2 UNIT TEST NUMBER 1

1996

Coordinate Geometry - Plane Geometry - Trigonometry.

QUESTION 1. (9 marks) **Marks**

(a) Plot the points $A(2,-1)$, $B(5,3)$ and $C(0,4)$ on a number plane. **1**

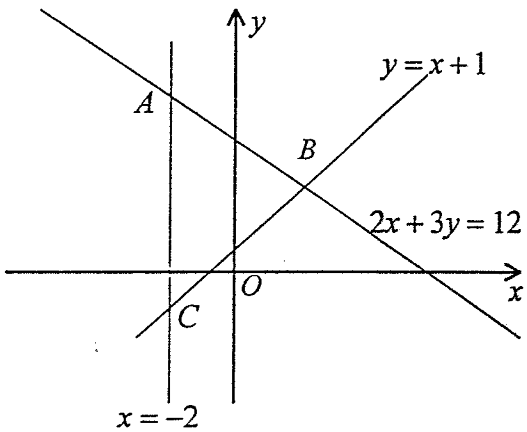
(b) Find the equation of AB in general form. **2**

(c) Find the perpendicular distance from C to AB. **2**

(d) Find the area of the triangle ABC. **2**

(e) Find the coordinates of D so that $ABCD$ is a parallelogram. **2**

QUESTION 2. (3 marks)

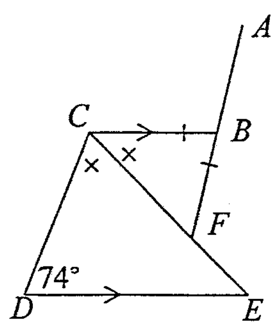


Write down the three inequations which simultaneously satisfy the region inside the triangle ABC. **3**

QUESTION 3. (13 marks)

(a) In the diagram, $CB \parallel DE$ and $BC = BF$.
CF bisects $\angle BCD$. $\angle CDE = 74°$.

Find the size of $\angle ABC$, giving reasons. **4**

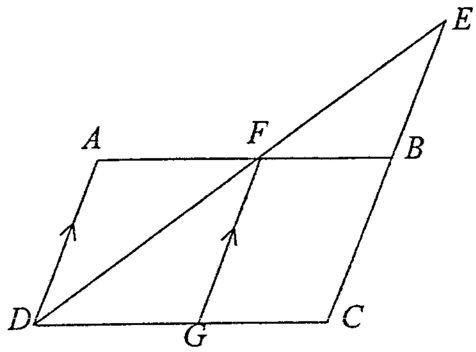


Marks

(b) *ABCD* is a parallelogram and $FG \parallel AD$. **5**
DFE and *CBE* are straight lines.

(i) Prove, using the properties of transversals cutting parallel lines, that $\dfrac{DG}{GC} = \dfrac{CB}{BE}$.

(ii) Hence prove that $AF \times BE = FB \times CB$.

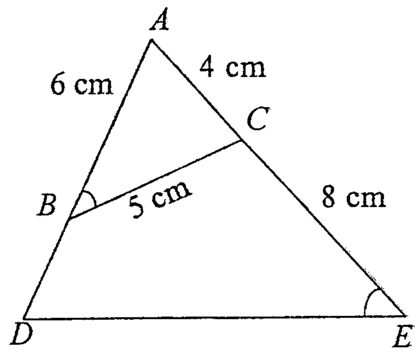


(c) In the diagram, $\angle ABC = \angle AED$. **4**
$AB = 6$ cm, $AC = 4$ cm,
$CE = 8$ cm, $BC = 5$ cm.

(i) Copy the diagram onto your answer sheet.

(ii) Prove $\Delta ABC \;|||\; \Delta ADE$.

(iii) Find the lengths of *BD* and *DE*.



QUESTION 4. (15 marks)

(a) Write down, in surd form, the values of: (i) $\cos 30°$ (ii) $\sec 45°$ (iii) $\cot 150°$. **3**

(b) Write down all values of θ, $0° \le \theta \le 360°$, for which $\sin^2\theta = \frac{3}{4}$. **3**

(c) Solve $3\sin\theta = 2\cos\theta$ for $0° \le \theta \le 360°$, to the nearest minute. **3**

(d) A point *Q* is 5.5 km south-west of a point *P*, and a point *R* is 8.2 km on a bearing of 147° from *P*. **6**

(i) What is the size of $\angle QPR$?

(ii) Find the distance *QR* (to 1 decimal place).

(iii) Find the size of $\angle PQR$ (to nearest degree), and hence find the bearing of *R* from *Q*.

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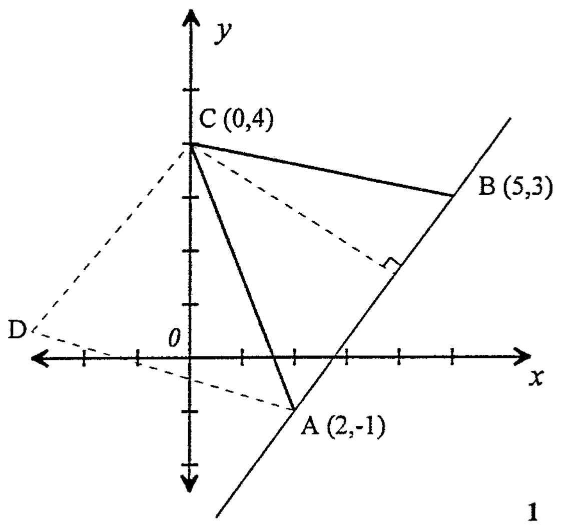
SUGGESTED SOLUTIONS

QUESTION 1

(a)



1

(b) Use $y - y_1 = m(x - x_1)$ to find equation of AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3+1}{5-2}$$

$$= \frac{4}{3} \qquad \mathbf{1}$$

$$y + 1 = \frac{4}{3}(x-2)$$

$$3y + 3 = 4x - 8$$

$$4x - 3y - 11 = 0 \qquad \mathbf{1} \qquad \textbf{Total} = \mathbf{2}$$

(c) Perp Dist. $= \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \dfrac{4 \times 0 - 3 \times 4 - 11}{\sqrt{4^2 + 3^2}} \right|$ **1**

$= \left| \dfrac{-23}{5} \right|$

$= \dfrac{23}{5}$ units. **1** **Total = 2**

(d) Length AB $= \sqrt{3^2 + 4^2}$

Note : Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= 5$ units **1**

Area of ΔABC $= \dfrac{1}{2} \times$ base $\times$ perp. height

$= \dfrac{1}{2} \times 5 \times \dfrac{23}{5}$

$= 11.5$ units^2 **1** **Total = 2**

(e) Plot D from C : go down 4, and back 3, because A is down 4 and back 3 from B, $DC // AB$.

Therefore D is $(-3, 0)$. **1,1** **Total = 2**

Alternative solution :

Diagonals have the same midpoint.

Midpoint of AC is $\left(1, 1\frac{1}{2}\right)$. If D is (x, y),

Note : Using the midpoint formula midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

midpoint of BD is $\left(\dfrac{x+5}{2}, \dfrac{y+3}{2}\right) = \left(1, 1\frac{1}{2}\right)$

$\therefore x = -3, \quad y = 0$.

QUESTION 2

The region inside the triangle ABC is given by

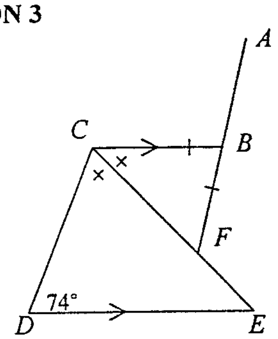
$x > -2$ **1**

$y > x + 1$ **1**

$2x + 3y < 12$. **1** **Total = 3**

QUESTION 3

(a)



$\angle BCD = 180° - \angle CDE$ (Cointerior angles are supplementary, *CB//DE*.)

$= 180° - 74°$

$= 106°$ **1**

$\angle BCE = \angle ECD$ (CF bisects $\angle BCD$)

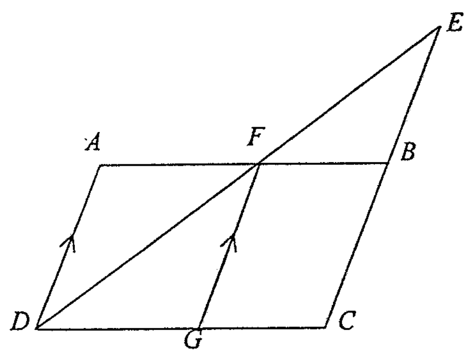
$\angle BCF = \frac{1}{2}(106°)$

$\angle BCF = 53°$ **1**

$\angle BFC = 53°$ (Angels opp. equal sides *BC*, *BF*.) **1**

$\angle ABC = 106°$ (Exterior angle of ΔBCF equals sum of interior opposite angles.) **1** **Total = 4**

(b)



(i) $\frac{DG}{GC} = \frac{DF}{FE}$ (Intercepts on transversals cut by parallel lines are in the same ratio *AD // FG // EC*.) **2**

$\frac{DF}{FE} = \frac{CB}{BE}$ (Intercepts on transversals cut by parallel lines are in the same ratio *FB // DC*.) **1**

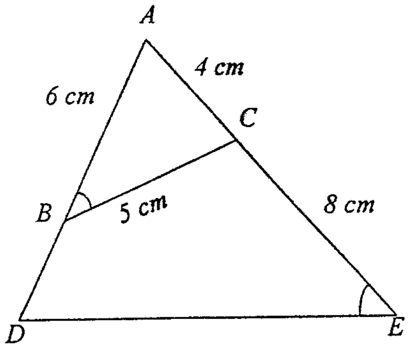
$\therefore \frac{DG}{GC} = \frac{CB}{BE}$. **Total = 3**

(ii) $\frac{DG}{GC} = \frac{AF}{FB}$ (Intercepts on transversals cut by parallel lines are in the same ratio $AD \parallel FG \parallel EC$.) **1**

$\therefore \frac{AF}{FB} = \frac{CB}{BE}$ (from (i)) **1**

$\therefore AF \times BE = FB \times CB$. **Total = 2**

(c) (i)



(ii) In ΔABC, ΔADE

$\angle BAC = \angle EAD$ (Common.) **1**

$\angle ABC = \angle AED$ (Given.)

$\therefore \Delta ABC \parallel\!\mid \Delta ADE$ (2 angles equal.) **1** **Total = 2**

(iii) $\frac{AB}{AC} = \frac{AE}{AD}$ (If two triangles are similar, the ratio of corresponding sides is equal.)

$\frac{6}{4} = \frac{12}{AD}$

$AD = 8$

$\therefore BD = 2$ cm. **1**

$\frac{AB}{BC} = \frac{AE}{DE}$ (If two triangles are similar, the ratio of corresponding sides is equal.)

$\frac{6}{5} = \frac{12}{DE}$

$\therefore DE = 10$ cm. **1** **Total = 2**

QUESTION 4

(a) (i) $\cos 30° = \frac{\sqrt{3}}{2}$. **1**

(ii) $\sec 45° = \sqrt{2}$. **1** *Note :* $\sec 45° = \frac{1}{\cos 45°}$ and $\cos 45° = \frac{1}{\sqrt{2}}$.

(iii) $\cot 150° = \cot(180° - 30°)$

$= -\cot 30°$

$= -\sqrt{3}$ **1** *Note :* $\cot 30° = \frac{1}{\tan 30°}$ and $\tan 30° = \frac{1}{\sqrt{3}}$

Total = 3

(b) $\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$ **1**

$\therefore \theta = 60°, \quad 180° - 60°, \quad 180° + 60°, \quad 360° - 60°.$

$\therefore \theta = 60°, 120°, 240°, 300°.$ **1,1** **Total = 3**

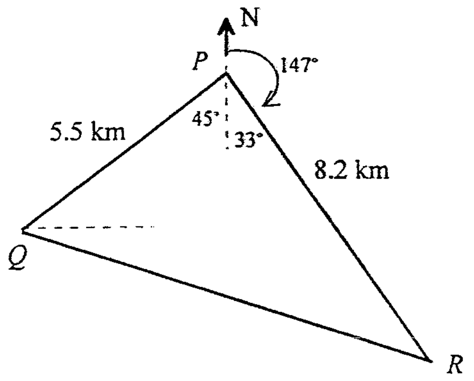
(c) $3\sin\theta = 2\cos\theta$ *Note :* Divide both sides by 3 and by $\cos\theta$.

$\tan\theta = \frac{2}{3}$ **1** *Note :* $\frac{\sin\theta}{\cos\theta} = \tan\theta$.

$\theta = 33°41'$ or $180° + 33°41'$

$\therefore \theta = 33°41'$ or $213°41'$. **1,1** **Total = 3**

(d)



(i) $\angle QPR = 78°$ **1**

(ii) To find QR use the cosine rule.

$QR^2 = 5.5^2 + 8.2^2 - 2 \times 5.5 \times 8.2 \times \cos 78°$ **1**

$QR^2 = 78.736$

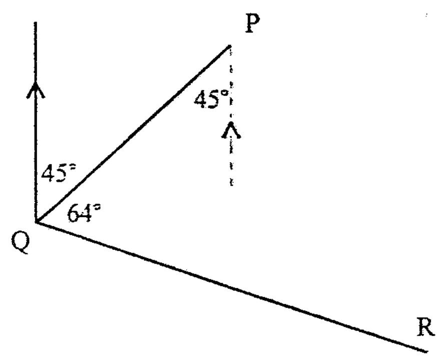
$QR = 8.9$ km (correct to 1d.p.) **1** **Total = 2**

(iii) To find angle PQR use the sine rule.

$$\frac{\sin \angle PQR}{8.2} = \frac{\sin 78°}{8.9}$$ **1**

$$\sin \angle PQR = \frac{8.2 \times \sin 78°}{8.9}$$

$\angle PQR = 64°$(correct to nearest degree) **1**



Bearing of R from Q is $45° + 64°$, i.e. $109°$ **1** **Total = 3**