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HSC Trial Examination 2006

Mathematics

This paper must be kept under strict security and may only be used on or after the morning of Tuesday 8 August, 2006 as specified in the Neap Examination Timetable.

General Instructions

Reading time 5 minutes

Working time 3 hours

Board-approved calculators may be used

Write using blue or black pen

A table of standard integrals is provided at the back of this paper

All necessary working should be shown in every question

Total marks – 120

Attempt questions 1–10

All questions are of equal value

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 HSC Mathematics Examination.

Total marks 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\log_e 70 - \log_e 10$ correct to 3 significant figures. **2**

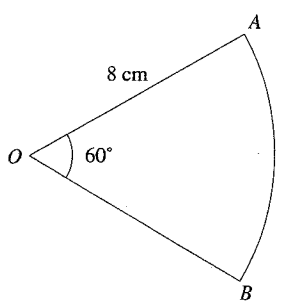
(b) Express $\frac{\sqrt{5}}{\sqrt{5}-2}$ with a rational denominator. **2**

(c) Solve $(x+1)(x-1)=15$. **2**

(d) Find the exact value of $\sec\frac{\pi}{4}$. **1**

(e) Simplify $\log_b a \times \log_e b$. **2**

(f) The diagram shows a sector of a circle with radius 8cm. The angle at the centre of the sector is 60°.



(i) Express 60° in radians. **1**

(ii) Find the exact length of the arc AB. **1**

(iii) What is the exact area of the sector? **1**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x:

(i) $y = 5x^8 + 4$ **1**

(ii) $y = \frac{2+3x}{x-4}$ **2**

(iii) $y = x^2 \sin x$. **2**

(b) Find:

(i) $\int 8x^3\,dx$ **1**

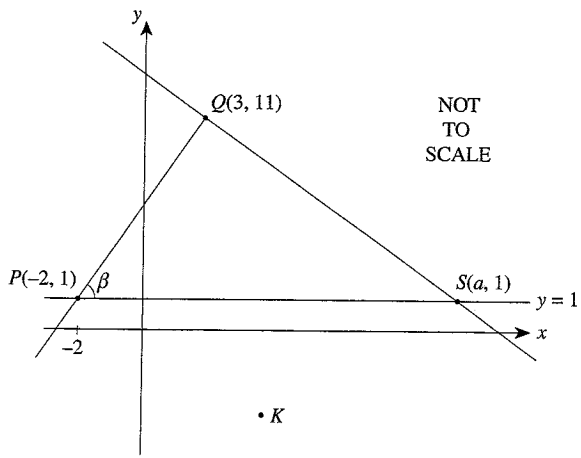
(ii) $\int \frac{1}{2x+3}\,dx$. **2**

(c) Find the exact value of the definite integral $\int_0^2 e^{3x}\,dx$. **2**

(d) What is the domain and range of $y = \log_{10}(x+1)$? **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.



In the diagram P is the point $(-2, 0)$ and Q is the point $(3, 11)$. Both points P and S lie on the line $y = 1$. Point S has coordinates $(a, 1)$. K is a point in the fourth quadrant. Line PQ makes an angle of β with the line $y = 1$.

(a) Find the gradient of PQ. **1**

(b) Find the equation of the line PQ. **2**

(c) Briefly explain why $\beta = 63°$ correct to the nearest degree. **1**

(d) $PQSK$ is a rhombus. Find the exact lengths of PQ and PK. Give a geometrical reason for your answer. **2**

(e) Calculate the size of $\angle PQS$ and give a reason for your answer. **2**

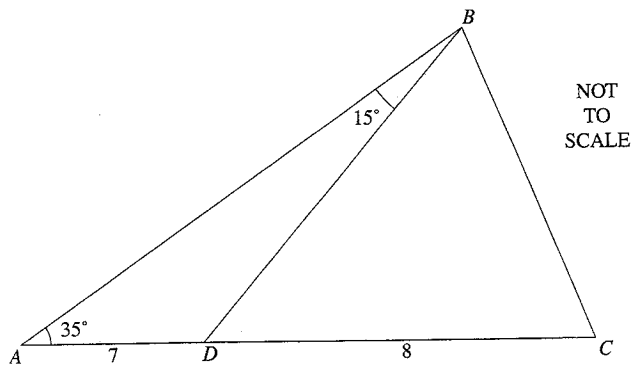
(f) Show that the value of a, the x-coordinate of point S, is 8. **2**

(g) Determine the area of the rhombus $PQSK$. **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a)

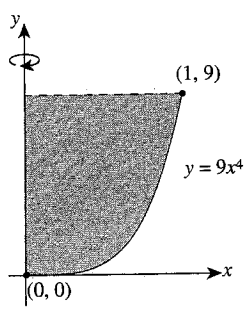


In the diagram $AD = 7$, $DC = 8$, $\angle ABD = 15°$ and $\angle DAB = 35°$.

(i) Show that $BD = 15.5$ correct to 1 decimal place. **2**

(ii) Find the length of BC correct to the nearest whole number. **2**

(b) **3**



The diagram shows the part of the graph of $y = 9x^4$ that is between $(0, 0)$ and $(1, 9)$.

The area bounded by the section of the curve between $(0, 0)$ and $(1, 9)$ and the y-axis is rotated about the **y-axis.**

Find the exact value of the volume of the solid of revolution.

(c) (i) Sketch a parabola which is concave up, has an axis of symmetry at $x = -2$ and a horizontal tangent at $y = -3$. **2**

(ii) When the equation of the parabola in (c) (i) is expressed in the form $y = ax^2 + bx + c$ the value of a = 1. **3**

Determine the equation of the parabola **and** show that this parabola has two irrational roots.

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the equation of the normal to $y = \log_e(3x - 2)$ at the point (1, 0). **3**

(b) In a triathlon athletes compete in three sections: a swim section, a bike riding section and a running section.

Erin is competing in her first challenging triathlon. In this event she has a 90% chance of completing the swimming section, a 65% chance of completing the bike riding section and an 85% chance of completing the running section.

(i) What is the probability that Erin will not be able to complete the swimming section of the triathlon? **1**

(ii) What is the probability that Erin will be able to complete the swim and run sections but be unable to complete the bike section? **1**

(iii) What is the probability that Erin will be able to complete at least one section of the triathlon? **2**

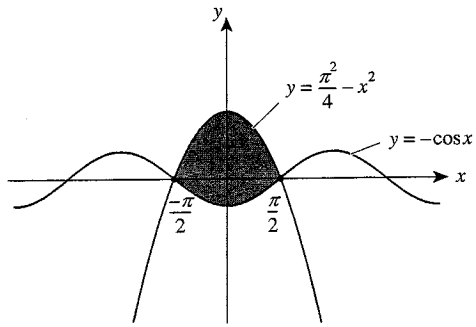
(iv) Is it true that Erin has close to a 50% chance of completing all three sections of the race successfully?
Support your answer with relevant working. **1**

(c) Prove the identity $\dfrac{1}{\sin\theta + 1} - \dfrac{1}{\sin\theta - 1} \equiv 2\sec^2\theta$ **4**

and hence find $\displaystyle\int_0^{\frac{\pi}{4}} \left(\frac{1}{\sin\theta + 1} - \frac{1}{\sin\theta - 1}\right) d\theta.$

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

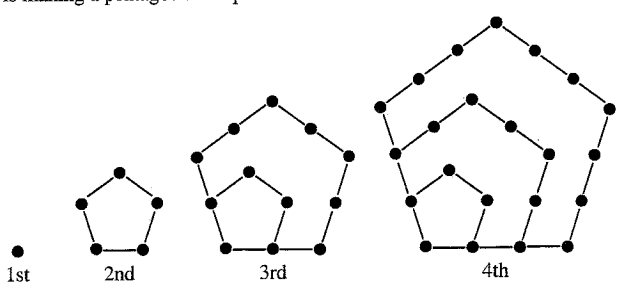
(a) **3**



The diagram shows the section of the graphs of $y = -\cos x$ and $y = \frac{\pi^2}{4} - x^2$.
The graphs intersect on the x-axis at $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$.

Find the exact value of the shaded area.

(b) Josh is making a pentagonal dot pattern.



The table below shows the number of dots on the first four patterns.

Diagram	1st	2nd	3rd	4th
Number of dots on the bottom row (n)	1	2	3	4
Total number of dots in the diagram (T)	1	5	12	22
Additional dots added to the previous pattern to make this pattern	1	4	7	10

(i) How many dots will Josh have to add to the 17th diagram to make the 18th diagram? **2**

(ii) How many dots are in the 18th diagram? **1**

Question 6 continues on page 8

Marks

Question 6 (Continued)

(c) A jar contains honey. A marble is placed on the surface of the honey and proceeds to sink. Its height, x cm, above the bottom of the jar at time t seconds is given by

$$x = 14 - 6e^{-\frac{t}{2}}.$$

(i) Show that the marble was dropped from a height of 8 cm above the bottom of the jar. **1**

(ii) Show that the velocity, v cm/s, of the marble is given by **1**

$$v = 3e^{-\frac{t}{2}}.$$

(iii) Sketch the graph of the velocity, v cm/s, of the marble at time t seconds. Clearly label the initial velocity and the velocity of the marble when $t = 2$ seconds. **2**

(iv) Determine an expression for the acceleration of the marble at t seconds and describe how the acceleration changes as time increases. **2**

End of Question 6

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) The rate at which Adam's investment, V, is growing is proportional to the value of the investment, i.e. $\frac{dV}{dt} = kV$.

(i) Show that $V = V_0e^{kt}$ satisfies the equation $\frac{dV}{dt} = kV$. **1**

(ii) Ten years ago Adam purchased a block of land for \$120 000. It is estimated that the land will be worth \$1 300 000 in **another** 15 years time. **2**

Use this information to find the value of V_0 and an expression for k.

(iii) Adam has been offered \$375 000 for the land today. **2**

Do you think Adam should accept the offer and sell the land today? Use calculations to justify your answer.

(b) A function $f(x)$ is defined by $f(x) = \frac{x^3}{4}(x - 8)$.

(i) Find the coordinates of the stationary points of the graph of $f(x)$ and determine their nature. **4**

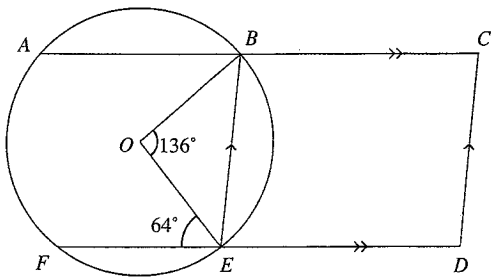
(ii) Hence sketch the graph of $y = f(x)$ showing all the essential features of the graph, including the stationary points and the intercepts on the x- and y-axes. **2**

(iii) For what values of x is the curve increasing? **1**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) 3



NOT TO SCALE

In the diagram O is the centre of a circle and points A, B, E and F lie on the circumference of the circle. Line ABC is parallel to line FED and BE is parallel to CD. $\angle BOE = 136°$ and $\angle OEF = 64°$.

Determine the size of $\angle CDE$, giving reasons for each statement.

(b) A resting adult's breathing cycle is 5 seconds long. For time t seconds, $0 \le t \le 2\frac{1}{2}$, air is taken into the lungs. For $2\frac{1}{2} < t < 5$ air is expelled from the lungs. The rate, R litres/second, at which air is taken in or expelled from the lungs can be modelled on the equation $R = \frac{1}{2}\sin\left(\frac{2\pi}{5}t\right)$. 3

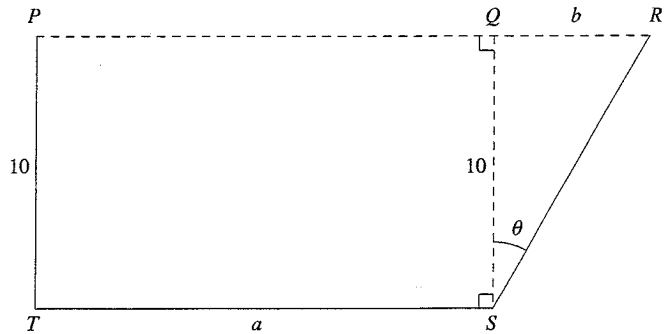
How many litres of air does a resting adult take **into** their lungs during one breathing cycle?

Question 8 continues on page 11

Marks

Question 8 (Continued)

(c) A roof gutter is made by folding a long piece of metal $PTSR$ 40 cm wide. The cross-section of the gutter is in the shape of a trapezium $PRST$.



The vertical side of the gutter is 10 cm long and the other side is at an acute angle of θ with the vertical. The sum of the three lengths in the gutter is 40 cm, i.e. $10 + a + RS = 40$.

(i) Show $b = 10\tan\theta$ and $RS = 10\sec\theta$. 1

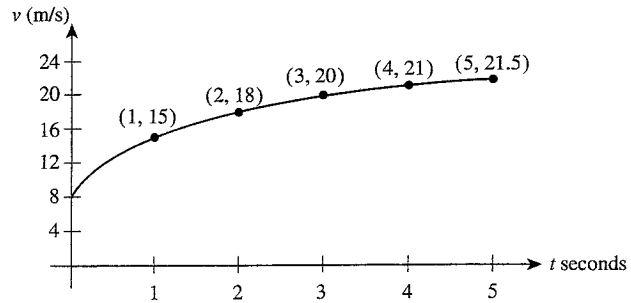
(ii) Show the area, A cm^2, of the cross section $PRST$ is given by $A = 300 - 100\sec\theta + 50\tan\theta$. 2

(iii) Hence show that the cross-sectional area is a maximum when $\theta = \frac{\pi}{6}$ radians. 3

End of Question 8

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

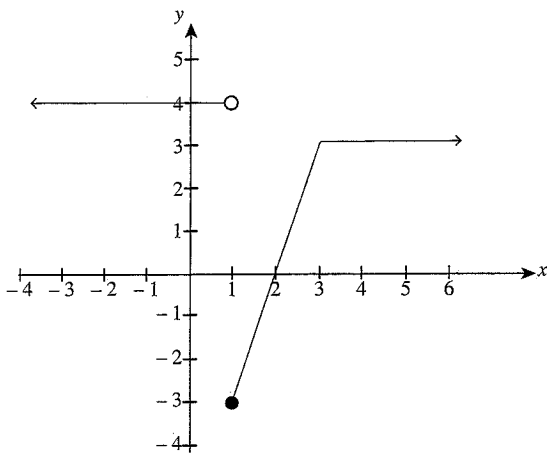


(a) The diagram shows the graph of a particle's velocity, v m/s, at time t seconds.

(i) Use the trapezoidal rule with 3 sub-intervals (4 function values) to approximate the distance the particle travelled in the first 3 seconds. **2**

(ii) Is the estimate you found for the distance the particle travelled more than or less than the exact answer? Give a reason for your answer. **1**

(b) .



The diagram shows the graph of $y = f(x)$

(i) For which values of x is the function not differentiable? **1**

(ii) Sketch the graph of $y = f'(x)$, i.e. the derivative of $y = f(x)$. Use open and/or closed circles to indicate clearly what happens at $x = 1$ and $x = 3$. **3**

Question 9 continues on page 13

Marks

Question 9 (Continued)

(c) Joseph borrowed \$26 000 (P) at monthly compounding interest of 9% p.a. He is to repay the loan in 8 equal instalments over 2 years.
(That is, he will make repayments of \$$R$ at 3-monthly intervals.)

Let $K = (1.0075)^3$ and
A_n = the amount owing on the loan immediately after the nth repayment.

(i) Show $A_n = PK^n - \dfrac{R(K^n - 1)}{K - 1}$. **2**

(ii) Show that the value of the repayments \$$R$ is \$3590.20. **1**

(iii) When Joseph made his 7th repayment he repaid an additional \$2000, i.e. he repaid \$5590.20. **2**

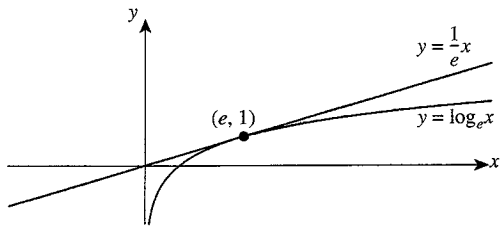
Determine the amount by which his final repayment will be reduced as a result of the additional \$2000 in his 7th repayment.

End of Question 9

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a)



The diagram shows the graphs of $y = \log_e x$ and the tangent to $y = \log_e x$ at the point $(e, 1)$.

The equation of the tangent is $y = \frac{1}{e}x$.

(i) Explain how you know that $\log_e x < \frac{x}{e}$ for all positive values of x except $x = e$. **1**

(ii) Hence show that $\pi^e < e^\pi$. **3**

(b) The average monthly temperature, T°C, in Canberra can be modelled on the formula

$$T = 7\sin(nx + 1.5) + 13,$$

where n = a constant value and

x = the number of the month of the year (i.e. January = 1, February = 2...).

(i) According to the model, what are the maximum and minimum average monthly temperatures in Canberra? **2**

(ii) The period of the function is 12. Determine the value of n correct to 2 decimal places. **1**

(iii) Which month has the lowest average monthly temperature? **2**

(iv) Graph the function $T = 7\sin(nx + 1.5) + 13$ for $1 \le x \le 12$. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

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Mathematics

Solutions and marking guidelines

Question 1

Sample answer	Syllabus outcomes and marking guide
(a) $\log_e 70 - \log_e 10 = \frac{4.248495242}{2.302585093}$ $= 1.945910149$ $= 1.95$ to 3 significant figures	H3 • Gives the correct answer ..... 2 • Gives a correct evaluation or indication of understanding of rounding to 3 significant figures OR • Correct use of log law 1
(b) $\frac{\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{5+2\sqrt{5}}{5-4}$ $= 5 + 2\sqrt{5}$	P3 • Gives the correct answer ..... 2 • Demonstrates some understanding of the process 1
(c) $(x+1)(x-1) = 15$ $x^2 - 1 = 15$ $x^2 = 16$ $\therefore x = +4$ or -4	P3, P4 • Gives the correct solutions..... 2 • Gives one correct answer or some attempt at expanding and/or re-factorising..... 1
(d) $\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}}$ $= \sqrt{2}$	P4, P5 • Gives the correct answer 1
(e) $\log_b a \times \log_e b = \frac{\log_e a}{\log_e b} \times \log_e b$ $= \log_e a$ **Alternative answer:** $\log_b a \times \log_e b = \log_b a \times \frac{\log_b b}{\log_b e}$ $= \frac{\log_b a}{\log_b e}$ $= \log_e a$	H3 • Gives the correct answer ..... 2 • Gives an answer which demonstrates some understanding of logarithms..... 1
(f) (i) $60° = \frac{\pi}{3}$ radians	H5, H9 • Gives the correct answer..... 1
(ii) $l = r\theta$ $= 8 \times \frac{\pi}{3}$ $= \frac{8\pi}{3}$ cm	H5, H9 • Gives the correct answer 1
(iii) $= \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 64 \times \frac{\pi}{3}$ $= \frac{32\pi}{3}$cm^2	H5, H9 • Gives the correct answer 1

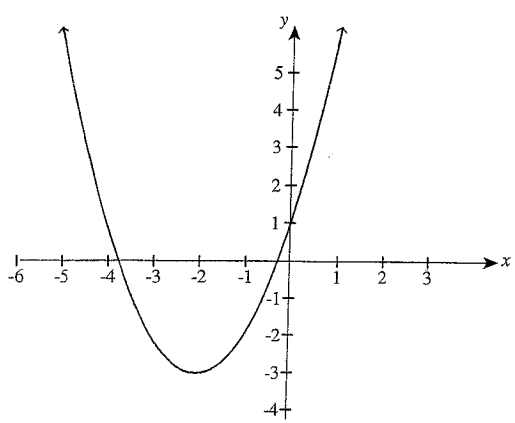
Question 2

Sample answer	Syllabus outcomes and marking guide
(a) (i) $y = 5x^8 + 4$ $y' = 40x^7$	H5, H9 • Gives the correct answer ... 1
(ii) $y = \frac{2+3x}{x-4}$ $y' = \frac{(x-4)\times 3 - (2+3x)\times 1}{(x-4)^2}$ $y' = \frac{3x-12-2-3x}{(x-4)^2}$ $y' = \frac{-14}{(x-4)^2}$	H5, H9 • Gives the correct answer ... 2 • Use of quotient rule or similar ... 1
(iii) $y = x^2 \sin x$ $y' = 2x\sin x + x^2\cos x$	H5, H9 • Gives the correct answer ... 2 • Use of product rule ... 1
(b) (i) $\int 8x^3\,dx = 2x^4 + C$	H5, H9 • Gives the correct primitive ... 1
(ii) $\int \frac{1}{2x+3}\,dx = \frac{1}{2}\log_e(2x+3) + C$	H5,H9 • Gives the correct primitive ... 2 • Gives an expression using $\log_e(f(x))$ ... 1
(c) $\int_0^2 e^{3x}\,dx = \frac{1}{3}[e^{3x}]_0^2$ $= \frac{1}{3}(e^6 - 1)$	H5, H8 • Gives the correct answer ... 2 • Demonstrates some understanding of the process ... 1
(d) $y = \log_{10}(x+1)$ domain: real x, $x > -1$ range: all real y	P5, H3 • Gives the correct answers ... 2 • Correct domain or range ... 1

Question 3

Sample answer	Syllabus outcomes and marking guide
(a) $m = \frac{11-1}{3-(-2)}$ $m = 2$	P3, P4 • Gives the correct answer ... 1
(b) $m = 2$ point (–2, 1) $y - 1 = 2(x-(-2))$ $\therefore y = 2x + 5$	P3, P4 • Gives the correct answer in any form OR • Gives the correct answer from previous answer ... 2 • Demonstrates some understanding of the process ... 1
(c) $\beta = 63°$ because $m = 2$, and $\tan 63° = 2$.	P2, H5, H9 • Gives the correct reason ... 1
(d) As these are the sides of a rhombus, $PQ = PR$. $\therefore PQ = \sqrt{(11-1)^2 + (3-(-2))^2}$ $= \sqrt{100+25}$ $= \sqrt{125}$	P2, H5, H9 • Gives the correct answer and reason ... 2 • Gives either correct answer or the correct reason ... 1
(e) In ΔPQS, $PQ = QS$ as they are the sides of a rhombus. $\therefore \Delta PQS$ is isosceles. $2 \times 63° + \angle PQS = 180°$ $\angle PQS = 54°$ **Alternative answer:** $\angle QPK = 2 \times 63°$, diagonals of rhombus bisect vertices $\angle QPK = \angle QSK$ and $\angle PKS = \angle PQS$ Opposite angles in a rhombus are equal. $\therefore 2 \times 126 + 2 \times \angle PQS = 360°$ $\angle PQS = 54°$	P2, H5, H9 • Gives the correct angle and a correct geometrical property ... 2 • Gives either angle or correct property ... 1
(f) The diagonals of a rhombus intersect at 90° $\therefore$ They will intersect at (3, 1). The length from (–2, 1) to (3, 1) is 5 units. $\therefore$ From (3, 1) to S is 5 units. $\therefore a = 3 + 5$ $a = 8$	H5, H9 • Gives a correct demonstration ... 2 • Demonstrates some progress, e.g. determines the point (3, 1) from geometrical properties ... 1
(g) Area $= 2 \times$ Area ΔPQS $= 2 \times \frac{1}{2} \times 10 \times 10$ $= 100$ units^2 **Alternative answer:** Area $= 2 \times \frac{1}{2} \times (\sqrt{125})^2 \times \sin 126°$ $\doteqdot 101$ units^2	H5, H9 • Gives the correct area ... 2 • Demonstrates some understanding of the process ... 1

Question 4

Sample answer	Syllabus outcomes and marking guide
(a) (i) $\frac{BD}{\sin 35°} = \frac{7}{\sin 15°}$ $\therefore BD = \frac{7\sin 35°}{\sin 15°}$ $= 15.51290421$ $= 15.5$ to 1 decimal place	P4 • Gives the correct answer . . . . . 2 • Correct substitution into correct Sine rule formula. 1
(ii) $\angle BDC = 50°$ (external angle ΔADB) $BC^2 = 15.5^2 + 8^2 - 2 \times 15.5 \times 8\cos 50°$ $BC = \sqrt{144.8386728}$ $BC = 12$ to the nearest whole number	H4, H5 • Gives the correct answer . . . . . 2 • Correct substitution into correct Cosine rule formula. 1
(b) $V = \pi \int_a^b x^2 dy$ $V = \frac{\pi}{3}\int_0^9 y^{\frac{1}{2}} dy$ $= \frac{\pi}{3}\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^9$ $= \frac{2\pi}{9}\left(9^{\frac{3}{2}} - 0\right)$ $= 6\pi$ units^3	H8, H9 • Gives the correct answer (units may be omitted) . . . . . 3 • Correct substitution into correct volume formula and correct integral or equivalent progress OR • Gives volume about the x-axis. . . . . 2 • Correct substitution into correct volume formula or equivalent progress 1
(c) (i)  	P4, P5 • Gives the correct diagram . . . . . 2 • Diagram has one correct feature 1

Question 4 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (ii) $y = x^2 + bx + c$ The equation of the axis is $x = -2,\ a = 1$ $\frac{-b}{2a} = -2,\ a = 1$ $-b = -4$ $b = 4$ $y = x^2 + 4x + c$ passes through $(-2, -3)$ $-3 = 4 - 8 + c$ $\Rightarrow c = 1$ The equation of the parabola is $y = x^2 + 4x + 1$ $\Rightarrow \Delta = 4^2 - 4 \times 1 \times 1$ $\Delta = 12$ As $\Delta > 0$, there are two unequal, real roots. As $\Delta \neq k^2$, the roots are irrational.	P4, P5 • Gives the correct answer . . . . . 3 • Correctly determines the equation of the parabola or equivalent merit OR • Gives a correct interpretation of non-trivial equation . . . . . 2 • Correctly evaluates either b or c or equivalent merit 1

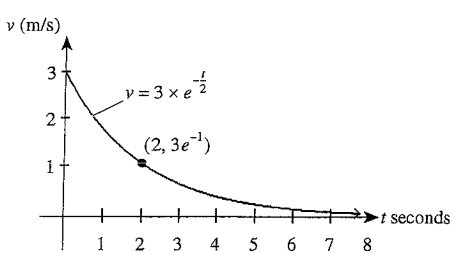
Question 5

Sample answer	Syllabus outcomes and marking guide
(a) $y = \ln(3x-2)$ $y' = \frac{3}{3x-2}$ At $(1, 0)$, $y' = m = 3$. $\therefore$ gradient of normal is $-\frac{1}{3}$ $\therefore$ equation of the normal is $y - 0 = -\frac{1}{3}(x-1)$ $x + 3y - 1 = 0$	H5, H6, H9 • $y - 0 = -\frac{1}{3}(x-1)$ or equivalent. ..... 3 • Gives the correct gradient of normal OR • Gives $y - 0 = m(x-1)$ where the value of m has been determined by a calculus process (not necessarily correctly) ..... 2 • Correctly determines the derivative OR • Finds the gradient of normal from their gradient of tangent 1
(b) (i) $P(\bar{S}) = \frac{1}{10}$	H4, H5 • Gives the correct answer 1
(ii) $P(S\bar{B}R) = \frac{9}{10} \times \frac{35}{100} \times \frac{85}{100}$ $= \frac{1071}{4000}$	H4, H5 • Gives the correct numerical expression .. 1
(iii) P(at least 1 section) $= 1 - P(\overline{SBR})$ $= 1 - \frac{1}{10} \times \frac{35}{100} \times \frac{15}{100}$ $= \frac{3979}{4000}$	H4, H5 • Gives the correct answer ..... 2 • Gives an answer which demonstrates some understanding of the process 1
(iv) Yes, since $P(SBR) = \frac{9}{10} \times \frac{13}{20} \times \frac{17}{20}$ $= \frac{1989}{4000}$ which is close to 50%.	H2, H4, H5 • Gives a correct answer with correct reasoning in the form of a calculation ... 1

Question 5 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) Required to prove $\frac{1}{\sin\theta + 1} - \frac{1}{\sin\theta - 1} \equiv 2\sec^2\theta$ LHS $= \frac{(\sin\theta - 1) - (\sin\theta + 1)}{\sin^2\theta - 1}$ $= \frac{-2}{\sin^2\theta - 1}$ $= \frac{-2}{-\cos^2\theta}$ $= \frac{2}{\cos^2\theta}$ $= 2\sec^2\theta$ = RHS $\therefore \int_0^{\frac{\pi}{4}} \left(\frac{1}{\sin\theta + 1} - \frac{1}{\sin\theta - 1}\right) d\theta$ $= 2\int_0^{\frac{\pi}{4}} \sec^2\theta \, d\theta$ $= 2[\tan\theta]_0^{\frac{\pi}{4}}$ $= 2(1 - 0)$ $= 2$	H2, H5, H8, H9 • Gives the correct proof and correct evaluation of integral. ..... 4 • Gives the correct proof and substantial progress with integral OR • Substantial progress with proof and correct evaluation of integral ..... 3 • Gives the correct proof or correct evaluation of integral or equivalent progress..... 2 • Makes some progress with either proof or integral 1

Question 6

Sample answer	Syllabus outcomes and marking guide
(a) Area $= 2\times\int_0^{\frac{\pi}{2}}\left(\frac{\pi^2}{4}-x^2--\cos x\right)dx$ $=2\left[\frac{\pi^2 x}{4}-\frac{1}{3}x^3+\sin x\right]_0^{\frac{\pi}{2}}$ $=2\left[\frac{\pi^3}{8}-\frac{\pi^3}{3\times 8}+\sin\frac{\pi}{2}-0\right]$ $=2\left[\frac{2\pi^3}{3\times 8}+1\right]$ $=\frac{\pi^3}{6}+2$ units^2	H8, H9 • Gives the correct answer . . . . . 3 • Gives the correct integral coming from correct area statement with correct limits of integration or equivalent progress . . . . . 2 • Gives the correct statement for area in terms of integral or equivalent progress 1
(b) (i) $T_n = a+(n-1)d$ of 1, 4, 7, 10... $T_{18}=1+(18-1)\times 3$ $T_{18}=52$	H4, H5 • Gives the correct answer . . . . . 2 • Recognises that a term in an arithmetic sequence is required. 1
(ii) $S_n=\frac{n}{2}\{2a+(n-1)d\}$ $\therefore S_{18}=\frac{18}{2}\{2+17\times 3\}$ $S_{18}=477$ There are 477 dots in the 18th diagram.	H4, H5 • Gives the correct answer 1
(c) (i) $x=14-6e^{-\frac{t}{2}}$ When $t=0$, $x=14-6\times 1$ $=8$ $\therefore$ It is dropped from a height of 8 cm above the bottom of the jar.	H3, H4 • Gives the correct answer 1
(ii) $x=14-6e^{-\frac{t}{2}} \Rightarrow v=-6\times\left(-\frac{1}{2}e^{-\frac{t}{2}}\right)$ $v=3e^{-\frac{t}{2}}$	H3, H4 • Correctly determines velocity equation . . 1
(iii)  	• H3, H4, H9 • Gives the correct sketch which is fully labelled. . . . . 2 • Gives the correct sketch or equivalent progress 1

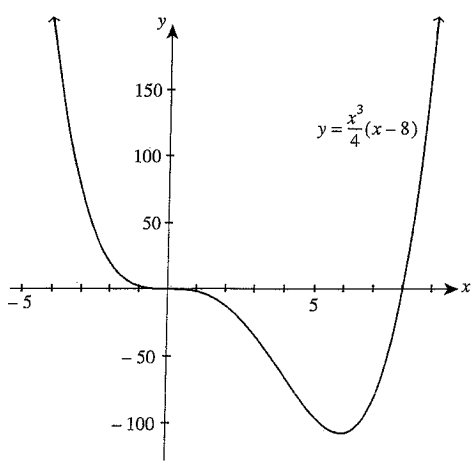
Question 6 (Continued)

Sample answer	Syllabus outcomes and marking guide
(iv) Acceleration: $a=-\frac{3}{2}e^{-\frac{t}{2}}$ $a=-\frac{3}{2e^{\frac{t}{2}}}$ As $t\to\infty$, $\ddot{x}\to 0$ i.e. As time increases the acceleration approaches zero.	H3, H4 • Gives the correct equation for acceleration and correct description. . . . . 2 • Gives the correct equation for acceleration OR • Gives the correct description from their acceleration 1

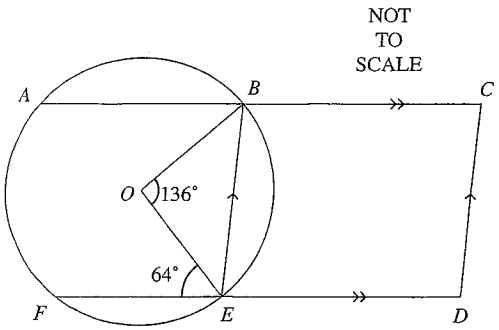
Question 7

Sample answer	Syllabus outcomes and marking guide
(a) (i) $V = V_0 e^{kt}$ $\frac{dV}{dt} = k(V_0 e^{kt})$ $= kV$ $\therefore V = V_0 e^{kt}$ is a solution.	H3, H4 • Uses the correct method 1
(ii) When $t = 0$, $V = 120\,000$ $\therefore 120\,000 = V_0 e^0$ $\therefore V_0 = 120\,000$ When $t = 25$, $V = 1\,300\,000$. $\therefore 1\,300\,000 = 120\,000 e^{25k}$ $\frac{130}{12} = e^{25k}$ $25k = \ln\left(\frac{130}{12}\right)$ $k \approx 0.095305$	H3, H4 • Correctly evaluates V_0 and finds an expression for k . . . . . 2 • Correctly evaluates V_0 1
(iii) When $t = 10$, $V = 120\,000 e^{10k}$ $= 120\,000 \times 2.593611018$ $= \$311\,233.32$ ∴Adam should take the offer of \$375 000 as the property has a current value of only \$311 233. **Alternative justification/interpretation:** $V = \$311\,233.32$ ∴While Adam has been offered more than the land is worth, he might decide not to sell because it could be worth a lot more than \$1.3 million in 15 years time. Maybe the person wanting to buy the land knows something Adam doesn't know.	H3, H4 • Gives the correct solution based on calculations with correct justification/interpretation . . . . . 2 • Significant progress toward a solution. . . 1

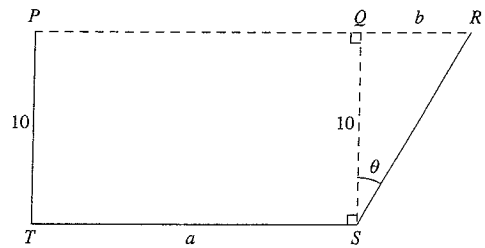
Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(b) (i) $f(x) = \frac{x^3}{4}(x-8)$ $f(x) = \frac{x^4}{4} - 2x^3$ $\Rightarrow f'(x) = x^3 - 6x^2$ Stationary points occur when $f'(x) = 0$, i.e. $x^2(x-6) = 0$. This occurs when $x = 0$ or $x = 6$ and $y = 0$ or $y = -108$. ∴Stationary points are $(0, 0)$ and $(6, -108)$. Test: $f''(x) = 3x^2 - 12x$ At $(0, 0)$, x | −1 | 0 | 1 $f''(x)$ | 15 | 0 | −9 ∴There is a change in concavity at $(0, 0)$, so the origin is a horizontal point of inflection. At $(6, -108)$, $f''(x) = 36$ which is positive, so at that point the curve is concave up and $(6, -108)$ is a minimum turning point.	H5, H6 • Correctly determines the nature of the 2 stationary points with appropriate testing for both . . . . . 4 • Correctly finds the coordinates of both points and determines the nature of one point correctly . . . . . 3 • Correctly finds the coordinates of the stationary points or equivalent merit OR • Finds one point correctly and determines the nature correctly . . . . . 2 • Correctly finds the coordinates of 1 stationary point or equivalent merit . . . 1
(ii)  	H6, H9 • Gives the correct sketch from previous answer showing both stationary points and x- and y-intercepts 2 • Gives the correct sketch from previous answer with some information missing or equivalent progress 1
(iii) Increasing for $x > 6$.	H6, H9 • Gives the correct answer OR • Correct from the previous answer 1

Question 8

Sample answer	Syllabus outcomes and marking guide
(a)  $OB = OE$ (radii of circle equal) ΔOBE is isosceles. $\angle OBE = \frac{1}{2}(180 - 136)$ (base angles of isosceles triangles are equal.) $\therefore \angle OEB = 22°$ $\therefore \angle BEF = 64° + 22°$ $= 86°$ $\therefore \angle EDC = 86°$ (corresponding angles, BE parallel to CD)	P4, H2, H5 • Gives the correct answer with appropriate reasons . . . . . 3 • Gives the correct answer without reasons or substantial progress . . . . . 2 • Correctly determines $\angle OEB$ or substantial progress 1
(b) $\frac{dV}{dt} = \frac{1}{2}\sin\left(\frac{2\pi}{5}t\right)$ Volume $= \int_0^{\frac{5}{2}} \frac{1}{2}\sin\left(\frac{2\pi}{5}t\right)dt$ $= -\frac{1}{2} \times \frac{5}{2\pi}\left[\cos\left(\frac{2\pi}{5}t\right)\right]_0^{\frac{5}{2}}$ $= -\frac{5}{4\pi}[\cos\pi - \cos 0]$ $= -\frac{5}{4\pi} \times -2$ $= \frac{5}{2\pi}$ litres	H4, H8, H9 • Gives the correct answer (omit units) . . . 3 • Correctly determines integral with correct limits . . . . . 2 • Correctly determines integral or equivalent progress or equivalent progress 1

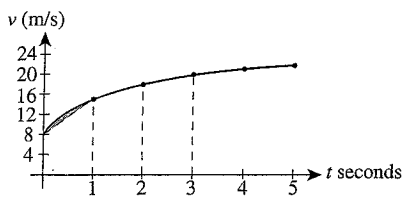
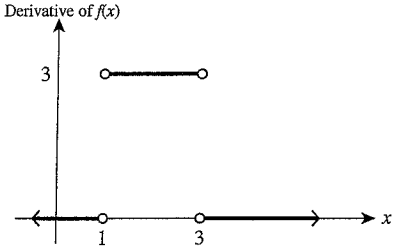
Question 8 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i)  In ΔQRS, $\tan\theta = \frac{b}{10}$ $\therefore b = 10\tan\theta$ In ΔQRS, $\sec\theta = \frac{RS}{10}$ $\therefore RS = 10\sec\theta$	P3, H4, H5, H9 • Gives the correct evaluation for b and RS 1
(ii) $a = 40 - 10 - RS$ $= 30 - 10\sec\theta$ Area $PQST = 10 \times a$ $= 10(30 - 10\sec\theta)$ $= 300 - 100\sec\theta$ Area $\Delta QRS = \frac{1}{2} \times b \times 10$ $= 5 \times 10\tan\theta$ $= 50\tan\theta$ Total area $= 300 - 100\sec\theta + 50\tan\theta$ **Alternative solution:** $A = \frac{1}{2}H(A + B)$ $A = \frac{1}{2} \times 10 \times (a + b + a)$ $A = 5(2a + b)$ $= 5(60 - 20\sec\theta + 10\tan\theta)$ $A = 300 - 100\sec\theta + 50\tan\theta$	P3, H4, H5, H9 • Gives the correct proof, any suitable method . . . . . 2 • Makes significant progress toward the proof. 1

Question 8 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(iii) $A = 300 - 100\sec\theta + 50\tan\theta$ $= 300 - \frac{100}{\cos\theta} + 50\tan\theta$ $A' = -\frac{\cos\theta \times 0 - 100 \times -\sin\theta}{\cos^2\theta} + 50\sec^2\theta$ $= \frac{-100\sin\theta}{\cos^2\theta} + 50\sec^2\theta$ $= -100\sin\theta\sec^2\theta + 50\sec^2\theta$ For a maximum, $\frac{dA}{d\theta} = 0$. $\therefore -100\sin\theta\sec^2\theta + 50\sec^2\theta = 0$ $50\sec^2\theta(1 - 2\sin\theta) = 0$ $\sec^2\theta \neq 0, \therefore \sin\theta = \frac{1}{2}$ As θ is acute, $\theta = \frac{\pi}{6}$. Test: θ (rads) | 0.5 | $\frac{\pi}{6}$ | 1 $\frac{dA}{d\theta}$ | 2.67 | 0 | –71.3 As the gradient changes from positive to negative about $\frac{\pi}{6}$, this indicates that there is a maximum turning point at $\frac{\pi}{6}$.	H4, H5, H9 • Gives a correct answer which includes a correct test ..... 3 • Makes substantial progress but omits test ..... 2 • Correctly differentiates $\sec\theta$ and $\tan\theta$ .. 1

Question 9

Sample answer	Syllabus outcomes and marking guide
(a) (i) t | 0 | 1 | 2 | 3 V | 8 | 15 | 18 | 20 Distance $= \frac{h}{2}$[first + last + 2 × others] $= \frac{1}{2}[8 + 20 + 2 \times (15 + 18)]$ $= \frac{1}{2} \times 94$ = 47 metres	H4, H8 • Gives the correct answer ..... 2 • Correct use of trapezoidal rule but wrong values substituted OR • Wrong number of function values used or equivalent merit 1
(ii)  The answer in part **i.** is less than the exact answer as you are forming trapeziums which lie beneath the function. The shaded sections are included in the exact answer but not in the approximation.	H2 • Correct reasoning 1
(b) (i) The function is **not** differentiable at $x = 1$ and $x = 3$.	H5, H7 • Gives the correct answers 1
(ii)  	H7, H9 • Gives the correct sketch from previous answer ..... 3 • Correctly sketches any 2 sections of $y = f'(x)$ OR • Correctly sketches the 3 horizontal sections and makes error(s) with open/closed circles..... 2 • Correctly sketches 1 section of $y = f'(x)$ from previous answer OR • Correctly sketches 2 of the 3 horizontal sections and makes errors with open/closed circles... 1

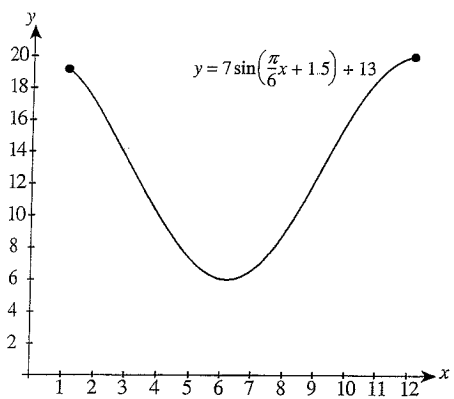
Question 9 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) $A_1 = P(1.0075)^3 - R$ $A_2 = \{P(1.0075)^3 - R\}(1.0075)^3 - R$ $= P[1.0075]^6 - R[(1.0075)^3 + 1]$ $= PK^2 - R(K+1)$ $A_n = PK^n - R(K^{n-1} + K^{n-2} \ldots + 1)$ $= PK^n - R \times \frac{a(r^n - 1)}{r-1}$ $= PK^n - R \times \frac{1(K^n - 1)}{K-1}$ $= PK^n - \frac{R(K^n - 1)}{K-1}$	H2, H4, H9 • Gives the correct demonstration . . . . . 2 • Makes some progress with the derivation, e.g. calculates an expression for the amounts owing 6 months into the loan. 1
(ii) When $n = 8$, $A_n = 0$. $\therefore PK^8 - \frac{R(K^8 - 1)}{K-1} = 0$ $\Rightarrow R\frac{\{1.0075^{24} - 1\}}{1.0075^3 - 1} = 26\,000 \times (1.0075)^{24}$ $\therefore R = \$3590.20$	H2, H4, H9 • Gives the correct demonstration by solving the equation OR • Substitution of 3590.20 into the equation 1
(iii) $A_7 = 26\,000(1.0075)^{21} - \frac{3590.20\{(1.0075)^{21} - 1\}}{1.0075^3 - 1}$ Because he repaid an additional \$2000 he only owes \$1510.64 for the remaining 3 months. Last payment $= \$1520.64 \times (1.0075)^3$ $= \$1544.88$ Thus his last payment is $= \$3590.20 - \$1544.88$ less $= \$2045.32$ less It is \$2045.32 less.	H4, H9 • Gives the correct answer . . . . . 2 • Substantial progress, e.g. correct payment amount owing after the additional repayment 1

Question 10

Sample answer	Syllabus outcomes and marking guide
(a) (i) From the diagram, it can be seen that the graph of $y = \log_e x$ is always less than the graph of the tangent, except for $x = e$, i.e. $\log_e x < \frac{x}{e}$.	H2, H3, H9 • Gives the correct reasoning 1
(ii) $\log_e x < \frac{x}{e}, x \neq e$ $\therefore e\log_e x < x$ $\therefore \log_e x^e < x$ $\therefore x^e < e^x$ As this is true for all values of x except $x = e$, substitute $x = \pi$, i.e. $\pi^e < e^\pi$. **Alternative answer:** $\log_e x < \frac{x}{e}$ $e^{\log_e x} < e^{\frac{e}{x}}$ $x < e^{\frac{1}{e}x}$ $x^e < \left(e^{\frac{1}{e}x}\right)^e$ $x^e < e^x$ Let $x = \pi$ $\pi^e < e^\pi$	H3, H5 • Gives the correct solution . . . . . 3 • Substantial progress . . . . . 2 • Makes some progress 1
(b) (i) $T = 7\sin(nx + 1.5) + 13$ The maximum value of $\sin(nx + 1.5) = 1$. $\therefore$ Max $T = 7 \times 1 + 13$ $= 20°C$ The minimum value of $\sin(nx + 1.5) = -1$. $\therefore$ Min $T = 7 \times -1 + 13$ $= 6°C$	P4, H4, H5 • Gives the correct maximum and correct minimum . . . . . 2 • Gives either the correct maximum or correct minimum 1
(ii) Period $= \frac{2\pi}{n}$ $= \frac{2\pi}{n} = 12$ $\therefore n = \frac{2\pi}{12}$ or $\frac{\pi}{6}$ $\therefore n = 0.52$ to 2 decimal places	H5 • Gives the correct answer (disregard rounding) 1

Question 10 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(b) (iii) The lowest temperature will be when $nx + 1.5 = \frac{3\pi}{2}, n = \frac{\pi}{6}$ $x = \frac{\left[\frac{3\pi}{2} - 1.5\right]}{n}$ $x \approx 6.1$ The 6th month is June, so June is the month with the coldest average temperature.	H5 • Gives the correct answer ..... 2 • Correctly equates $nx + 1.5$ to $\frac{3\pi}{2}$ or equivalent merit 1
(iv) $y = 7\sin\left(\frac{\pi}{6}x + 1.5\right) + 13$, $0 \le x \le 12$  	H5, H9 • Correct sketch ..... 3 • Substantial progress ..... 2 • Gives the correct range or domain OR • Equivalent progress 1