Neap:

HSC Trial Examination 2007

Mathematics

This paper must be kept under strict security and may only be used on or after the morning of Monday 13 August, 2007 as specified in the Neap Examination Timetable

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

Attempt questions 1–10

All questions are of equal value

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2007 HSC Mathematics Examination.

Total marks 120

Attempt Questions 1–10

All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $2\log_e 3 + \log_e 5$ correct to 3 significant figures. **2**

(b) Fully factorise $x^3 - x^2 - x + 1$. **2**

(c) Find the sum of the first hundred terms of the series $-23 + -20 + -17 + -14 \ldots$ **2**

(d) Express $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$ with a rational denominator. **2**

(e) Calculate the gradient of the tangent to the curve $y = e^{3x-6}$ at the point (2, 1). **2**

(f) Shade the region in the Cartesian plane where $y < 3x + 1$ and $x \le 2$ hold simultaneously. **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate the following with respect to x.

(i) $x^2\tan x$ **2**

(ii) $\sin^3 x$ **2**

(b) (i) Find $\int \sqrt{1-x}\, dx$. **2**

(ii) Evaluate $\int_0^1 \frac{x^2}{x^3+1}\, dx$. **2**

(c) For the parabola $y^2 = 12(x+1)$, find

(i) the coordinates of the vertex. **1**

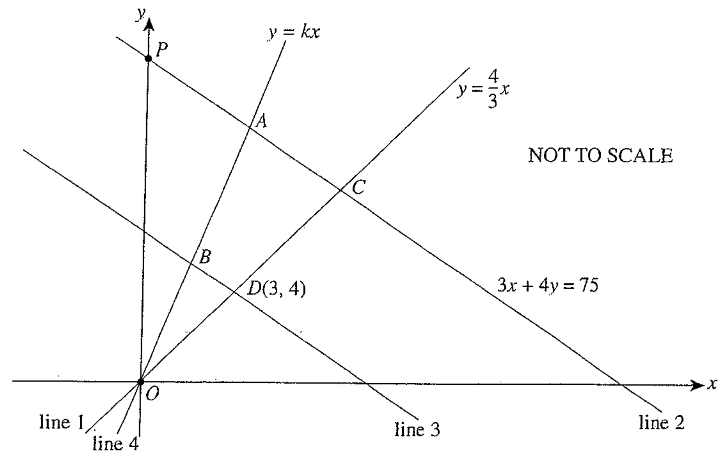
(ii) the coordinates of the focus. **1**

(iii) the length of the chord through the focus perpendicular to the axis of the parabola. **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

In the diagram below, line 1 has the equation $y = \frac{4}{3}x$, line 2 has the equation $4y + 3x = 75$, line 4 has the equation $y = kx$ and the coordinates of point D are (3, 4).



(a) Show that line 1 and line 2 are perpendicular. **1**

(b) Determine the coordinates of point P, the point where line 2 crosses the y-axis. **1**

(c) Determine the equation of line 3, which passes through D (3, 4) and is parallel to line 2 and perpendicular to line 1. **2**

(d) Show that the perpendicular distances from the origin to line 3 and the origin to line 2 are 5 and 15 units respectively. **2**

(e) Line 4, $y = kx$, intersects lines 1 and 2 at points A and B respectively. Determine the ratio $OB : BA$. **1**

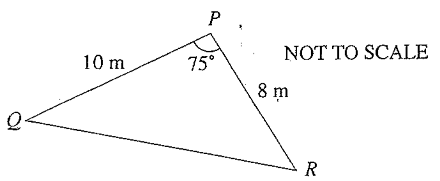
(f) Show that $\Delta\, OAC$ and $\Delta\, OBD$ are similar. **2**

(g) The area of $\Delta\, OBD$ is 2 square units. What is the area of the trapezium $BACD$? **3**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a)



(i) Determine the length of QR, correct to 2 decimal places. **2**

(ii) What is the area of triangle PQR? Answer correct to 2 decimal places. **1**

(b) A function $f(x)$ is defined as $f(x) = x^4 - 8x^2$.

(i) Locate all stationary points and any points of inflexion. Distinguish between them. **4**

(ii) Determine the coordinates of the points where $y = f(x)$ crosses the x-axis. **2**

(iii) On a half-page diagram, sketch the function $y = f(x)$. Clearly label the stationary points, points of inflexion and intercepts with the x-axis. **2**

(iv) What is the maximum value of $f(x)$ in the interval $-2 \le x \le 3$? **1**

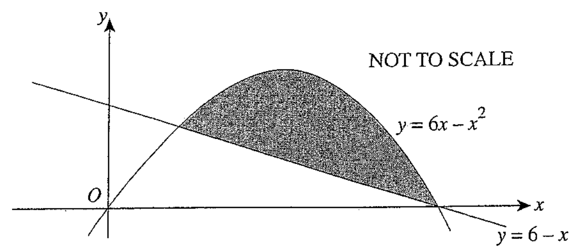
Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Simplify $\log_b a^m \div \log_m a$ as a single expression in a logarithm of base b. **2**

(b) The roots of the quadratic equation $2x^2 + kx + D = 0$ are α and β. $\alpha\beta = -5$ and $\alpha + \beta = 3$. Determine the values of k and D. **2**

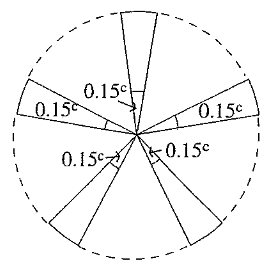
(c) The diagram shows the graph of $y = 6x - x^2$ and $y = 6 - x$.



(i) Use simultaneous equations to show that $y = 6x - x^2$ and $y = 6 - x$ intersect at $(1, 5)$ and $(6, 0)$. **1**

(ii) Use calculus to determine the size of the shaded area. **3**

(d)



The five blades on a windmill are identical sectors of the same circle. The angle of each blade at the centre of the circle is 0.15^c and the radius is 1.2 metres.

(i) All the edges on each of the blades are to be covered by a protective metal strip. Calculate the total length of metal strip required to protect the edges of all five blades. **2**

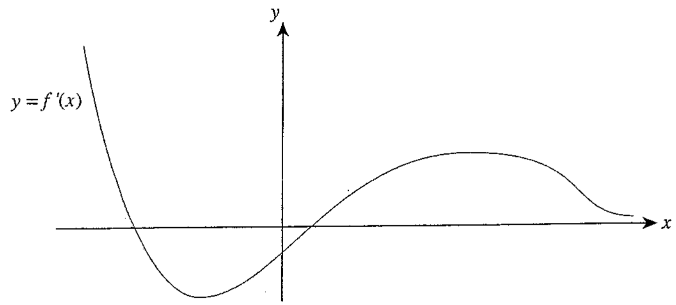
(ii) The front and back surface of each blade is to be painted with a metal protector. A 100 mL container of the metal protector covers 400 cm^2. Calculate the quantity of metal protector required. **2**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $\sin\theta = \frac{-\sqrt{3}}{2}$ for $0 \le \theta \le 2\pi$. **2**

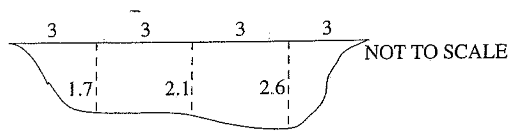
(b)



The diagram shows the gradient function $y = f'(x)$. Copy or trace the diagram into your answer booklet.

The curve $y = f(x)$ passes through the origin. Sketch the function $y = f(x)$ on the same set of axes. Clearly indicate any turning points, points of inflexion, and the behaviour of the graph for very large positive and negative values of x. **3**

(c)

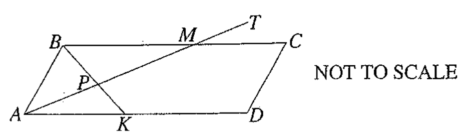


The diagram shows the cross-section of a 12-metre-wide pond. The depths are taken every 3 metres.

(i) Use Simpson's rule with five function values to find an approximate value for the area of the cross-section. **2**

(ii) The pond is 25 metres long. Calculate the approximate quantity of water in the pond. Express the volume in cubic metres. **1**

(d)



$ABCD$ is a parallelogram. Line AT bisects $\angle BAD$ and cuts BC at M. Line BK bisects $\angle ABC$. AT and BK intersect at P.

Copy the diagram onto your answer page and prove that

(i) $\angle BPA = 90°$. **2**

(ii) $AB = BM$. **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Solve the equation $e^{2x} - 28e^{x} + 27 = 0$. Leave your answer in exact form **3**

(b) An ambulance is delivering a patient to the hospital who is unconscious from a drug overdose. The medical staff don't know how much of the drug the unconscious patient has taken.

The rate of change of the concentration of the drug (C) in the blood is proportional to the concentration, i.e. $\frac{dC}{dt} = kC$.

(i) Prove that $C = C_0e^{kt}$ is a solution to $\frac{dC}{dt} = kC$. **1**

(ii) Three hours after the patient took the overdose, the blood concentration of the drug was 2.45 mg/L. Half an hour later the concentration was 1.84 mg/L. Determine the initial concentration of the drug in the patient's blood. Give your answer correct to two decimal places. **3**

(iii) If the medical staff don't give the patient any further medication, when will the drug concentration fall below the critical value of 0.5 mg/L? **1**

(c) Two particles moving in a straight line are initially at the origin. The velocity of one particle is $\frac{2}{\pi}$ m/s and the velocity of the other particle at time t seconds is given by $v = -2\cos t$ m/s.

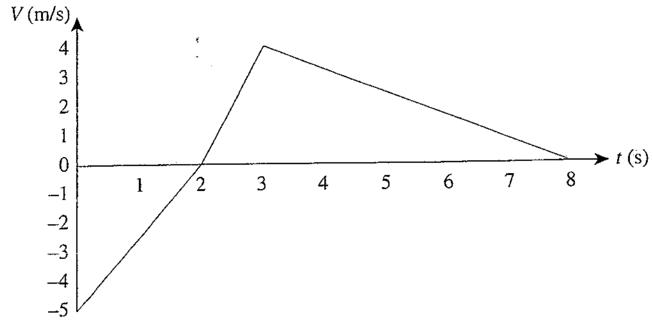
(i) Determine equations that give the displacements, x_1 and x_2 metres, of the particles from the origin at time t seconds. **2**

(ii) Hence, or otherwise, show that the particles will never meet again. **2**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a)



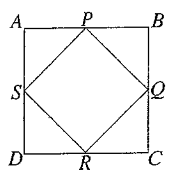
The graph shows the velocity of a particle moving in a straight line for 8 seconds.

(i) When does the particle change direction? **1**

(ii) Determine the total distance covered by the particle during the 8 seconds. **2**

(iii) What is the particle's position relative to its starting position when $t = 8$ seconds? **1**

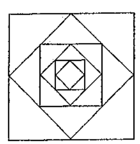
(b)



ABCD is a square with sides 16 cm long. *P*, *Q*, *R* and *S* are the midpoints of the sides of the square *ABCD*. *P*, *Q*, *R* and *S* are joined to make another square.

(i) Show that $PS = 8\sqrt{2}$ and that the area of *PQRS* is 128 cm^2. **2**

A 'squares within squares' pattern is produced by joining midpoints of the sides of successive squares.



(ii) *ABCD* is the first square and *PQRS* is the second square. What is the area of the 10th square? **2**

(iii) Which square has a perimeter of $\sqrt{2}$ cm? **2**

(iv) Imagine the pattern can be repeated infinitely. What is the relationship between the sum of the areas of all the squares and the original square *ABCD*? Use a calculation to justify your answer. **2**

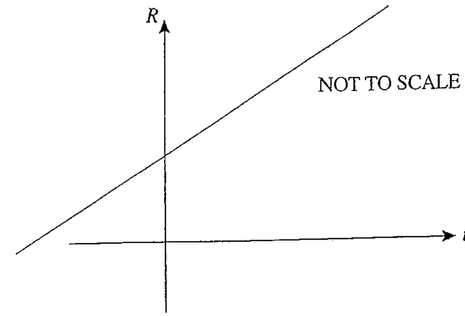
Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) During a famine in Europe in the 19th century people in a small rural city ate an increasing quantity of potatoes each month as other food became increasingly scarce.

The rate at which potatoes were eaten (R) was given by $R = 15 + 2t$ tonnes per month, where t is the time in months after the beginning of the famine.

(i)



The diagram shows the graph of $R = 15 + 2t$. Copy the graph onto your answer page and show on the graph the region representing the total quantity of potatoes eaten in the city in 12 months. **1**

(ii) Calculate the total amount of potatoes that were eaten in the city during the 12-month famine. **3**

(b) Beth and Cathy are best friends who work in the same office. Each year on January 1, they each receive a cash bonus of \$5000. They received their first bonuses in 1997. Every year Beth invests her \$5000 in superannuation at 9% p.a. compounding interest. Each year Cathy spends her bonus on an overseas trip.

(i) Show that the expression $5000(1.09^{10} + 1.09^{9} + 1.09^{8} + \ldots + 1.09)$ represents the amount in Beth's superannuation account on January 1, 2007, immediately before her 2007 bonus was added to the account. **2**

(ii) Show that Beth had almost \$88 000 in her superannuation account on January 1, 2007, after her 2007 bonus was credited to her account. **2**

Cathy decides that on January 1, 2007, she will start saving for her retirement, which will occur in 20 years' time. She would like to have the same amount that Beth will have in 20 years' time from saving her annual \$5000 bonus. Cathy's account also pays 9% p.a. compounding interest.

(iii) How much will Cathy need to save each year to have the same total amount as Beth will have in 20 years' time (i.e. including the amounts Beth invested in the first 10 years)? **3**

(iv) How much more will Cathy have to invest over the 20 years than Beth will have invested over the 30 years? **1**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

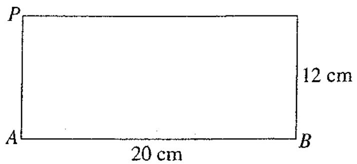
(a) The probability that a biased coin lands heads up is p. I have a pair of identical biased coins. The coins show heads more often than normal coins show heads.

(i) Draw a tree diagram to show the possible outcomes when I toss the pair of biased coins together. Write the probabilities on each branch of the tree. **1**

(ii) When I toss the coins, 25% of the time they land showing a head and a tail. Determine the probability that, on the next toss of the pair of coins, they will land with at least one of the coins showing a head. Give your answer correct to three decimal places. **3**

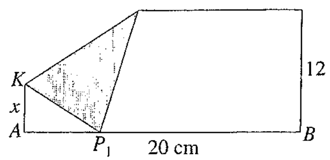
(iii) Let the probability that a pair of biased coins will land showing one head and one tail be K. Prove that it is impossible for the value of K to be greater than 50%. **2**

(b)



NOT TO SCALE

I have a rectangular sheet of paper 12 cm high by 20 cm long. I take the vertex labelled P and place it on the side AB. P now lies on top of P_1.



NOT TO SCALE

At the bottom left of the rectangle there is a small triangle AKP_1. Let the length of KA be x cm.

(i) Explain why KP_1 is $(12 - x)$ cm long. **1**

(ii) Show that the area of ΔAKP_1 is given by $A = x\sqrt{36 - 6x}$. **2**

(iii) Hence show that when x is one-third the length of PA the area of ΔAKP_1 is a maximum. **3**

End of paper

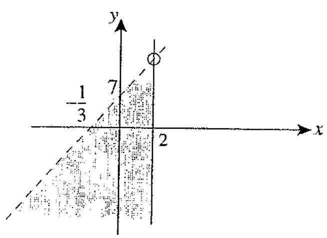
Neap:

HSC Trial Examination 2007

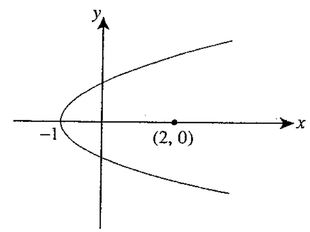
Mathematics

Solutions and marking guidelines

Question 1

	Sample answer	Syllabus outcomes and marking guide
(a)	3.80666249 = 3.81 to 3 significant figures	P3, H3 • Gives the correct answer .... 2 • Gives any answer correct to 3 significant figures 1
(b)	$x^2(x-1)-1(x-1)$ $=(x-1)(x^2-1)$ $=(x-1)(x-1)(x+1)$	P3 • Gives the correct answer .... 2 • Makes some attempt to factorise by grouping 1
(c)	arithmetic, $a=-23$, $d=3$, $n=100$ $S_n=\frac{n}{2}\{2a+(n-1)d\}$ $S_{100}=\frac{100}{2}\{2\times -23+(100-1)\times 3\}$ $= 12\ 550$	H5 • Gives the correct answer .... 2 • Correctly determines the value of a, d and n OR • Obtains the correct value for two of a, d and n, and correctly determines the value of S_n for their three values 1
(d)	$\frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}$ $=(2+\sqrt{3})^2$ or $7+4\sqrt{3}$	P3 • Gives the correct answer .... 2 • Makes an attempt to rationalise by multiplication by conjugate surd 1
(e)	$y'=3e^{3x-6}$ $M_T=3e^{3\times 2-6}$ $=3$	H3 • Gives the correct answer .... 2 • Gives the correct derivative or equivalent merit 1
(f)		P4 • Gives the correct solution .... 2 • Graphs one line correctly 1

Question 2

	Sample answer	Syllabus outcomes and marking guide
(a) (i)	$2x\tan x + x^2\sec^2 x$	P7, H9 • Gives the correct answer .... 2 • Makes an attempt to use product rule ... 1
(ii)	$3\sin^2 x\cos x$	P7, H9 • Gives the correct answer .... 2 • Makes some progress 1
(b) (i)	$\int(1-x)^{\frac{1}{2}}dx$ $=-\frac{1}{(\frac{3}{2})}(1-x)^{\frac{3}{2}}+C$ $=-\frac{2}{3}(1-x)^{\frac{3}{2}}+C$	H5 • Gives the correct solution .... 2 • Makes some progress 1
(ii)	$\frac{1}{3}\int_0^1\frac{3x^2}{x^3+1}dx$ $=\left[\frac{1}{3}\log_e(x^3+1)\right]_0^1$ $=\frac{1}{3}(\log_e 2-\log_e 1)$ $=\frac{1}{3}\log_e 2$	H5 • Gives the correct solution .... 2 • Gives the correct integral 1
(c) (i)	$y^2=4aX$ $y^2=12(x+1)$ vertex = (−1, 0) $a=3$ 	P4 • Gives the correct answer 1
(ii)	focus = (2, 0)	P4 • Gives the correct answer 1
(iii)	when $x=2$, $y^2=36$ ∴ the points at the end of the chord are (2, 6) and (2, −6) ∴ the chord is 12 units long (alternatively, the chord is $4a$ units long $4\times 3=12$)	P4 • Gives the correct answer .... 2 • Makes some progress 1

Question 3

Sample answer / **Syllabus outcomes and marking guide**

(a) $m_1 = \frac{4}{3}$ $\quad m_2 = \frac{-3}{4}$

$m_1 \; m_2 = \frac{4}{3} \times \frac{-3}{4}$

$= -1$ $\therefore$ lines are perpendicular

P4
- Gives the correct answer 1

(b) $4y + 3x = 75$

when $x = 0$, $4y = 75$

$\therefore y = 18.75$

Therefore, the coordinates of P are $(0, 18.75)$.

P4
- Gives the correct answer 1

(c) $m = \frac{-3}{4}$ $\qquad D(3, 4)$

Therefore, the equation of line 3 is $y - 4 = \frac{-3}{4}(x - 3)$

$3x + 4y - 25 = 0$

P4
- Gives the correct answer 2
- Correctly identifies that the gradient is $\frac{-3}{4}$ or equivalent merit. 1

(d) to line 3: $\left|\frac{3 \times 0 + 4 \times 0 - 25}{\sqrt{3^2 + 4^2}}\right| = \frac{25}{5} = 5$

to line 2: $\left|\frac{0 \times 3 + 4 \times 0 - 75}{\sqrt{3^2 + 4^2}}\right| = \frac{75}{5} = 15$

P4
- Gives both correct answers 2
- Gives one correct perpendicular distance 1

(e) $OD : DC = 5 : 10 = 1 : 2$

$\therefore OB : BA = 1 : 2$ (ratios of intercepts on parallel lines are equal)

H5, H9
- Gives the correct answer (with or without reason) 1

(f) In ΔOAC and ΔOBD,

$\angle AOC$ is common.

$\angle OAC = \angle OBD$ (corresponding angles, $AC \parallel BD$)

$\therefore \Delta AOC \;|||\; \Delta BOD$ (equal angles)

H2, H5, H9
- Gives the correct proof 2
- Correctly identifies a pair of equal angles 1

(g) Method 1

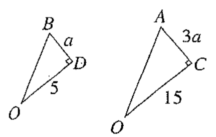
The enlargement factor of ΔBOD to ΔAOC is 3 (from part (d))

$\therefore$ area of $\Delta AOC = 9$ times the area of ΔBOD

$= 18$ square units

But area ΔBOD + area of trapezium $ABCD = 18$, therefore area of trapezium = 16 square units.

Method 2



$\frac{1}{2} \times a \times 5 = 2 \qquad a = \frac{4}{5}$

$\therefore$ area of ΔAOC $= \frac{1}{2} \times \left(3 \times \frac{4}{5}\right) \times 15$

$= 18$

$\therefore$ area of trapezium = 16 square units

H5, H9
- Gives the correct answer (by any appropriate method). 3
- Makes significant progress. 2
- Makes some progress. 1

Question 4

Sample answer / **Syllabus outcomes and marking guide**

(a) (i) $x^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \cos 75$

$\therefore x = 11.07$ to 2 decimal places

P4
- Gives the correct answer 2
- Makes the correct substitution into the correct formula. 1

(ii) area $= \frac{1}{2} \times 10 \times 8 \times \sin 75 = 38\;64\text{ m}^2$

- Gives the correct answer 1

(b) (i) $f(x) = x^4 - 8x^2$

$f'(x) = 4x^3 - 16x$

stationary points occur when $f'(x) = 0$

i.e. $4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$\therefore$ stationary points occur at (0, 0), (2, −16) and (−2, −16)

Testing

$f''(x) = 12x^2 - 16$

At (0, 0), $f''(x) = -16 < 0$. $\therefore$ maximum turning point.

At $x = \pm 2$, $f''(x) = 32 > 0$. $\therefore$ minimum turning points.

For inflexions, $f''(x) = 0$ and a change in concavity occurs.

$\therefore x = \pm\sqrt{\frac{16}{12}} = \pm\frac{2}{\sqrt{3}}$

x	-2	$-\frac{2}{\sqrt{3}}$	0	$\frac{2}{\sqrt{3}}$	2
$f''(x)$	>0	0	<0	0	>0

Therefore, points of inflexion are at $x = \pm\frac{2}{\sqrt{3}}$.

H6, H9
- Gives the correct solutions 4
- Locates stationary points and determines nature or equivalent progress 3
- Locates stationary points or equivalent progress 2
- Correctly identifies the x-values of a cubic derivative 1

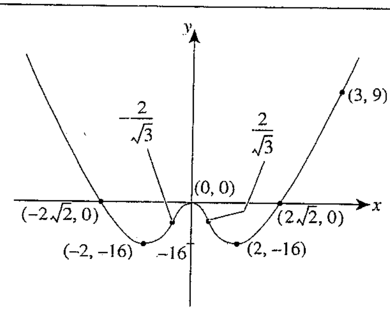
(ii) crosses x-axis at $x^4 - 8x^2 = 0$

$x^2(x^2 - 8) = 0$

$x = 0$ or $\pm 2\sqrt{2}$

P4, H9
- Gives the correct answers 2
- Gives one correct answer, or attempts to solve $x^4 - 8x^2 = 0$ 1

(iii)

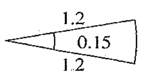


H6, H9
- Gives a correct sketch showing all features (or correct from previous answer) (point (3, 9) not required) 2
- Gives any quartic-shaped sketch (or correct from previous answer) 1

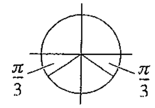
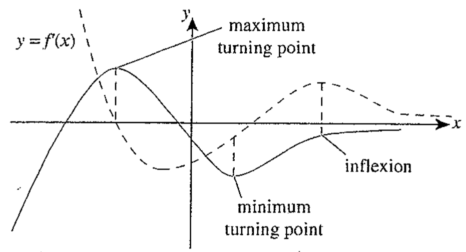
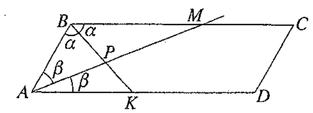
(iv) maximum value in $-2 \le x \le 3$ is 9

P4, H9
- Gives correct answer (or correct from previous answer) 1

Question 5

Sample answer	Syllabus outcomes and marking guide
(a) $\log_b a^m \div \log_m a$ $= m\log_b a \div \dfrac{\log_b a}{\log_b m}$ $= m\log_b a \times \dfrac{\log_b m}{\log_b a}$ $= m\log_b m$	H3, H9 • Gives the correct answer ..... 2 • Uses the change of base law 1
(b) $\alpha + \beta = -\dfrac{b}{a}$ $\quad \alpha\beta = \dfrac{c}{a}$ $3 = -\dfrac{k}{2}$ $\quad -5 = \dfrac{D}{2}$ $k = -6$ $\quad D = -10$	P4 • Gives the correct answers ..... 2 • Gives the correct answer for either D or k. 1
(c) (i) $6 - x = 6x - x^2$ $x^2 - 7x + 6 = 0$ $(x-1)(x-6) = 0$ $\therefore x = 1$ or 6 $y = 6-1, 6-6$ Therefore, the points are (1, 5) and (6, 0).	P4, H9 • Gives the correct answers 1
(ii) $\int_1^6 [6x - x^2 - (6-x)]dx$ $= \int_1^6 (7x - x^2 - 6)dx$ $= \left[\frac{7}{2}x^2 - \frac{1}{3}x^3 - 6x\right]_1^6$ $= 7\times 18 - \frac{1}{3}\times 6\times 36 - 36 - \left(\frac{7}{2} - \frac{1}{3} - 6\right)$ $= 20\frac{5}{6}$ square units	H8 • Gives the correct solution ..... 3 • Makes significant progress ..... 2 • Gives the correct expression for area or equivalent merit 1
(d) (i)  length for 1 blade = 2 × 1.2 + 1.2 × 0.15 = 2.58 m length for 5 blades = 12.9 m	H4 • Gives the correct answer ..... 2 • Gives the correct length for one blade OR • Gives 0.18 × 5 as the length for the five arcs 1
(ii) area = $10 \times \frac{1}{2} \times (120)^2 \times 0.15$ $= 10\ 800$ cm^2 quantity = 10 800 ÷ 400 × 100 mL = 2.7 L	H4 • Gives the correct quantity ..... 2 • Gives the correct area or equivalent merit 1

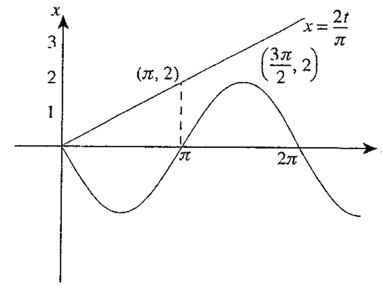
Question 6

Sample answer	Syllabus outcomes and marking guide
(a) $\sin\theta = \dfrac{-\sqrt{3}}{2}$  $\theta = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{4\pi}{3}, \frac{5\pi}{3}$	H5 • Gives the correct solutions ..... 2 • Gives one correct solution 1
(b) 	H7 • Gives a correct sketch which shows all features ..... 3 • Gives a sketch which shows two correct features ..... 2 • Gives a sketch showing one correct feature 1
(c) (i) $A = \frac{h}{3}\{(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\}$ $= \frac{3}{3}\{0 + 0 + 4(1.7 + 2.6) + 2\times 2.1\}$ $= 21.4$ m^2	H8, H9 • Gives the correct answer ..... 2 • Gives a correct evaluation for h or equivalent merit 1
(ii) $V = Ah = 21.4 \times 25 = 535$ m^3	H8 • Gives the correct answer 1
(d) (i)  Let $\angle BAP = \beta$, $\angle ABP = \alpha$ $\therefore \angle MBP = \alpha$ $\angle PAK = \beta$ (given BK and PA bisect $\angle ABM$ and $\angle BAK$ respectively) Now $2\alpha + 2\beta = 180°$ (co-int angles $BM \parallel AK$ $ABCD$ parm) $\therefore \alpha + \beta = 90°$ $\therefore \angle BPA = 90°$ (angles in a triangle add to 180°)	P2, P4 • Gives the correct proof ..... 2 • Makes some progress 1
(ii) In ΔBAP and ΔBPM, $\angle BPA = \angle BPM = 90°$ (proved in (i)) BP is common $\angle ABP = \angle MBP$ (given PB bisects ΔABC) $\therefore \Delta ABP \equiv MPB$ (AAS) $\therefore AB = BM$ (corresponding sides in congruent triangles)	P2, P4 • Gives the correct proof ..... 2 • Makes some progress (e.g. establishes congruency without reasons) 1

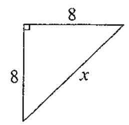
Question 7

Sample answer	Syllabus outcomes and marking guide
(a) Let $k = e^x$ $k^2 - 28k + 27 = 0$ $(k-27)(k-1) = 0$ $\therefore k = 27$ or 1 Hence, $e^x = 27$ or $e^x = 1$ $x = \log_e 27$ or $x = 0$	H3 • Gives the correct solutions . . . . 3 • Reduces equation to quadratic, and correctly factorises and solves for k . . . 2 • Reduces equation to a quadratic or equivalent merit 1
(b) (i) $C = C_0 e^{kt}$ $\frac{dC}{dt} = k \times C_0 e^{kt}$ $= kC$ as required	H3 • Gives the correct proof 1
(ii) $t = 3$ $C = 2.45$ $t = 3.5$ $C = 1.84$ $2.45 = C_0 e^{3k} \Rightarrow C_0 = \frac{2.45}{e^{3k}}$ $1.84 = C_0 e^{3.5k} \Rightarrow C_0 = \frac{1.84}{e^{3.5k}}$ $\frac{2.45}{e^{3k}} = \frac{1.84}{e^{3k} \times e^{0.5k}}$ $e^{0.5k} = \frac{1.84}{2.45}$ $0.5k = \log_e \frac{1.84}{2.45}$ $k = 2\log_e \frac{1.84}{2.45}$ $= -0.5726$ $\therefore C_0 = \frac{2.45}{e^{3 \times -0.2487}}$ = 13.65 mg/L	H3, H4 • Gives the correct solution (ignore rounding) . . . . 3 • Makes significant progress . . . . 2 • Establishes two values for C_0 or equivalent merit 1
(iii) $t = ?$ $C = 0.5$ $0.5 = 13.65 \times e^{-0.5726t}$ $0.03663 = e^{-0.5726t}$ $\therefore t = \log_e 0.09677 \div -0.5726$ $= 5.78$ hours $\therefore$ after 5.78 hours	H3, H4 • Gives the correct answer 1

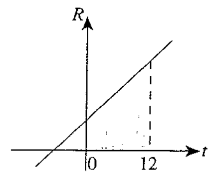
Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) $t = 0, x = 0$ $v_1 = \frac{2}{\pi}$ $v_2 = -2\cos t$ $\therefore x_1 = \frac{2t}{\pi} + C_1$ $x_2 = -2\sin t + C_2$ when $t = 0, x = 0 \Rightarrow C_1 = 0$ when $t = 0, x = 0 \Rightarrow C_2 = 0$ $\therefore x_1 = \frac{2t}{\pi}$ $\therefore x_2 = -2\sin t$	H4, H5 • Gives the correct answers . . . . 2 • Gives one correct answer 1
(ii)  The graphs don't intersect again. $x = \frac{2t}{\pi}$ has a value greater than 2 for $x > \pi$, and the maximum value of $x = -2\sin t$ is 2.	H2, H4, H5 • Gives the correct justification and explanation . . . . 2 • Draws graphs, with no justification or explanation 1

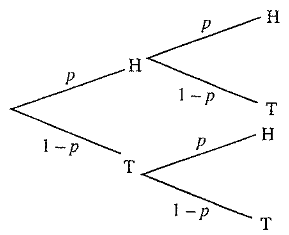
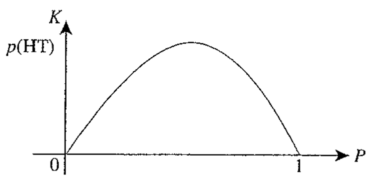
Question 8

	Sample answer	Syllabus outcomes and marking guide
(a) (i)	At $t = 2$, because $v = 0$.	H4, H5 • Gives the correct answer 1
(ii)	Total distance covered equals the area under the v–t graph. Therefore, the distance covered $= \frac{1}{2} \times 5 \times 2 + \frac{1}{2} \times 6 \times 4$ $= 5 + 12$ $= 17$ m	H4, H5, H8 • Gives the correct distance .... 2 • Correctly calculates one area or equivalent merit 1
(iii)	7 metres on the positive side of the starting position.	H4, H5 • Gives the correct answer 1
(b) (i)	 $x^2 = 64 + 64$ $= 64 \times 2$ $x = 8\sqrt{2}$ area of $PQRS = (8\sqrt{2})^2$ $= 128$ cm^2	H4, H5 • Gives the correct proof and the correct area .... 2 • Gives one correct proof 1
(ii)	The areas are 256, 128, ... This is a geometric sequence: $a = 256$, $r = \frac{1}{2}$. $T_{10} = ar^9$ $= 256 \times \left(\frac{1}{2}\right)^9$ $= \frac{1}{2}$ cm^2	H5 • Gives the correct answer .... 2 • Identifies the correct $r = \frac{1}{2}$ 1
(iii)	The perimeters are 64, $32\sqrt{2}$, 32 ... $T_n = \sqrt{2} = 64 \times \left(\frac{1}{\sqrt{2}}\right)^{n-1}$ $(\sqrt{2})^n = 64$ $2^{\frac{1}{2}n} = 2^6$ $n = 12$	H5 • Gives the correct answer .... 2 • Determines a correct equation, or solves an incorrect, non-trivial exponential equation for n..................... 1
(iv)	areas = 256, 128 ..., $r = \frac{1}{2}$ $S_\infty = \frac{a}{1 - \frac{1}{2}}$ $= 2a$ The sum of the areas of all the squares is twice the area of the original square.	H5 • Gives the correct answer .... 2 • Uses limiting sum 1

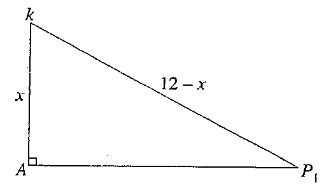
Question 9

	Sample answer	Syllabus outcomes and marking guide
(a) (i)		P4, H4, H9 • Gives the correct answer (12 must be shown)........................ 1
(ii)	$\int_0^{12} (15 + 2t)dt = [15t + t^2]_0^{12}$ $\frac{15 + 39}{2} \times 12$ $= 15 \times 12 + 12^2$ $= 324$ tonnes OR area $= \frac{15 + 39}{2} \times 12$ $= 324$ tonnes	H4, H8, H9 • Gives the correct answer .... 3 • Makes significant progress .... 2 • Makes limited progress 1
(b) (i)	Let a_n = amount in the account at the end of n years immediately before the next addition. $a_1 = 5000(1.09)$ $a_2 = [5000(1.09) + 5000](1.09)$ $= 5000(1.09)^2 + 5000(1.09)$ $= 5000[(1.09)^2 + 1.09]$ $a_3 = [5000\{(1.09)^2 + 1.09\} + 5000](1.09)$ $= 5000[1.09^3 + 1.09^2 + 1.09]$ $a_n = 5000[1.09^n + 1.09^{n-1} \ldots 1.09]$ $\therefore a_{10} = 5000[1.09^{10} + 1.09^9 \ldots 1.09]$	H5, H9 • Gives the correct demonstration .... 2 • Makes some progress.................. 1
(ii)	$A_{10} + 5000$ $= 5000 + 5000 \times \frac{1.09(1.09^{10} - 1)}{1.09 - 1}$ $= \$87\,801.46$ Beth has almost \$88 000 in her account.	H5, H9 • Gives the answer \$87 801 46.......... 2 • Uses the sum of geometric series...... 1
(iii)	In 20 more years, Beth will have $5000 \times \frac{1.09(1.09^{30} - 1)}{1.09 - 1} = \$742\,876.09$ Cathy $742\,876.09 = A \times \frac{1.09(1.09^{20} - 1)}{1.09 - 1}$ $= A \times 55.7645$ $\therefore A = 13\,321.66$ Cathy will need to invest \$13 321.66 each year.	H5, H9 • Gives the answer \$13 321.66.......... 3 • Makes significant progress .... 2 • Makes some progress (e.g. determines \$742 876.09) 1
(iv)	$\$13\,321.66 \times 20 - \$5000 \times 30 = \$116\,433.19$ Cathy will have to invest \$116 433.19 more than Beth.	• Gives the correct answer (accept correct from previous answer)............... 1

Question 10

Sample answer	Syllabus outcomes and marking guide
(a) (i) 	H5 • Gives a totally correct diagram 1
(ii) $p(\text{HT}) = \frac{1}{4}$ $p(\text{HH or HT or TH}) = 1 - p(\text{TT})$ from $p(\text{HT}) = \frac{1}{4}$ $p(1-p) + (1-p)p = \frac{1}{4}$ $8p^2 - 8p + 1 = 0$ $p = \frac{8 \pm \sqrt{64 - 4 \times 8}}{16}$ $= 0.1464466$ or 0.85355 But $p > 0.5$ as coin shows heads more often than normal. $\therefore p(\text{H}) = 0.85355 \quad p(\text{T}) = 0.1464466$ $p(\text{at least 1H}) = 1 - (0.1464466)^2$ $= 0.979$	H5 • Gives the correct solution (ignore decimal places) .... 3 • Makes significant progress .... 2 • Makes some progress (e.g. $p(1-p) + (1-p)p = \frac{1}{4}$) 1
(iii) $p(\text{HT}) = 2p(1-p)$ let $K = 2p(1-p)$ The graph will be a parabola with roots 0, 1.  The maximum turning point is at $p = \frac{1}{2}$. $K = 2 \times \frac{1}{2}\left(1 - \frac{1}{2}\right)$ $= \frac{1}{2}$ $= 50\%$ The maximum probability for HT or TH is 50%. It is impossible for the probability to be greater than 50%.	H2, H5, H9 • Gives the correct proof .... 2 • Makes some progress 1

Question 10 (Continued)

Sample answer	Syllabus outcomes and marking guide
(b) (i) $kA + kP_1 = \text{length of } AP$ $kA + kP_1 = 12$ $kP_1 = 12 - kA$ $= 12 - x$	H2 • Gives the correct explanation. 1
(ii)  $(AP_1)^2 = (12-x)^2 - x^2$ $= 4(36 - 6x)$ $AP_1 = 2\sqrt{36 - 6x}$ area $= \frac{1}{2} \times Ak \times AP_1$ $= \frac{1}{2}x \times 2\sqrt{36 - 6x}$ $= x\sqrt{36 - 6x}$	H2, H4 • Gives the correct demonstration .... 2 • Makes progress (e.g. shows $(AP_1)^2 = 4(36 - 6x)$) 1
(iii) For a maximum $\frac{dA}{dx} = 0$ $\frac{dA}{dx} = \sqrt{36 - 6x} + \frac{1}{2} \times x(36 - 6x)^{-\frac{1}{2}} \times -6 = 0$ $\sqrt{36 - 6x} - \frac{3x}{\sqrt{36 - 6x}} = 0$ $36 - 6x - 3x = 0$ $9x = 36$ $x = 4$ test in the first derivative x: 3, 4, 5 y′: $\frac{9}{\sqrt{18}}$, 0, $\frac{-9}{\sqrt{6}}$ ↗, —, ↘ Therefore it is a maximum. When $x = 4$, which is $\frac{1}{3}$ of AP. The area of the triangle is a maximum.	H4, H5, H9 • Gives the correct proof. .... 3 • Makes significant progress .... 2 • Makes some progress, e.g. equates the correct expression for $\frac{dA}{dx}$ to 0 1