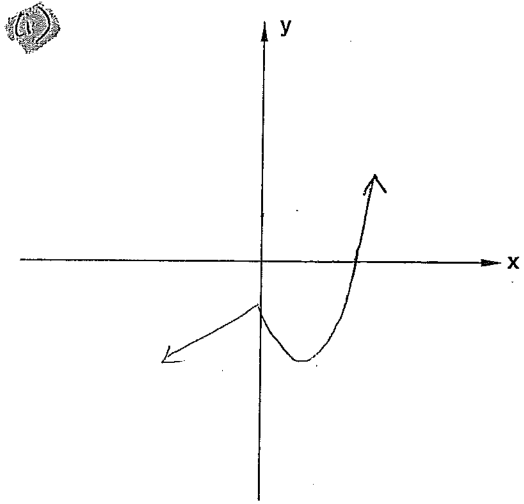
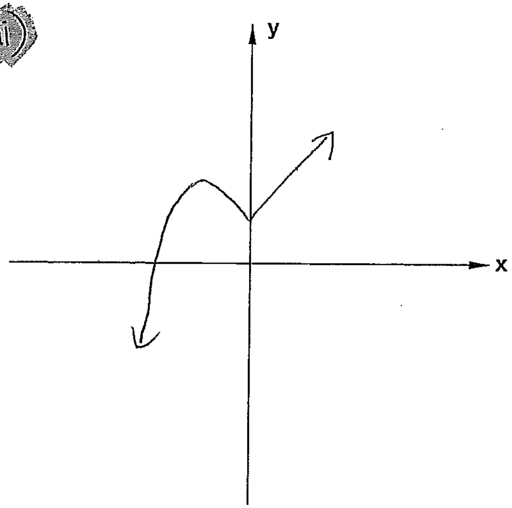
Question 4   d)

(i)



(ii)



Question 5

a) $3^{n-2} \times 9^{n+1}$

$= 3^{n-2} \times 3^{2(n+1)}$

$= 3^{n-2+2n+2}$

$= 3^{3n}$

b) $x^2+y^2+4x+6y=12$

$x^2+4x+y^2+6y=12$

$\left(\frac{4}{2}\right)^2 = 2^2 = 4$

$x^2+4x+4+y^2+6y+9=12+4+9$

$(x+2)^2+(y+3)^2=25$

and $\left(\frac{6}{2}\right)^2 = 3^2 = 9$

Centre $(-2,-3)$

radius 5

c) $T_4+T_9=109$, $T_6+T_{15}=181$

$a+3d+a+8d=109$

$2a+11d=109$ ①

$a+5d+a+14d=181$

$2a+19d=181$ ②

$2a+11d=109$ ①

$2a+19d=181$ ②

② − ① we get,

$8d=72$

$d=9$

Subst. $d=9$ into ① we obtain $2a+11\times9=109$

$2a=10$

so

$S_{15}=\frac{15}{2}\left[2\times5+14\times9\right]$

$S_{15}=1020$

d) Assume that $\sqrt{7}$ is rational

ie $\sqrt{7}=\frac{a}{b}$ where $b\neq0$ and a and b have no common factors

$7=\frac{a^2}{b^2}$

$7b^2=a^2$ ①

$\therefore$ a^2 is divisible by 7

Hence a is divisible by 7

$\therefore$ Let $a=7k$ where k is an integer

Subst. $a=7k$ into ① we get,

$7b^2=(7k)^2$

$7b^2=49k^2$

$\therefore b^2=7k^2$

$\therefore$ b^2 is divisible by 7

$\therefore$ b is divisible by 7

Which means a and b have a common factor of 7, which contradicts our original assumption that $\sqrt{7}$ is rational

$\therefore \sqrt{7}$ is irrational

Question 6

a) $S_n = 5n - 3n^2$

$S_{n-1} = 5(n-1) - 3(n-1)^2$

$= 5n - 5 - 3(n^2 - 2n + 1)$

$= \cancel{5n} - 5 - 3n^2 + \cancel{6n} - 3$

$S_{n-1} = 11n - 3n^2 - 8$

So $T_n = S_n - S_{n-1}$

$T_n = 5n - 3n^2 - (11n - 3n^2 - 8)$

$T_n = 5n - 3n^2 - 11n + 3n^2 + 8$

$T_n = 8 - 6n$

b) $a = 16$
$r = -\frac{1}{2}$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{16}{1 - -\frac{1}{2}}$

$S_\infty = \frac{16}{\frac{3}{2}}$

$S_\infty = \frac{32}{3}$ OR $10\frac{2}{3}$

c) If a, b, c are in arithmetic sequence,

$d = c - b = b - a$

so $c - b = b - a$ ①

If $2^a, 2^b, 2^c$ are in geometric sequence

then

$r = \frac{T_3}{T_2} = \frac{T_2}{T_1}$

so $\frac{2^c}{2^b} = \frac{2^b}{2^a}$

$2^{c-b} = 2^{b-a}$

$\therefore c - b = b - a$

Now since we know from above, $c - b = b - a$

$\therefore 2^a, 2^b, 2^c$ are in geometric sequence.

d) $\log_2\left(\frac{1}{x}\right)$

$= \log_2 1 - \log_2 x$

$= -\log_2 x$

$\log_2\left(\frac{1}{x^2}\right)$

$= \log_2 1 - \log_2 x^2$

$= \log_2 1 - 2\log_2 x$

$= -2\log_2 x$

$\log_2\left(\frac{1}{x^3}\right)$

$= \log_2 1 - \log_2 x^{+3}$

$= 0 - +3\log_2$

$= 3\log_2 x$

Arithmetic Sequence $a = \log_2 x$

$d = -\log_2 x$

since, $S_{10} = -110$ then

$-110 = \frac{10}{2}\left[2 \times \log_2 x + 9 \times \log_2 x\right]$

$-110 = 5\left[-2\log_2 x - 9\log_2 x\right]$

$-22 = -11\log_2 x$

$+2 = \log_2 x$

$2^{+2} = x$

$\therefore x = 4$