2006
HSC COURSE
MID YEAR EXAMINATIONS

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Start a new page for each question
- Approved calculators may be used

Total marks – 86

Question 1 - 20 marks
Question 2 - 20 marks
Question 3 - 20 marks
Question 4 - 20 marks
Question 5 - 6 marks

Allow approximately 27 minutes for questions 1 to 4 and 12 minutes for question 5

Question 1 (20 marks) – Start a new booklet

a. Differentiate each of the following

i. $3x^3 - 7x$ — 1

ii. $(2x-7)^5$ — 1

iii. $6\sqrt{x}$ — 1

iv. $\dfrac{2x}{3x^2-7}$ — 2

c. Given that $f(x) = (x+1)(x-1)^5$ find the values of a and b such that $f'(x) = (ax+b)(x-1)^4$ — 2

d. Solve the equation $x^2 + 4x = 1$. Leave your answer in surd form. — 2

e. For the quadratic equation $x^2 - x - m = 0$

i. Find the discriminant, Δ — 1

ii. Using your answer to part i. or otherwise, find the values of m for which this quadratic equation has no real solutions — 3

f. If α and β are the roots of the equation $2x^2 - 5x + 1 = 0$, without solving the equation find

i. $\alpha + \beta$ — 1

ii. $\alpha\beta$ — 1

iii. $\dfrac{2}{\alpha} + \dfrac{2}{\beta}$ — 1

iv. $\alpha^2 + \beta^2$ — 2

g. Express $2x^2 - 5x + 3$ in the form $a(x-2)^2 + b(x-2) + c$ — 2

Question 2 (20 marks) – Start a new booklet

a. Consider the function $f(x)=3x^2-x^3$

i. Find the values of x for which $f'(x)=0$ 2

ii. Find the coordinates of the turning points of the curve $y=f(x)$ and determine their nature 2

iii. Sketch the graph of the curve $y=f(x)$ showing these turning points 2

iv. Determine the values of x for which $f'(x)\geq 0$ 1

b. Sketch a graph of a continuous function $f(x)$ with the following properties: 4

- $f(x)$ is odd
- $f(3)=0$ and $f'(1)=0$
- $f'(x)>0$ for $x>1$
- $f'(x)<0$ for $0\leq x\leq 1$

c.

"Although the world's population continues to increase, the rate of population growth is decreasing."

Draw a graph of population as a function of time that fits this description. 2

d. A transport company runs a truck from Batemans Bay to Sydney, a distance of 250km, at a constant speed of v km/h. For a given speed v, the cost per hour is $6400+v^2$ cents.

i. If the trip costs C cents, use the formula time$=\dfrac{\text{distance}}{\text{speed}}$ to show that 2

$$C=250\left(\frac{6400}{v}+v\right)$$

ii. Find the speed for which the cost of the trip is minimised. 2

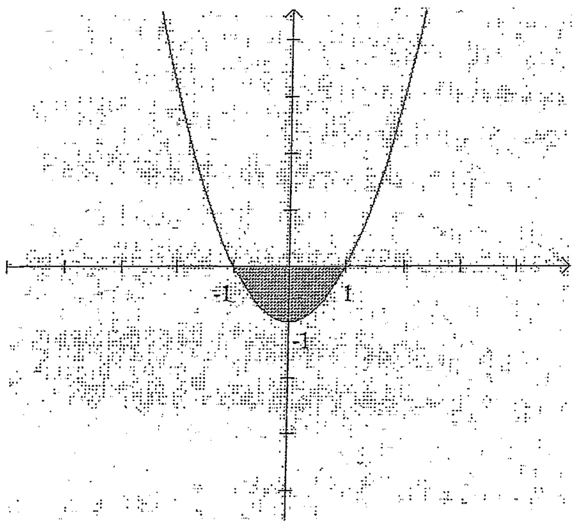
iii. Find the minimum cost of the journey 1

e. The curve $y=f(x)$ has gradient function $\dfrac{dy}{dx}=3x^2+4x-1$. 2

The curve passes through the point $P(1,0)$. Find its equation

Question 3 (20 marks) – Start a new booklet

a. The graph below is of a quadratic function. Determine what the function is, and write an integral that could be used to evaluate the shaded area below. 2



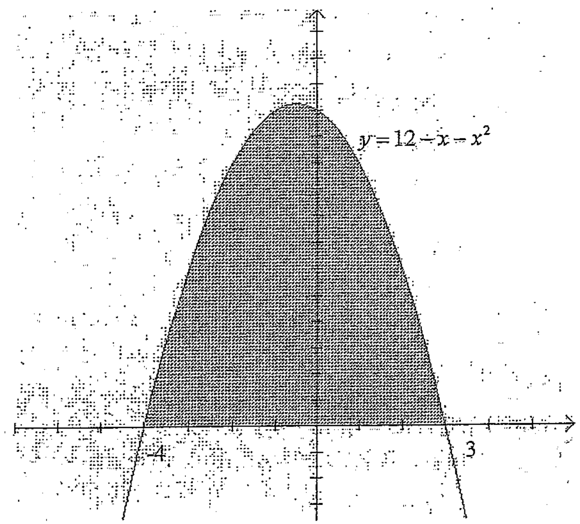
b. Evaluate

i. $\int_{-1}^{1}\left(x^3 - x + 1\right)dx$ 2

ii. $\int x\sqrt{x}\ dx$ 2

c. Find the value of k if $\int_{2}^{k} 3\ dx = 18$ 2

d. Find the exact area of the shaded region. 2



e.

i. Let P be the point where the line $y = x - 4$ touches the parabola $y = x^2 - 5x + 5$. Show that the normal to the parabola at P is $y = -x + 2$ 3

ii. Find the area of the region enclosed between the parabola and the line $y = -x + 2$ 3

f. A cone of height h and radius r is generated by rotating the line $y = \frac{r}{h}x$ between $x = 0$ and $x = h$ about the x axis. Show that the cone has volume $\frac{1}{3}\pi r^2 h$ 2

g. Use the trapezoidal rule with five function values to approximate the integral $\int_{2}^{6} \frac{1}{x}\ dx$ .Give your solution correct to 3 decimal places. 2

Question 4 (20 marks) Start a new booklet

a. A letter is chosen from the word TASMANIA. Find the probability that it is

i. the letter A — 1

ii. a vowel — 1

iii. a letter from the word HOBART — 2

b. Comment on the following argument. Identify any fallacies in the argument and if possible give some indication of how to correct them. — 2

"On every day of the year it either rains or it doesn't. Therefore the chance that it will rain tomorrow is $\frac{1}{2}$"

c.

i. A coin is tossed three times. Find the probability of obtaining three heads. — 1

ii. A coin is tossed n times. Find the probability of obtaining n heads. — 2

d. John has an exciting job checking light bulbs in a light bulb factory. John selects two light bulbs from a large batch in which 5% are defective.

i. Find the probability that John chooses two bulbs that both work — 1

ii. Find the probability that John chooses at least one working bulb. — 2

e. From a standard pack of 52 cards, a card is drawn from the pack at random. Find the probability of drawing

i. a black or a picture card — 2

ii. is neither a jack nor a ten — 2

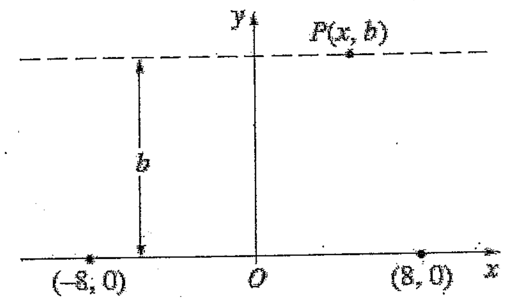
f. Sixty five kangaroos were tagged and released into a national park known to contain untagged kangaroos.

i. Letting n be the number of untagged kangaroos in the park, derive an expression for the proportion of tagged kangaroos to the entire kangaroo population in the park. — 1

ii. Some time later a sample of 32 kangaroos were captured from the park and released back. It was noted that of the 32 kangaroos captured, 5 were tagged. Using this information estimate the total number of kangaroos in the park. State any assumptions you need to make in calculating this. — 3

Question 5 (6 marks) Start a new booklet

On a dark night, two ships, Saga and Hero, sail parallel to a straight coastline on which there are two lights of equal brightness, 16 kilometres apart..



Suppose the coastline is represented by the x axis where the origin O is chosen to be the midpoint of the light sources. It is known that the (total) brightness from the lights on a ship at point $P(x,b)$ is

$$I=\frac{1}{b^2+(x+8)^2}+\frac{1}{b^2+(x-8)^2}$$

i. Show that $\frac{dI}{dx}=-\frac{2P}{Q}$ where — 2

$$P=\left[(x+8)\left(b^2+(x-8)^2\right)^2+(x-8)\left(b^2+(x+8)^2\right)^2\right] \text{ and}$$

$$Q=\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2$$

To answer parts (ii) and (iii), you may assume the following factorisation, given by a computer package, that

$$P=2x\left(x^2+64+b^2+16\sqrt{64+b^2}\right)\left(x^2+64+b^2-16\sqrt{64+b^2}\right)$$

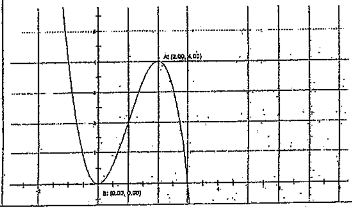
ii. Saga sails parallel to the coast at a distance 15 km from the coast. By considering $\frac{dI}{dx}$ show that, as Saga sails from left to right, the brightness on Saga increases to a maximum when $x=0$, then decreases. — 2

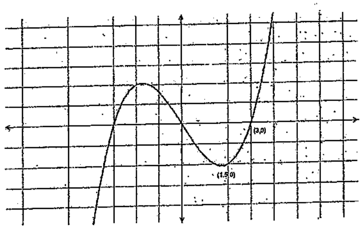
iii Hero sails parallel to the coast at a distance 6 km from the coast. Describe how you're ane brightness on Hero changes as Hero sails from left to right. Give clear reasons for you're answer. — 2

End of paper

	Question	Solution	Rubric
1	Differentiate each of the following		
ai	$3x^3-7x$	$9x^2-7$	1 correct answer
aii	$(2x-7)^5$	$10(2x-7)^4$	1 correct answer
aiii	$6\sqrt{x}$	$\frac{d}{dx}(6\sqrt{x})$ $=\frac{d}{dx}(6x^{0.5})$ $=3x^{-0.5}$ $=\frac{3}{\sqrt{x}}$	1 $3x^{-0.5}$ or $\frac{3}{\sqrt{x}}$
aiv	$\frac{2x}{3x^2-7}$	$\frac{d}{dx}\left(\frac{2x}{3x^2-7}\right)$ $=\frac{(3x^2-7)(2)-(2x)(6x)}{(3x^2-7)^2}$ $=\frac{6x^2-14-12x^2}{(3x^2-7)^2}$ $=\frac{-6x^2-14}{(3x^2-7)^2}$	1 correct expansion of derivative 2 $\frac{-6x^2}{(3x^2-7)^2}$

c	Given that $f(x)=(x+1)(x-1)^5$ find the values of a and b such that $f'(x)=(ax+b)(x-1)^4$	$f(x)=(x+1)(x-1)^5$ $f'(x)=5(x+1)(x-1)^4+(x-1)^5$ $=(x-1)^4(5(x+1)+(x-1))$ $=(x-1)^4(5x+5+x-1)$ $=(6x+4)(x-1)^4$ $\therefore a=6, b=4$	1 Derivative correctly determined 2 $f'(x)=(6x+4)(x-1)^4$ or $a=6, b=4$ No one got this correct.
d	Solve the equation $x^2+4x=1$. Leave your answer in surd form.	$x^2+4x=1$ $x^2+4x-1=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-4\pm\sqrt{16+4}}{2}$ $=\frac{-4\pm\sqrt{20}}{2}$ $=\frac{-4\pm2\sqrt{5}}{2}$ $=-2\pm\sqrt{5}$	1 correct values substituted into the quadratic formula 2 $x=\frac{-4\pm\sqrt{20}}{2}$ or $-2\pm2\sqrt{5}$ many had great difficulties with this question because of the unusual format. You must be able to spot a quadratic and then manipulate it into the right format to work with.
	For the quadratic equation $x^2-x-m=0$		
ei	Find the discriminant, Δ	$x^2-x-m=0$ $\Delta=1+4m$	1 correct answer Many had great difficulty with this band 2 question. Learn the fomula.
eii	Using your answer to part i. or otherwise, find the values of m for which this quadratic equation has no real solutions	For no real solutions $\Delta<0$ $1+4m<0$ $4m<-1$ $m<-\frac{1}{4}$	2 $1+4m<0$ 3 correct answer
	If α and β are the roots of the equation $2x^2-5x+1=0$, without solving the equation find		
fi	$\alpha+\beta$	$\alpha+\beta=-\frac{b}{a}=\frac{5}{2}$	1 correct answer
fii	$\alpha\beta$	$\alpha\beta=\frac{1}{2}$	1 correct answer
fiii	$\frac{2}{\alpha}+\frac{2}{\beta}$	$\frac{2}{\alpha}+\frac{2}{\beta}$ $=\frac{2(\alpha+\beta)}{\alpha\beta}$ $=\frac{2\left(\frac{5}{2}\right)}{\frac{1}{2}}$ $=10$	1 correct answer
fiv	$\alpha^2+\beta^2$	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{5}{2}\right)^2-1$ $=\frac{25}{4}-1$ $=\frac{21}{4}$	2 correct answer
g	Express $2x^2-5x+3$ in the form $a(x-2)^2+b(x-2)+c$	$2x^2-5x+3\equiv a(x-2)^2+b(x-2)+c$ $\equiv a(x^2-4x+4)+bx-2b+c$ $\equiv ax^2-4ax+4a+bx-2b+c$ $\equiv ax^2+(-4a+b)x+(4a-2b+c)$ equating coefficients gives... $a=2$ $-4a+b=-5$ $-8+b=-5$ $b=3$ $4a-2b+c=3$ $8-6+c=3$ $c=1$ Therefore... $2x^2-5x+3\equiv2(x-2)^2+3(x-2)+1$	1 $a(x-2)^2+b(x-2)+c$ correctly expanded 2 $2x^2-5x+3\equiv2(x-2)^2+3(x-2)+1$ or $a=2, b=3, c=1$ I was surprised at how many correct answers I got here when so many had difficulties with the easier parts of question 1.

2				
a	Consider the function $f(x)=3x^2-x^3$			
ai	Find the values of x for which $f'(x)=0$	$f(x)=3x^2-x^3$ $f'(x)=6x-3x^2=0$ $3x(2-x)=0$ $x=0$ or 2	1 2	Correctly finding derivative Incorrectly finding derivative but solving for $f'(x)=0$ $x=0$ or 2
aii	Find the coordinates of the turning points of the curve $y=f(x)$ and determine their nature	Coordinates of turning points are $(0,0)$ and $(2,4)$ $f''(x)=6-6x$ $f''(0)=6-6(0)=6$ which is positive $\therefore$ $(0,0)$ is a minimum turning point $f''(2)=6-6(2)=-6$ which is negative $\therefore$ $(2,4)$ is a maximum turning point	1 2	Finding x values or coordinates of both turning points Finding both turning points and determining their nature
aiii	Sketch the graph of the curve $y=f(x)$ showing these turning points		1 2	Graph has right shape and correct intercepts Correct graph with turning points shown

aiv	Determine the values of x for which $f'(x)\geq 0$	The gradient is positive between $0<x<2$ therefore $f'(x)\geq 0$ for $0\leq x\leq 2$	1	correct answer
b	Sketch a graph of a continuous function $f(x)$ with the following properties: • $f(x)$ is odd • $f(3)=0$ and $f'(1)=0$ • $f'(x)>0$ for $x>1$ • $f'(x)<0$ for $0\leq x<1$		1 2 3 4	1 criterion correct 2 criteria correct 3 criteria correct completely correct This question was not done well.
c	"Although the world's population continues to increase, the rate of population growth is decreasing" Draw a graph of population as a function of time that fits this description		1 2	axes correctly labelled or positive gradient graph must have positive gradient and be concave down This question was done well – good job!

d	A transport company runs a truck from Batemans Bay to Sydney, a distance of 250km, at a constant speed of v km/h. For a given speed v, the cost per hour is $6400+v^2$ cents.			
di	If the trip costs C cents, show that $C=250\left(\frac{6400}{v}+v\right)$	Time taken$=\frac{\text{distance}}{\text{velocity}}$ $=\frac{250}{v}$ Cost of trip $=(6400+v^2)\times$ time $=(6400+v^2)\times\frac{250}{v}$ $=250\left(\frac{6400}{v}+v\right)$	1 1	finding time taken $\frac{250}{v}$ minor error in working

dii	Find the speed for which the cost of the trip is minimised.	$C=250\left(\frac{6400}{v}+v\right)$ $=\frac{1600000}{v}+250v$ $\frac{dC}{dv}=-\frac{1600000}{v^2}+250$ $-1600000+250v^2=0$ $-6400+v^2=0$ $v^2=6400$ $v=80$ km/h At 80.1 km/h $\frac{dC}{dv}>0$ At 79.9 km/h $\frac{dC}{dv}<0$ So $v=80$ km/h will minimise the cost of the journey	1	correctly determining derivative You must show that $v=80$ is a minimum turning point. It is not good enough just to assume this.
diii	Find the minimum cost of the journey	$C=250\left(\frac{6400}{v}+v\right)$ $=250\left(\frac{6400}{80}+80\right)$ $=160\times 250$ cents $=\$400$		

e	The curve $y=f(x)$ has gradient function $\frac{dy}{dx}=3x^2+4x-1$. The curve passes through the point $P(1,0)$. Find its equation	$\frac{dy}{dx}=3x^2+4x-1$ $y=x^3+2x^2-x+C$ $0=1+2-1+C$ $\therefore C=-2$ $\therefore y=x^3+2x^2-x-2$	1 correctly finding primitive function $y=x^3+2x^2-x+C$
3			
a	Write an integral that would be used to evaluate the shaded area below.	$A=\left\|\int_{-1}^{1} x^2-1\,dx\right\|$ or $A=2\left\|\int_{0}^{1} x^2-1\,dx\right\|$	1 determining the correct function 1 correct integral but forgot the absolute value
bi	$\int_{-1}^{1}(x^3-x+1)\,dx$	$\int_{-1}^{1}(x^3-x+1)\,dx$ $=\left[\frac{x^4}{4}-\frac{x^2}{2}+x\right]_{-1}^{1}$ $=\left(\frac{1}{4}-\frac{1}{2}+1\right)-\left(\frac{1}{4}-\frac{1}{2}-1\right)$ $=2$	1 correct integration 1 incorrect integration but correct evaluation
bii	$\int x\sqrt{x}\,dx$	$\int x\sqrt{x}\,dx$ $=\int x\times x^{\frac{1}{2}}\,dx$ $=\int x^{\frac{3}{2}}\,dx$ $=\frac{2}{5}x^{\frac{5}{2}}+C$ $=\frac{2\sqrt{x^5}}{5}+C$	1 recognising that $x\sqrt{x}=x^{\frac{3}{2}}$ 1 forgetting the constant
c	Find the value of k if $\int_{2}^{k}3\,dx=18$	$\int_{2}^{k}3\,dx=18$ $[3x]_2^k=18$ $(3k)-(6)=18$ $3k-6=18$ $3k=24$ $k=8$	1 correct integration 1 an equation in k obtained but incorrectly solved
d	Find the exact area of the shaded region.	$y=12-x-x^2$ $A=\int_{-4}^{3}12-x-x^2\,dx$ $=\left[12x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-4}^{3}$ $=\left(36-\frac{9}{2}-9\right)-\left(-48-8+\frac{64}{3}\right)$ $=57\frac{1}{6}$ square units	1 correct integral expression 1 minor error in evaluation
ei	Let P be the point where the line $y=x-4$ touches the parabola $y=x^2-5x+5$. Show that the normal to the parabola at P is $y=-x+2$	First, we need to find P. This is done by solving these two equations simultaneously... $x^2-5x+5=x-4$ $x^2-6x+9=0$ $(x-3)^2=0$ $x=3$ When $x=3$ $y=-3+2=-1$ Therefore P is the point $(3,-1)$	1 finds P 2 finds gradient at $x=3$
eii	find the area of the region between these two curves.	hmm...I think I made this too hard	
f	A cone of height h and radius r is generated by rotating the line $y=\frac{r}{h}x$ between $x=0$ and $x=h$ about the x axis. Show that the cone has volume $\frac{1}{3}\pi r^2h$	$V=\pi\int_0^h\left\{\frac{r}{h}x\right\}^2dx$ $=\pi\int_0^h\frac{r^2}{h^2}x^2\,dx$ $=\pi\frac{r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h$ $=\pi\frac{r^2}{h^2}\left[\left(\frac{h^3}{3}\right)-(0)\right]$ $=\frac{1}{3}\pi r^2h$	1 correct integration
g	Use the trapezoidal rule with five function values to approximate the integral $\int_2^6\frac{1}{x}\,dx$. Give your solution correct to 3 decimal places.	$\int_2^6 y\,dx\approx\frac{1}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$ where $h=$ width of each trapezium$=\frac{b-a}{n}$ here $h=\frac{6-2}{4}=1$ $y=\frac{1}{x}$ $y_0=\frac{1}{2};\ y_1=\frac{1}{3};\ y_2=\frac{1}{4};\ y_3=\frac{1}{5};\ y_4=\frac{1}{6}$ $\int_2^6 y\,dx\approx\frac{1}{2}\left[\left(\frac{1}{2}+\frac{1}{6}\right)+2\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)\right]$ $=1\frac{7}{60}$ square units $\doteq 1.117$ (correct to 3 decimal places).	1 finding function values

4			
a	A letter is chosen from the word TASMANIA. Find the probability that it is		
ai	the letter A	$\frac{3}{8}$	
aii	a vowel	$\frac{4}{8}$ or $\frac{1}{2}$	
aiii	a letter from the word HOBART	$P(\text{a letter from the word HOBART}) = \frac{3}{8}+\frac{1}{8}=\frac{4}{8}$	
b	Comment on the following argument. Identify any fallacies in the argument and if possible give some indication of how to correct them. "On every day of the year it either rains or it doesn't. Therefore the chance that it will rain tomorrow is $\frac{1}{2}$"	Although there are only two possible outcomes here, they are not equally likely. Therefore the probability shown is not necessarily correct.	
ci	A coin is tossed three times. Find the probability of obtaining three heads.	$P(HHH)=\left(\frac{1}{2}\right)^3=\frac{1}{8}$	
cii	A coin is tossed n times. Find the probability of obtaining n heads.	$P(n \text{ heads})=\left(\frac{1}{2}\right)^n=\frac{1}{2^n}$	
d	John has an exciting job checking light bulbs in a light bulb factory. John selects two light bulbs from a large batch in which 5% are defective.		
di	Find the probability that John chooses two bulbs that both work	$P(\text{defective})=0.05$ $P(\text{working})=0.95$ $\therefore P(\text{two bulbs working})=0.95\times0.95=0.9025$	
dii	Find the probability that John chooses at least one working bulb	$P(\text{At least one working})=P(1\text{ working})+P(2\text{ working})$ $=2\times0.95\times0.05+0.95\times0.95$ $=0.9975$	
e	From a standard pack of 52 cards, a card is drawn from the pack at random. Find the probability of drawing		
ei	a black or a picture card	$P(\text{black})=\frac{1}{2}$ $P(\text{picture card})=\frac{16}{52}$ $P(\text{picture card and black})=\frac{8}{52}$ $P(\text{picture card or black})$ $=P(\text{black})+P(\text{picture card})-P(\text{picture card and black})$ $=\frac{1}{2}+\frac{16}{52}-\frac{8}{52}$ $=\frac{34}{52}$	
eii	is neither a jack nor a ten	$P(\text{is neither a jack nor a ten})=1-P(\text{jack})-P(\text{ten})$ $=1-\frac{4}{52}-\frac{4}{52}$ $=\frac{44}{52}$	
f	Sixty five kangaroos were tagged and released into a national park known to contain untagged kangaroos.		
fi	Letting n be the number of untagged kangaroos in the park, derive an expression for the proportion of tagged kangaroos to the entire kangaroo population in the park.	Proportion of tagged kangaroos to the entire kangaroo population $=\frac{65}{n+65}$	
fii	Some time later a sample of 32 kangaroos were captured from the park and released back. It was noted that of the 32 kangaroos captured, 5 were tagged. Using this information estimate the total number of kangaroos in the park. State any assumptions you need to make in calculating this.	The proportion of tagged kangaroos to untagged should roughly be constant. This is an assumption. Given that this is the case... $\frac{65}{n+65}=\frac{5}{32}$ $5(n+65)=2080$ $5n+325=2080$ $5n=1755$ $n=351$	
5			
i	Show that $\frac{dI}{dx}=-\frac{2P}{Q}$ where $P=\left[(x+8)\left(b^2+(x-8)^2\right)^2+(x-8)\left(b^2+(x+8)^2\right)^2\right]$ and $Q=\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2$	$I=\frac{1}{b^2+(x+8)^2}+\frac{1}{b^2+(x-8)^2}$ $\frac{dI}{dx}=\frac{\left(b^2+(x+8)^2\right)\times0-1\times\left(2(x+8)\right)}{\left(b^2+(x+8)^2\right)^2}+\frac{\left(b^2+(x-8)^2\right)\times0-1\times\left(2(x-8)\right)}{\left(b^2+(x-8)^2\right)^2}$ $=\frac{-2(x+8)}{\left(b^2+(x+8)^2\right)^2}-\frac{2(x-8)}{\left(b^2+(x-8)^2\right)^2}$ $=\frac{-2(x+8)\left(b^2+(x-8)^2\right)^2-2(x-8)\left(b^2+(x+8)^2\right)^2}{\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2}$ $=\frac{-2\left((x+8)\left(b^2+(x-8)^2\right)^2-2(x-8)\left(b^2+(x+8)^2\right)^2\right)}{\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2}$ $=-\frac{2P}{Q}$	You need to realise that b is a constant – not a variable. x is the variable.

ii	Saga sails parallel to the coast at a distance 15 km from the coast. By considering $\frac{dI}{dx}$ show that, as Saga sails from left to right, the brightness on Saga increases to a maximum when $x=0$, then decreases.	$\frac{dI}{dx}=\frac{-2\left((x+8)\left(b^2+(x-8)^2\right)^2-2(x-8)\left(b^2+(x+8)^2\right)^2\right)}{\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2}$ $P=2x\left(x^2+64+b^2+16\sqrt{64+b^2}\right)\left(x^2+64+b^2-16\sqrt{64+b^2}\right)$ $\frac{dI}{dx}=\frac{-4x\left(x^2+64+b^2+16\sqrt{64+b^2}\right)\left(x^2+64+b^2-16\sqrt{64+b^2}\right)}{\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2}$ $=0$ when $x=0$ Now when $b=15$ and $x=-0.1$ $\frac{dI}{dx}<0$ When $x=0.1$ $\frac{dI}{dx}>0$ Therefore $x=0$ is a maximum turning point for I. Therefore as Saga sails from left to right, the brightness on Saga increases to a maximum when $x=0$, then decreases.	
iii	Hero sails parallel to the coast at a distance 6 km from the coast. Describe how the brightness on Hero changes as Hero sails from left to right. Give clear reasons for you're answer.	So here $b=6$. $I=\frac{1}{b^2+(x+8)^2}+\frac{1}{b^2+(x-8)^2}=\frac{1}{36+(x+8)^2}+\frac{1}{36+(x-8)^2}$ $\frac{dI}{dx}=\frac{-4x\left(x^2+64+b^2+16\sqrt{64+b^2}\right)\left(x^2+64+b^2-16\sqrt{64+b^2}\right)}{\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2}$ $=\frac{-4x\left(x^2+260\right)\left(x^2-60\right)}{\left(36+(x+8)^2\right)^2\left(36+(x-8)^2\right)^2}$ $=0$ when $-4x\left(x^2+260\right)\left(x^2-60\right)=0$ Solving… $-4x\left(x^2+260\right)\left(x^2-60\right)=0$ $x=0$ or $\pm\sqrt{60}$ At $x=-\sqrt{60}^-$ $\frac{dI}{dx}>0$ At $x=-\sqrt{60}^+$ $\frac{dI}{dx}<0$ Therefore Hero has maximum brightness at $x=-\sqrt{60}$	
		At $x=0^-$ $\frac{dI}{dx}<0$ and at $x=0^+$ $\frac{dI}{dx}>0$ so the brightness is minimised at $x=0$ At $x=\sqrt{60}^-$ $\frac{dI}{dx}>0$ and at $x=\sqrt{60}^+$ $\frac{dI}{dx}<0$ so therefore the brightness at $x=\sqrt{60}$ is a maximum for Hero at $x=\sqrt{60}$	