2 UNIT TEST NUMBER 4

1996

The Tangents to a Curve and the Derivative of a Function.

QUESTION 1. (8 marks) **Marks**

Differentiate:

(a) $2x^3+x+4$ 1

(b) $(7x^2-2)^5$ 2

(c) $x^2(x-2)^4$ 2

(d) $\dfrac{3x}{2x+5}$. 3

QUESTION 2. (8 marks)

Differentiate without using the product rule or quotient rule.

(a) $\dfrac{1}{3x^2}$ 2

(b) $x\sqrt{x}$ 2

(c) $\dfrac{3}{\sqrt{2x+1}}$ 2

(d) $\dfrac{3x^2-1}{x^2}$. 2

QUESTION 3. (9 marks)

The tangent to the curve $y=3x^3-8x^2$ at the point of contact, P (2, –8), cuts the x-axis at A, and the normal to the curve at the same point of contact cuts the y-axis at B.

(a) Find the equation of the tangent at P. 3

(b) Find the equation of the normal at P. 2

(c) Find the coordinates of A and B. 2

(d) Find the length of the interval AB. 2

QUESTION 4. (8 marks) **Marks**

(a) A function $f(x)$ is defined as 5

$$f(x) = \begin{cases} x^2 & \text{for } x < 2 \\ 2^x & \text{for } x = 2 \\ 5 - \frac{1}{2}x & \text{for } x > 2 \end{cases}$$

(i) Investigate whether $f(x)$ is continuous at $x = 2$.

(ii) Sketch the graph of the function $f(x)$ in the domain $-1 \leq x \leq 6$.

(b) A tangent to the curve $y = 3x^2 - 5x + 2$ has a gradient of 4. Find the coordinates of the point of contact. 3

QUESTION 5. (7 marks)

(a) Find: 3

(i) $\lim\limits_{x \to 2} \dfrac{2x-4}{x+2}$

(ii) $\lim\limits_{x \to 2} \dfrac{2x^2-8}{x-2}$

(b) Consider the function $f(x) = 3x^2 - 2x + 1$. 4

(i) Find the gradient of the chord joining the points whose x-coordinates are 1 and $(1+h)$ respectively.

(ii) Hence determine the gradient of the tangent to the curve at $x = 1$.