SYDNEY GRAMMAR SCHOOL

2010 Progress Test Examination

FORM I
MATHEMATICS

Wednesday 12th May 2010

General Instructions
- Writing time — 1 hours 30 minutes
- Write using black or blue pen.
- Calculators are not to be used.
- All necessary working should be shown in every question.
- Start each question on a new page.

Structure of the paper
- Total marks — 99
- All nine questions may be attempted.
- All nine questions are of equal value.

Collection
- Write your name, class and master clearly on each page of your answers.
- Staple your answers in a single bundle.
- Write your name and master on this question paper and submit it with your answers.

1BR/ADS:	SO	1RMF/CJW:	BR	1DBD/MW:	MW
1JAG/CDS:	TCW	1RBCH/JSH:	FMW	1SD/PKR:	JMR
1PGM/AHWD:	SJE	1WTR/AGY:	LYL		

Checklist
- Writing paper required.
- Candidature — 189 boys

Test writer
FMW

QUESTION ONE (11 marks) Start a new page.

(a) Find the value of:

(i) $32 + 16 - 11$

(ii) $543 - 248$

(iii) 14×30

(iv) $11 + 6 \times 3$

(b) Write 28 in Roman numerals.

(c) Simplify:

(i) $8x - 2x$

(ii) $10x \times 4y$

(iii) $4 \times q + 3 \times p$

(iv) $20xy \div 5x$

(v) $(5w)^2$

(d) Write the numeral for four hundred and twenty two thousand and thirty nine.

Exam continues next page ...

QUESTION TWO (11 marks) Start a new page.

(a) Between which two consecutive whole numbers does $\frac{9}{2}$ lie?

(b) Evaluate:

(i) $\frac{3}{7}+\frac{6}{7}$

(ii) $4-\frac{5}{8}$

(iii) $\frac{2}{9}\times\frac{5}{11}$

(iv) $\frac{4}{15}\div\frac{7}{3}$

(c) Which of the signs $<$, $=$ or $>$ should be placed in each box to make a true statement?

(i) $\frac{5}{9}\square\frac{19}{36}$

(ii) $\frac{3}{8}\square\frac{3}{4}$

(iii) $2\frac{15}{23}\square\frac{61}{23}$

(d) Fred drinks $\frac{2}{3}$ of a 375 mL can of soft drink. How much drink remains in the can? Give your answer in millilitres.

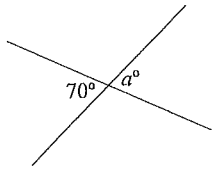
Exam continues overleaf ...

QUESTION THREE (11 marks) Start a new page.

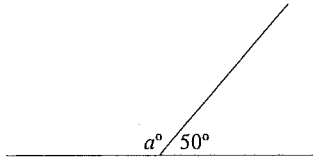
(a) What is the complement of $47°$?

(b) Find the value of a in each diagram below, giving a reason for each answer.

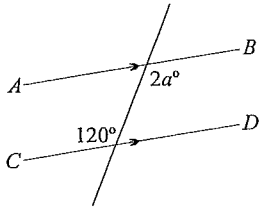
(i)



(ii)



(iii)



(c) (i) List the factors of 18.

(ii) List the factors of 42.

(iii) Hence find the highest common factor of 18 and 42.

Exam continues next page ...

QUESTION FOUR (11 marks) Start a new page.

(a) Rewrite the following statements as algebraic expressions, simplifying where necessary:

(i) the number which is y less than 10

(ii) half of b

(iii) 7 more than the product of w and 13

(b) Simplify the following:

(i) $8t - t$

(ii) $36 \times g \div 4 + 3$

(iii) $9a + 8b - 2a + 3b$

(c) Using the rule $m = 3n - 2$, copy and complete the following table of values.

n	1	2	3
m			

(d) Given $x = 5$ and $y = 8$, find the value of:

(i) $y - x$

(ii) $\dfrac{xy}{2}$

(iii) $3x^2$

Exam continues overleaf ...

QUESTION FIVE (11 marks) Start a new page.

(a) Sachi is paid an allowance of 33 cents per kilometre to drive to and from work. She lives 23 km from work and works a five day week. Calculate her allowance for one week. Show your working.

(b) Sam pays \$270 for nine pairs of shoes. How much would she pay if she only bought five pairs? Show your working.

(c) Simplify:

(i) $6\frac{1}{3} + 3\frac{1}{4}$

(ii) $2\frac{3}{8} - 1\frac{1}{3}$

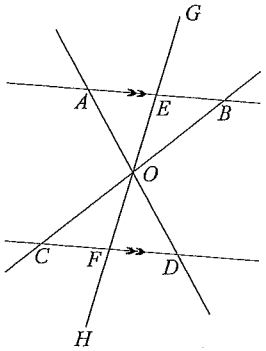
(iii) $1\frac{3}{7} \div 1\frac{7}{8}$

(iv) $\left(2\frac{1}{2}\right)^3$

Exam continues next page ...

QUESTION SIX (11 marks) Start a new page.

(a)



Using the diagram above, answer the following questions.

(i) Where do rays CO and AE intersect?

(ii) Name three collinear points.

(iii) Name an angle which is equal to $\angle AEG$. Give a reason for your answer.

(iv) Write down a geometrical word that completes the following sentence. Lines AO, BC and FG are _____.

(b) Find two numbers x, y that are less than 10 and which make the following statement true:

$$\frac{x}{3y} > 2$$

(c) What number is 3 more than the square root of 49?

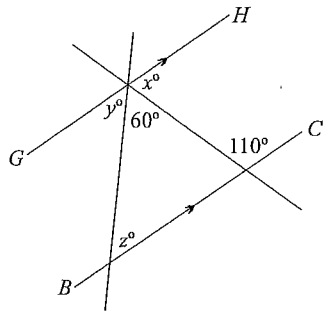
(d) Use long division to find $3906 \div 14$. Show your working.

(e) $\frac{3}{5}$ of a number is 72. Find the number. Show your working.

Exam continues overleaf ...

QUESTION SEVEN (11 marks) Start a new page.

(a)



Find the value of the pronumerals x, y and z in the diagram above. Give reasons.

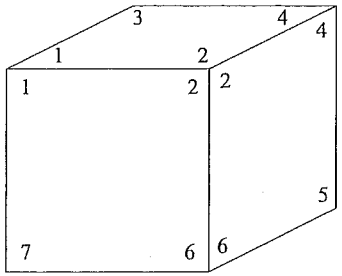
(b) Simplify:

(i) $8m + 10m^2 + 35m^2 \div 7 - 4m$

(ii) $\dfrac{y + y + y}{y \times y \times y}$

(c) The sum of three consecutive whole numbers is 30 and their product is 990. Find the three numbers. You are required to show some working.

(d)



Consider the diagram above. The vertices of the cube are numbered 1 to 8 as shown. Find the sum of all the numbers on the cube.

Exam continues next page ...

QUESTION EIGHT (11 marks) Start a new page.

(a) Zachary has one \$2, one \$1, one 20 cent and one 5 cent coin in his pocket. Write down a list which shows how many different sums of money he can make.

(b) Given the prime factorisation of 9800 is $2^3 \times 5^2 \times 7^2$ and the prime factorisation of 123 750 is $2 \times 3^2 \times 5^4 \times 11$, answer the following questions.

(i) Find the highest common factor of 9800 and 123 750. Leave your answer as a product of primes.

(ii) Find the lowest common multiple of 9800 and 123 750. Leave your answer as a product of primes.

(c) (i) Use a factor tree, or some other method, to find the prime factorisation of 3375.

(ii) Hence find the cube root of 3375.

(d) Simplify:

(i) $(56 - 12) \div 2 - 2 \times 8$

(ii) $10 - (10 - (9 - (10 - (9 - (10 - 9)))))$

QUESTION NINE (11 marks) Start a new page.

(a) Find the value of the expression below, when $m = 3$, $n = 4$, $p = 1$, $q = 2$ and $r = \frac{3}{4}$.

$$\frac{r(m - rp)}{(p + r)} - \frac{(m - r)^2}{2(m + n + q)}$$

(b) $[x]$ is defined as the largest whole number n such that $n \leq x$. For example $[1{\cdot}67] = 1$.

(i) Write down $[3{\cdot}42]$.

(ii) Write down $[\sqrt{2}]$.

(iii) Calculate:

$$1 + [\sqrt{2}] + [\sqrt{3}] + ... + [\sqrt{99}] + 10$$

(c) (i) How many odd seven digit numbers are there?

(ii) How many odd seven digit numbers contain 9 as one of their digits at least once? Show your working.

END OF TEST

FORM I – HY PROGRESS TEST ~~99~~ + M.A. /110

Q1

(a) (i) $32 + 16 - 11 = 37$ ✓

(ii) $^{4}\cancel{5}^{13}3 - 248 = 295$ ✓

(iii) $14 \times 30 = 42 \times 10 = 420$ ✓

(iv) $11 + 6 \times 3 = 11 + 18 = 29$ ✓

(b) XXVIII ✓

(c) (i) $8x - 2x = 6x$ ✓

(ii) $10x \times 4y = 40xy$ ✓

(iii) $4 \times q + 3 \times p = 4q + 3p$ ✓

(iv) $20xy \div 5x = 4y$ ✓

(v) $(5w)^2 = 25w^2$ ✓

(d) 422039 ✓

/11

Q2

(a) 4 and 5 ✓

(b) (i) $\frac{3}{7} + \frac{6}{7} = \frac{9}{7}$ ✓

(ii) $4 - \frac{5}{8} = 3\frac{3}{8}$ ✓

(iii) $\frac{2}{9} \times \frac{5}{11} = \frac{10}{99}$ ✓

(iv) $\frac{4}{15} \div \frac{7}{3} = \frac{4}{15_5} \times \frac{3^1}{7}$ ✓

$= \frac{4}{35}$ ✓

(c) (i) > ✓

(ii) < ✓

(iii) = ✓

(d) $\frac{1}{3} \times 375 = 125$ mL ✓

Q3

(a) 43° ✓

(b) (i) $a = 70$ (vertically opposite) ✓

(ii) $a + 50 = 180$ (straight angle) ✓

$a = 130$ ✓

(iii) $2a = 120$ (alternate angles, AB ∥ CD) ✓

$a = 60$ ✓

(c) (i) 18: 1, 2, 3, 6, 9, 18 ✓

(ii) 42: 1, 2, 3, 6, 7, 14, 21, 42 ✓

(iii) HCF = 6 ✓

/11

Q4

(a) (i) $10 - y$ ✓

(ii) $\frac{b}{2}$ ✓

(iii) $13w + 7$ ✓

(b) (i) $8t - t = 7t$ ✓

(ii) $36 \times g \div 4 + 3 = 9g + 3$ ✓

(iii) $9a + 8b - 2a + 3b = 7a + 11b$ ✓

(c)

n	1	2	3
m	1	4	7

✓✓ (Give ✓x for one or two correct)

(d) (i) $y - x = 8 - 5 = 3$ ✓

(ii) $\frac{xy}{2} = \frac{5 \times 8}{2} = 20$ ✓

(iii) $3x^2 = 3 \times 5^2 = 3 \times 25 = 75$ ✓

Q5

(a)

$2 \times 23 \times 33 \times 5$ ✓
$= 230 \times 33$
$= 7590$
$= \$75.90$ ✓

(b) 9 pairs cost \$270
1 pair costs $\frac{\$270}{9} = \$30$ ✓
5 pairs cost $30 \times 5 = \$150$ ✓

(c)(i) $6\frac{1}{3} + 3\frac{1}{4} = 9 + \frac{4+3}{12}$ ✓
$= 9\frac{7}{12}$ ✓

(ii) $2\frac{3}{8} - 1\frac{1}{3} = 2\frac{9}{24} - 1\frac{8}{24}$ ✓
$= 1\frac{1}{24}$ ✓

(iii) $1\frac{3}{7} \div 1\frac{7}{8}$
$= \frac{10}{7} \div \frac{15}{8}$ ✓
$= \frac{10^{2}}{7} \times \frac{8}{15_{3}}$
$= \frac{16}{21}$ ✓

(iv) $\left(2\frac{1}{2}\right)^3 = \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$
$= \frac{125}{8}$ ✓

11

Q6

(a)(i) at point B ✓

(ii) eg: A, E, B ✓ etc.

(iii) $\angle CFO$ ✓ (corresponding angles, AB ∥ CD) ✓
or $\angle OEB$ (vertically opposite angle) etc

(iv) concurrent ✓

(b) eg $x = 9$ ✓
$y = 1$

(c) $\sqrt{49} + 3 = 10$ ✓

(d)
$$1.4 \overline{)\,3906} = 279$$
28 ××
110
98
126
126
0

(e) $\frac{3}{5}$ of the number is 72
$\frac{1}{5}$ of the number is $\frac{72}{3} = 24$
the number is $5 \times 24 = 120$ ✓✓

11

Q7

(a)(i) $x = 70$ (corresponding angles, GH ∥ BC) ✓
$y + 60 + 70 = 180$ (straight angle) ✓
$y = 50$
$z = 50$ (alternate angles, GH ∥ BC) ✓

(ii) $8m + 10m^2 + 35m^2 \div 7 - 4m$
$= 4m + 10m^2 + 5m^2$ ✓✓
$= 4m + 15m^2$

(ii) $\frac{y+y+y}{y \times y \times y} = \frac{3y}{y^3}$ ✓
$= \frac{3}{y^2}$ ✓

(c) $9 + 10 + 11 = 30$
$9 \times 10 \times 11 = 990$ ✓
the numbers are 9, 10 & 11 (some working)

(d) $3 \times (1+2+3+4+5+6+7+8)$ ✓
$= 3 \times 36$
$= 108$ ✓

11

Q8

(a) one coin: \$2, \$1, 20c, 5c

two coins: $2 + 1 = \$3$
$2 + 20c = \$2.20$
$2 + 5c = \$2.05$
$1 + 20c = \$1.20$
$1 + 5c = \$1.05$
✓✓ (one for some idea!) $20c + 5c = \$0.25$

three coins:
$2 + 1 + 20c = \$3.20$
$2 + 1 + 5c = \$3.05$
$2 + 20c + 5c = \$2.25$
$1 + 20c + 5c = \$1.25$

four coins:
$2 + 1 + 20 + 5 = \$3.25$

(b)(i) HCF $= 2 \times 5^2$ ✓

(ii) LCM $= 2^3 \times 3^2 \times 5^4 \times 7^2 \times 11$ ✓

(c)(i) 3375
5 | 675
25 | 27
5 | 5 | 3 | 3 | 3
$3375 = 3^3 \times 5^3$ ✓

(ii) $\sqrt[3]{3375} = \sqrt[3]{3^3 \times 5^3}$
$= 3 \times 5$
$= 15$ ✓

Q8 cont'd

(d) (i) $(56-12) \div 2 - 2\times 8$
$= 44 \div 2 - 16$ ✓
$= 22 - 16$
$= 6$ ✓

(ii) $10-(10-(9-(10-(9-(10-9)))))$
$= 10-(10-(9-(10-(9-1))))$
$= 10-(10-(9-(10-8)))$ ✓
$= 10-(10-(9-2))$
$= 10-(10-7)$
$= 10-3$
$= 7$ ✓

11

Q9

(a) $\frac{r(m-rp)}{(p+r)} - \frac{(m-r)^2}{2(m+n+q)}$

$= \frac{\frac{3}{4}\left(3-\frac{3}{4}\times 1\right)}{\left(1+\frac{3}{4}\right)} - \frac{\left(3-\frac{3}{4}\right)^2}{2.(3+4+2)}$ ✓ (for correct substitution)

$= \frac{\frac{3}{4}\left(2\frac{1}{4}\right)}{1\frac{3}{4}} - \frac{\left(2\frac{1}{4}\right)^2}{2\times 9}$

$= \frac{3}{4}\times\frac{9}{7}\times\frac{1}{7} - \frac{9}{4}\times\frac{9}{4}\times\frac{1}{18}$ ✓ (or similar)

$= \frac{27}{28} - \frac{9}{32}$ $\quad 28 = 2^2\times 7$, $32 = 2^5$, LCM $= 2^5\times 7 = 224$

$= \frac{216-63}{224}$

$= \frac{153}{224}$ ✓

(b) (i) $[3.42] = 3$ ✓
(ii) $[\sqrt{2}] = 1$ ✓

(iii) $1 + [\sqrt{2}] + [\sqrt{3}] + \ldots + [\sqrt{99}] + 10$

$= 1 + [\sqrt{2}] + [\sqrt{3}] + [\sqrt{4}] + [\sqrt{5}] + [\sqrt{6}] + [\sqrt{7}] + \ldots + [\sqrt{99}] + [\sqrt{100}]$

$= 1+1+1+2+2+2+2+2+3+\ldots\ldots+10$ ✓

$= 3\times 1 + 5\times 2 + 7\times 3 + 9\times 4 + 11\times 5 + 13\times 6 + 15\times 7 + 17\times 8 + 19\times 9 + 10$ ✓

$= 629$ ✓

(c)(i) the smallest seven digit number is 1000000
the largest is 9999999

there are 9000000 seven digit numbers of
half of them will be odd. ✓

so there are 4500000 odd seven digit numbers.

(ii) We can construct an odd seven digit number such that none of its digits is 9 in $8 \times 9^5 \times 4 = 1889568$ ways ✓

↑ first digit ↑ last digit

the number of seven digit numbers with at least one 9 will be $4500000 - 1889568 = 2610432$ ✓