SYDNEY GRAMMAR SCHOOL

2009 Progress Test Examination

FORM I
MATHEMATICS

Tuesday 19th May 2009

General Instructions
- Writing time — 1 hours 30 minutes
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper
- Total marks — 100
- All ten questions may be attempted.
- All ten questions are of equal value.

Collection
- Write your name, class and master clearly on each page of your answers.
- Staple your answers in a single bundle.
- The question papers will be collected separately.

1BR/ADS:	SJE	1DBD/CJW:	BR	1FHB/MW:	MW
1JAG/DJM:	REP	1JFC/PKR:	JMR	1PGM/AHWD:	RCF
1RDWL/LDR:	SO	1WTR/AGY:	BR		

Checklist
- Writing paper required.
- Candidature — 186 boys

Test writer
SO

QUESTION ONE (10 marks) Start a new page.

(a) Find the value of:
- (i) $37 + 58$
- (ii) $42 - 17$
- (iii) $153 \div 9$
- (iv) 31×13
- (v) $9 + 27 \div 3$

(b) Write the numeral for seven hundred and fifteen thousand and nine.

(c) Simplify:
- (i) $9w - 3w$
- (ii) $9w \div 3w$
- (iii) $9w \times 3w$

(d) Write the number 319 in Roman numerals.

QUESTION TWO (10 marks) Start a new page.

(a) Write $\frac{18}{54}$ in lowest terms.

(b) Calculate:
- (i) $\frac{3}{15} + \frac{2}{3}$
- (ii) $3\frac{5}{6} - 1\frac{5}{12}$
- (iii) $\frac{6}{5} \times \frac{2}{9}$
- (iv) $\frac{4}{5} \div \frac{2}{15}$

(c) George has \$98 to spend at the Royal Easter Show. He spends $\frac{3}{7}$ of his money on rides. What amount of money does he have left?

Exam continues next page ...

QUESTION THREE (10 marks) Start a new page.

(a) Rewrite the following calculation, replacing the words by mathematical symbols:
The quotient of sixty three and nine is seven.

(b) Write the following expressions in simplest form, without using $\times$ and $\div$ symbols.

(i) $w \times 7 + 3 \times x$

(ii) $a \times a \times a \div (3 \times b)$

(c) If $p = 3$ and $q = \frac{1}{2}$, find the value of:

(i) $5p - 1$

(ii) $4p \times q$

(d) Using the rule $y = x^2 + 3$, copy and complete the following table of values:

x	0	1	2
y			

(e) Which of the signs $<$, $=$ or $>$ should be placed in each box to make a true statement?

(i) $\frac{45}{24} \square 1\frac{7}{8}$

(ii) $\frac{7}{12} \square \frac{12}{48}$

(f) Copy the statement $7 \times 8 - 11 \div 15 = 3$ and insert brackets where necessary to make it true.

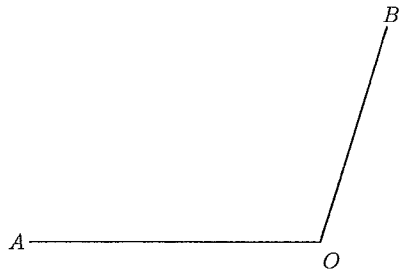
Exam continues overleaf ...

QUESTION FOUR (10 marks) Start a new page.

(a) (i) List all the factors of 84.

(ii) The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
Find the highest common factor of 24 and 84.

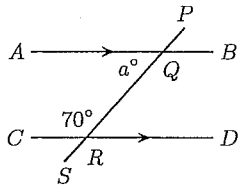
(b)



Use a protractor to measure the size of $\angle AOB$ shown above. Write your answer to the nearest degree.

(c) Write down the value of the pronumerals in the following diagrams, giving reasons.

(i)



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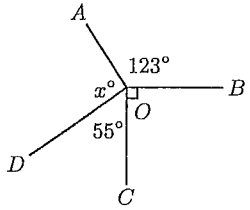

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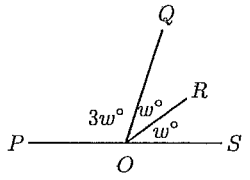

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QUESTION FIVE (10 marks) Start a new page.

(a) By using a factor tree or some other method, find the prime factorisation of 560.

(b) Given the prime factorisation of 13 608 is $2^3 \times 3^5 \times 7$ and the prime factorisation of 14 580 is $2^2 \times 3^6 \times 5$, answer the following questions.

(i) Find the highest common factor of 13 608 and 14 580. Leave your answer as a product of primes.

(ii) Find the lowest common multiple of 13 608 and 14 580. Leave your answer as a product of primes.

(c) Calculate:

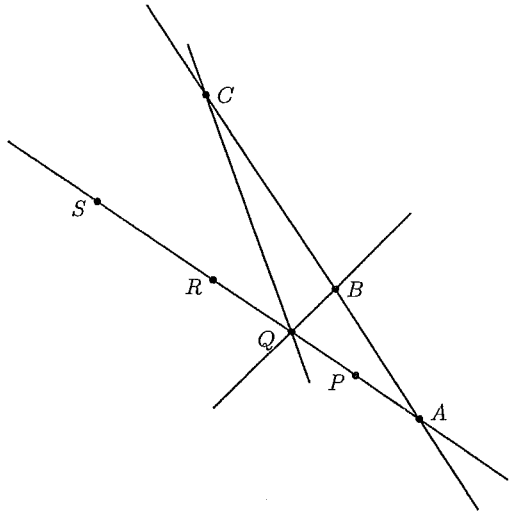
(i) $3\frac{4}{5} + 2\frac{7}{10}$

(ii) $5\frac{1}{4} - 3\frac{7}{12}$

(iii) $3\frac{1}{3} \div \frac{5}{9}$

QUESTION SIX (10 marks) Start a new page.

(a)



Using the diagram above, answer the following questions.

(i) Name three concurrent lines.

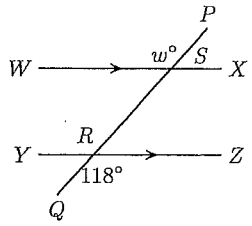
(ii) Name all points on the ray PQ.

(iii) Name all the intervals that contain both points P and Q.

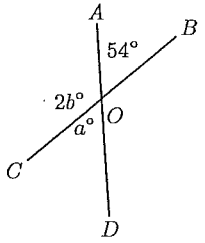
Exam continues overleaf ...

(b) Find the value of the pronumerals, giving full reasons.

(i)



(ii)



QUESTION SEVEN (10 marks) Start a new page.

(a) Simplify:

(i) $7a + 3b + 5a - 2b + 1$

(ii) $72x^2y \div 12xy$

(b) If 6 hotdogs cost \$21, how much will it cost to buy 9 hotdogs?

(c) Use long division to calculate $712 \div 29$. Show your working and express your answer as a mixed numeral.

(d) Find the number that is halfway between $\frac{2}{5}$ and $\frac{13}{20}$.

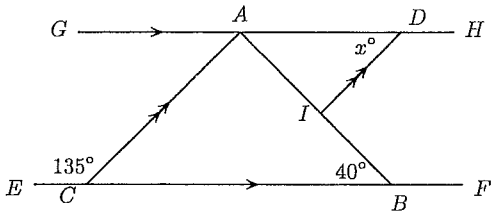
Exam continues next page ...

QUESTION EIGHT (10 marks) Start a new page.

(a) Simplify $4\frac{2}{5} - 1\frac{2}{3} \div \frac{6}{5} + 3\frac{3}{5}$.

(b) Calculate $736 \times 98 \div (\sqrt{49} + 42)$.

(c)



Find the value of x in the diagram above, giving reasons.

(d) Find 3 integers greater than one that multiply to 165.

(e) Simplify $\dfrac{3m \times 3m \times 3m}{3m + 3m + 3m}$.

(f) A packet of gum from a certain vending machine costs \$1·65. If a customer inserts \$2·00, what possible combinations of 5 cent, 10 cent and 20 cent coins could he get in change? List them all. This may involve combinations of 1, 2 or 3 coin types.

QUESTION NINE (10 marks) Start a new page.

(a) By using prime factors or otherwise, find the cube root of 9261.

(b) Jordan takes 60 seconds to run a lap of a 400 m oval, while Kieran takes 70 seconds to do the same. Assuming that they start together and continue to run at a constant rate, find the distance each boy has run when Jordan laps Kieran a second time.

(c) A boy ate 80 apples in five days. Each day he ate 5 more than the day before. How many apples did he eat on the first day?

(d) Sophie, Emma, John and Natasha received many chocolate eggs over the Easter holiday. To make them last longer, they decide to ration how many they will eat each day. Sophie, Emma and John will eat two eggs per day, while Natasha will eat only one egg per day. With this plan, there are enough eggs to last exactly 24 days.

Instead they don't follow this plan and they all eat two eggs per day. How many days will the eggs last?

Exam continues overleaf ...

QUESTION TEN (10 marks) Start a new page.

(a) I am thinking of a number.

The number is not an odd number.
It has exactly four factors.
If you reverse the digits, a prime number is formed.
The sum of the digits is a two digit prime number.
The number is less than the square root of 10^4.
One of the digits is a square number.

What is the number?

(b) Suppose we define the operation $\star$ to mean $a \star b = (a + b)(a \times b)$.

(i) Find $6\frac{1}{2} \star 3$.

(ii) Given $a \star a = 16$, find a.

(c) Find the value of the expression below, if $a = 2, b = 1, c = 0$ and $d = 4$.

$$\frac{\sqrt{5a - b}}{2d - (c + 4b)} + \frac{(a + dc)^2}{3a + 2b}$$

(d) Odd numbers can usually be written as the sum of a prime number and a power of two. This is true for all odd numbers greater than 3 and less than 100.
For example, $23 = 2^2 + 19$ or $23 = 2^4 + 7$.
There are some odd numbers that cannot be expressed as the sum of a prime number and a power of two. These have been called obstinate numbers. There are two obstinate numbers between 100 and 150. Find them.

END OF TEST

FIRST FORM SOLUTIONS

QUESTION 1

a)(i) 95 ✓
(ii) 25 ✓
(iii) 17 ✓
(iv) 403 ✓
(v) 18 ✓
b) 715009 ✓
c)(i) $6w$ ✓
(ii) 3 ✓
(iii) $27w^2$ ✓
d) CCCXIX ✓

QUESTION 2

a) $\frac{1}{3}$ ✓
b) (i) $\frac{2}{15} + \frac{2}{3}$
$= \frac{2}{15} + \frac{10}{15}$
$= \frac{12}{15}$ ✓
(ii) $3\frac{5}{6} - 1\frac{5}{12}$
$= 3\frac{10}{12} - 1\frac{5}{12}$ ✓
$= 2\frac{5}{12}$ ✓
(iii) $\frac{6}{5} \times \frac{2}{9}$
$= \frac{12}{45}$ ✓
$= \frac{4}{15}$ ✓
(iv) $\frac{4}{5} \div \frac{2}{15}$
$= \frac{4}{5} \times \frac{15}{2}$ ✓
$= 6$ ✓
c) Money spent on rides $= \frac{3}{7} \times 98$
$= \$42$ ✓
Remaining money $= 98 - 42$
$= \$56$ ✓

QUESTION 3

(a) $63 \div 9 = 7$ ✓
(b)(i) $7w + 3x$ ✓
(ii) $\frac{a^3}{36}$ ✓
c(i) $15 - 1$
$= 14$ ✓
(ii) $4 \times 3x\frac{1}{2}$
$= 6$ ✓
(d)

x	0	1	2
y	3	4	7

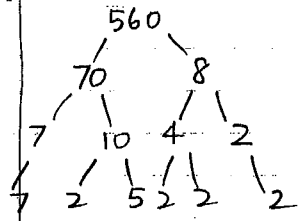
✓✓
(1 mark for two correct) ✓
(e) (i) $=$
(ii) $>$ ✓
(f) $(7 \times 8 - 11) \div 15 = 3$ ✓

QUESTION 4

(a)(i) Factors of 84: ✓✓
1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
(ii) HCF = 12 ✓
(b) $106° - 109°$ ✓
(c)(i) $a = 110°$ ✓✓
(cointerior angles are supplementary, AB||CD)
(ii) $x = 92°$ ✓
(angle of revolution) ✓
(iii) $5w = 180$ (straight angle) ✓
$w = 36°$ ✓

QUESTION 5

(a)



$560 = 2 \times 2 \times 2 \times 2 \times 5 \times 7$ ✓✓
$= 2^4 \times 5 \times 7$
(does not need to be in index form)
(b) (i) $2^2 \times 3^5$ ✓
(ii) $2^3 \times 3^6 \times 5 \times 7$ ✓
(c) (i) $3\frac{4}{5} + 2\frac{7}{10}$
$= 3\frac{8}{10} + 2\frac{7}{10}$ ✓
$= 5\frac{15}{10}$
$= 6\frac{1}{2}$ ✓
(ii) $5\frac{1}{4} - 3\frac{7}{12}$
$= 5\frac{3}{13} - 3\frac{7}{12}$
$= 4\frac{15}{12} - 3\frac{7}{12}$ ✓
$= 1\frac{8}{12}$
$= 1\frac{2}{3}$ ✓
(iii) $3\frac{1}{3} \div \frac{5}{9}$
$= \frac{10}{3} \times \frac{9}{5}$ ✓
$= \frac{2}{1} \times \frac{3}{1}$
$= 6$ ✓

QUESTION 6

(a)(i) SQ, CQ, BQ ✓
(ii) P, Q, R, S ✓
(iii) PQ, PR, PS, QA, RA, SA ✓✓
(1 mark for two correct)
(b)(i) $\angle YRP = 118°$
(vertically opposite angles are equal)
$\angle YRP = w°$
$= 118°$ ✓
(corresponding angles are equal, WX||YZ) ✓
(ii) $a = 54°$ ✓
(vertically opposite angles) ✓
$26 + 54 = 180°$ (straight angle) ✓
$b = 63°$ ✓

QUESTION 7

(a)(i) $12a + b + 1$ ✓✓
(ii) $6x$ ✓✓
(lose one mark for an error with one of the terms)
(b) 6 hotdogs cost \$21
1 hot dog cost \$3·50 ✓
9 hotdogs cost \$3·50 × 9
$= \$31·50$ ✓
(c)
$$\begin{array}{r} 24 \text{ r. } 16 \\ 29\overline{)712} \\ \underline{58} \\ 132 \\ \underline{116} \\ 16 \end{array}$$
$24\frac{16}{29}$ ✓✓
(d) $\left(\frac{2}{5} + \frac{13}{20}\right) \div 2$
$= \frac{8+13}{20} \times \frac{1}{2}$ ✓
$= \frac{21}{40}$ ✓

QUESTION 8

(a) $4\frac{2}{5} - \left(1\frac{2}{3} \div \frac{6}{5}\right) + \frac{3}{5}$

$= 8 - \left(\frac{5}{3} \times \frac{5}{6}\right)$ ✓

$= 8 - \frac{25}{18}$

$= 6\frac{11}{18}$ ✓

(b) $736 \times 98 \div (\sqrt{49} + 42)$

$= 736 \times 98 \div (7 + 42)$

$= 736 \times 98 \div 49$

$= 736 \times 2$

$= 1472$ ✓

(c) $\angle GAC = 45°$

(corinterior angles are supplementary, GH ∥ EF) ✓

$x = 45°$ (corresponding angles, AC ∥ DE) ✓

(d) $11 \times 3 \times 5$ ✓

(e) $\frac{3m \times 3m \times 3m}{3m + 3m + 3m}$

$= \frac{27m^3}{9m}$ ✓

$= 3m^2$ ✓

(f)

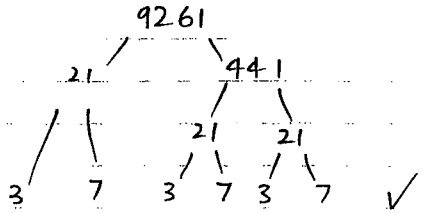
5 cent	10 cent	20 cent
7	0	0
5	1	0
3	2	0
3	0	1
1	3	0
1	1	1

✓✓

(1 mark for a partial list)

QUESTION 9

(a)



✓

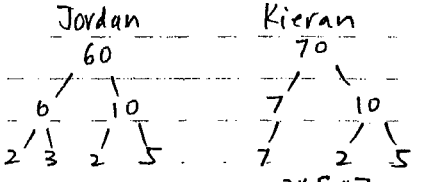
$\sqrt[3]{9261}$

$= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}$

$= 3 \times 7$

$= 21$ ✓

(b)



$2 \times 2 \times 3 \times 5$ $\quad 2 \times 5 \times 7$

$LCM = 2 \times 2 \times 3 \times 5 \times 7$

$= 420$ ✓

Jordan laps Kieran a 2nd time at 840s.

Jordan $= 840 \div 60$ $\quad$ Kieran $= 840 \div 70$

$= 14$ laps $\quad = 12$ laps

$= 5600m$ ✓ $\quad = 4800m$ ✓

(c) Day 1 □

Day 2 □ + 5

Day 3 □ + 10 ✓

Day 4 □ + 15

Day 5 □ + 20

5□ + 50 = 80 apples

5□ = 30 ✓

5 × 6 = 30

∴ On the first day the boy ate 6 apples ✓

(Guess and check method with working

(d) First plan $= 7 \times 24$

$= 168$ eggs ✓

Second plan $= 168 \div 8$

$= 21$ days ✓

QUESTION 10

(a) 74 ✓✓

(1 mark for systematic method to eliminate numbers)

(b)(i) $6\frac{1}{2} \star 3$

$= (6\frac{1}{2} \star 3)(6\frac{1}{2} \times 3)$

$= (\frac{13}{2} + 3)(\frac{13}{2} \times 3)$

$= \frac{19}{2} \times \frac{39}{2}$

$= \frac{741}{4}$ ✓

(i) $(a \times a)(a + a) = 16$

$2a \times a^2 = 16$ ✓

$2a^3 = 16$

$a^3 = 8$

$a = 2$ ✓

(c) $\frac{\sqrt{5 \times 2 - 1}}{2 \times 4 - (6 + 4)} + \frac{(2+6)^2}{3 \times 2 + 2 \times 1}$

$= \frac{\sqrt{9}}{4} + \frac{4}{8}$ ✓

$= \frac{3}{4} + \frac{4}{8}$

$= \frac{6}{8} + \frac{4}{8}$

$= \frac{10}{8}$

$= 1\frac{1}{4}$ ✓

(d) $101 = 2^6 + 37$

$103 = 2 + 101$

Not obstinate. Award 1 mark for some attempt at work.

$127 = 2^1 + 125$

$= 2^2 + 123$

$= 2^3 + 119$

$= 2^4 + 111$

$= 2^5 + 95$

$= 2^6$ ✓

$149 = 2^1 + 147$

$= 2^2 + 145$

$= 2^3 + 141$

$= 2^4 + 133$

$= 2^5 + 117$ ✓

$= 2^6 + 85$

$= 2^7 + 21$

∴ 127 and 149 are obstinate numbers