Neap:

HSC Trial Examination 2008

Mathematics

This paper must be kept under strict security and may only be used on or after the morning of Monday 11 August, 2008 as specified in the Neap Examination Timetable

General Instructions

Reading time – 5 minutes

Working time – 3 hours

Write using black or blue pen

Board-approved calculators may be used

A table of standard integrals is provided at the back of this paper

All necessary working should be shown in every question

Total marks – 120

Attempt questions 1–10

All questions are of equal value

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2008 HSC Mathematics Examination.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\ x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note: $\ln x = \log_e x,\quad x>0$

Total marks 120
Attempt Questions 1–10
All questions are of equal value
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Express 1.26 radians in degrees, correct to the nearest minute. **2**

(b) Solve $8 - 2x > 17$. **2**

(c) Simplify $16\sqrt{5} + \sqrt{45}$. **2**

(d) Simplify $\frac{a^3 - 8}{a - 2}$. **2**

(e) Given that $\log_k 5 = 0.627$ and $\log_k 2 = 0.270$, determine the value of

(i) $\log_k 10$ **1**

(ii) $\log_k 25$ **1**

(f) The minimum turning point of the parabola $y = x(x - k)$ is (3, –9). **2**

Determine the value of k.

Marks

Question 2 (12 Marks) Use a SEPARATE writing booklet.

(a) Differentiate the following with respect to x.

(i) $y = 12x - x^3$ **1**

(ii) $y = \frac{\sin x}{x + 1}$ **2**

(iii) $y = x\log_e x$ **2**

(b) Find an expression for each of the following integrals.

(i) $\int \frac{8}{x^2} dx$ **1**

(ii) $\int \sec^2 \pi x \, dx$ **1**

(c) Evaluate $\int_0^1 (e^{2x} - e^{-x}) dx$. **3**

(d) The gradient function of a curve is given by $\frac{dy}{dx} = 6x^2 - 4$. **2**

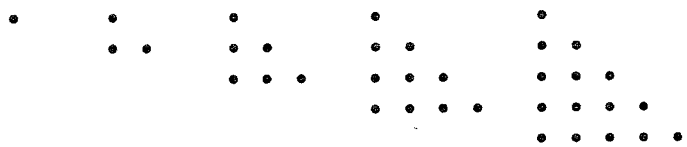
The curve passes through (1, 8). Determine the equation of the curve.

Marks

Question 3 (12 Marks) Use a SEPARATE writing booklet.

(a) **2**

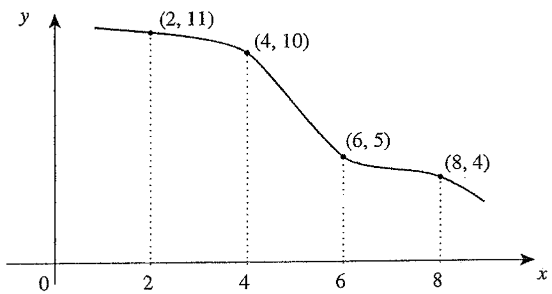
1st	2nd	3rd	4th	5th



	1 + 2	1 + 2 + 3	1 + 2 + 3 + 4	1 + 2 + 3 + 4 + 5
1	=3	=6	=10	=15

The diagram shows the first five triangular numbers. Calculate the value of the 78th triangular number.

(b) **3**



The diagram shows $y = f(x)$.

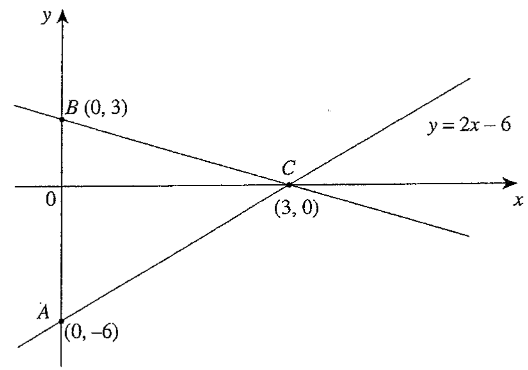
Use the trapezoidal rule with four function values (i.e. three applications) to approximate the area enclosed between the curve $y = f(x)$, the x-axis and the lines at $x = 2$ and $x = 8$.

Question 3 continues on page 5

Marks

Question 3 (continued)

(c)



NOT TO SCALE

(i) Determine the equation of the line containing points B and C. **2**

(ii) Calculate the length of the interval AC. **1**

(iii) Give three inequalities that specify the region inside the triangle. **2**

(iv) Calculate the area of ΔABC. **2**

End of Question 3

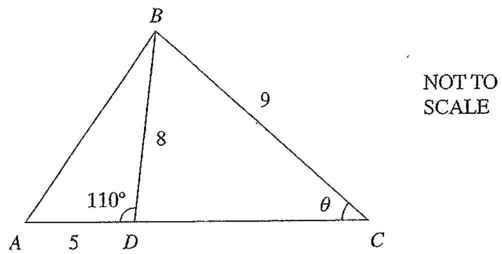
Marks

Question 4 (12 Marks) Use a SEPARATE writing booklet.

(a) (i) Determine the equation of the directrix of the parabola $y = x^2 - 2$. **2**

(ii) Determine the equation of the normal to the parabola $y = x^2 - 2$ at the point on the parabola where $x = 3$. **3**

(b)



In the diagram $AD = 5$ cm, $BC = 9$ cm, $DB = 8$ cm and $\angle ADB = 110°$.

(i) Determine the size of $\angle BDC$. Give a reason for your answer. **1**

(ii) Calculate the size of $\angle DCB$ (θ) correct to the nearest minute. **2**

(iii) Calculate the area of ΔABD in cm^2 correct to one decimal place. **1**

(c) A hospital patient has a very high temperature. The formula $T = \dfrac{120m^2 - m^3}{60\ 000}$ for $0 \le m \le 120$ gives the change in a patient's temperature in °C as a reaction to taking m milligrams of medicine. **3**

Determine the number of milligrams of medicine that will produce the largest change in the patient's temperature.

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

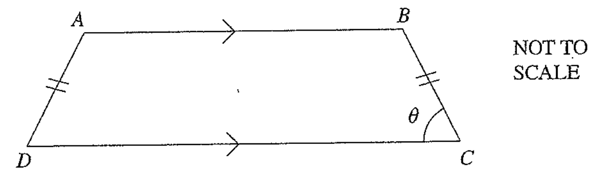
(a) A pile driver is hitting the top of a large pole. The first hit drives the pole 60 cm into the ground. The second hit drives the pole another $60 \times 0.75 = 45$ cm into the ground. The additional distance the pile goes into the ground with each drive is 75% of the previous distance.

(i) How far will the pole be driven into the ground on the 6th drive? **2**

(ii) Determine the total distance the pile will be driven on the 6th drive. **2**

(iii) Calculate the maximum distance the driver can drive the pole into the ground. **1**

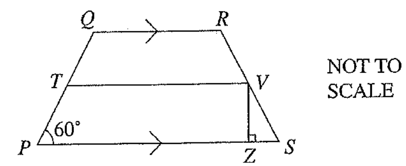
(b) **3**



$ABCD$ is a trapezium in which $AB \parallel DC$ and $AD = BC$. Let $\angle BCD = \theta$.

By constructing a line through A parallel to BC, or otherwise, prove that $\angle ADC = \angle BCD$.

(c)



In the trapezium $PQRS$, $PS \parallel QR$, $PQ = QR = RS$ and $\angle QPS = 60°$. Points T and V are the midpoints of PQ and RS respectively. Z is the foot of the perpendicular from V to PS.

Let $QR = 2x$.

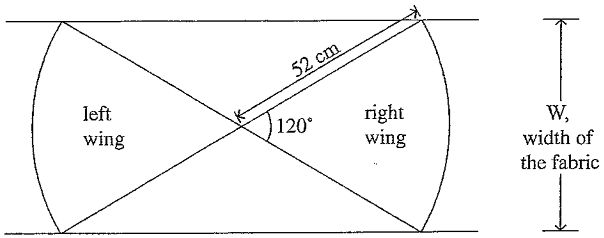
(i) Show that ZS is $\frac{x}{2}$ units long. **2**

(ii) Hence, or otherwise, determine an expression for the length of TV. **2**

Marks

Question 6 (12 Marks) Use a SEPARATE writing booklet.

(a)



NOT TO SCALE

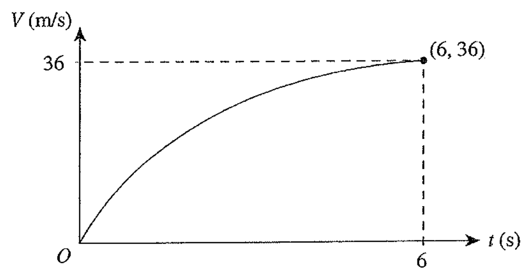
Jenny's mum is making a pair of fairy wings for the ballet concert, i.e. one left and one right wing. Each wing is in the shape of a sector of a circle. The angle at the centre of the sector is 120°.

(i) The radius of each sector is 52 cm. Calculate the width of fabric required to make the wings, W. Give your answer correct to the nearest cm. **2**

(ii) Calculate the area covered by the set of two wings. Give your answer correct to the nearest cm^2. **2**

(iii) The set of wings has a feather trim along each radius and each arc. Calculate the total length of trim required for the set of wings. Give your answer correct to one decimal place. **2**

(b) David is testing the acceleration of a new motorbike. The graph shows the speed David can make the bike travel during the first six seconds of accelerating.



The equation of the graph is $V = -t^2 + 12t$, for $0 \le t \le 6$, where V = speed in metres per second (m/s) and t = time in seconds (s).

(i) At what speed is the bike travelling when $t = 4$ s? **1**

(ii) How long did it take for the bike to reach a speed of 20 m/s? **1**

(iii) Calculate the distance the bike travelled during the six-second acceleration test. **2**

(iv) When was the acceleration greatest? Justify your answer. **2**

Marks

Question 7 (12 Marks) Use a SEPARATE writing booklet.

(a) The exterior angle of a regular polygon is $\frac{\pi^c}{10}$.

(i) What is the size of each interior angle? Give your answer in radians in exact form. **1**

(ii) How many sides does the polygon have? **1**

(b) Sharyn and Brad regularly play tennis against each other. Sharyn wins 75% of the games.

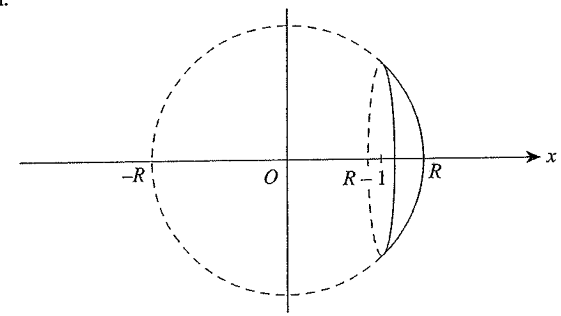
(i) What is the probability that Brad will beat Sharyn in a game? **1**

(ii) Sharyn and Brad are going to play three games of tennis. Draw a tree diagram representing the three games. Label each branch and show the probabilities on each branch. **2**

(iii) What is the probability that Sharyn will win two games and Brad will win one game? **2**

(iv) Determine the probability that Brad will win at least one of the three games. **1**

(c) A section 1 cm deep is cut off the right-hand side of a sphere radius R as shown in the diagram.



(i) Show that the volume of the section which has been cut off can be determined by simplifying $\pi\int_{R-1}^{R}(R^2 - x^2)dx$. **1**

(ii) Show that the volume of the section is given by $\frac{\pi}{3}(3R - 1)$. **3**

Marks

Question 8 (12 Marks) Use a SEPARATE writing booklet.

(a) One root of the quadratic equation $4x^2 - 24x + k = 0$ is twice the other root. **2**

Determine the value k.

(b) Consider the curve $y = ax^3 - 75ax + 2$, where a is a constant, negative value.

(i) Determine the coordinates of the two stationary points and determine which point is the local maximum turning point. **3**

(ii) Sketch the curve. Clearly label the coordinates of the stationary points and the y-intercept (x-intercepts are not required). **1**

(iii) Hence find an expression for the maximum value the function takes in $-1 \leq x \leq 1$. **1**

(c) John has just started work and he is very keen to buy the latest 'Aussie Ute'. The price of the ute is $24 000.

(i) John can borrow the money to buy the ute. To repay the money he will make monthly repayments of $600 for 4 years. **1**

Calculate the total amount John will pay for the ute if he borrows the money to pay for it.

(ii) John's father told him that the ute would lose 20% of its previous year's value each year. **1**

Show that the ute will lose $8640 in value during the first two years.

(iii) John's father recommended that, instead of borrowing money to buy the ute, at the end of each month John should deposit $600 into a special savings account that pays 6% p.a. monthly compounding interest. **2**

John's father said that at the end of two years John could buy a two year old ute.

Let A_n = the value of the account at the end of n months.

Show that if John invests $600 at the end of the month at 6% p.a. monthly compounding interest then the amount in the savings account at the end of n months is given by $A_n = 120\,000(1.005^n - 1)$.

(iv) If John decides to save $600 at the end of each month for two years will he have sufficient funds to buy a two-year-old ute? Use calculations to justify your answer. **1**

HSC Mathematics Trial Examination

Marks

Question 9 (12 Marks) Use a SEPARATE writing booklet.

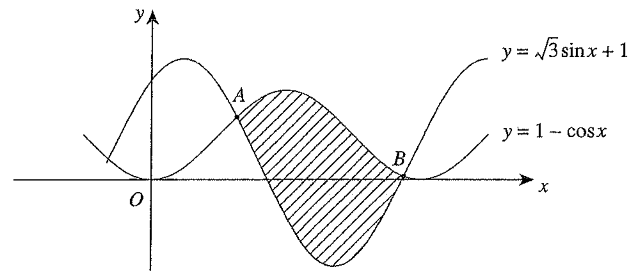
(a) Find all solutions in $0 \leq x \leq 2\pi$ to the equation $2\sin^2 x = 1 + \sin x$. **3**

(b) An archaeologist found a wooden axe. Only 35% of the original carbon-14 remained in the axe. The half-life of carbon-14 is 5730 years, that is, it takes 5730 years for half of the original carbon to decay.

(i) Use the half-life of carbon-14 to determine the value of k in the formula $P = P_0 e^{-kt}$. Give your answer correct to five significant figures. **2**

(ii) How old is the axe? Give your answer correct to the nearest ten years. **2**

(c)



The diagram shows the graphs of $y = \sqrt{3}\sin x + 1$ and $y = 1 - \cos x$.

(i) Solve the equation $\sqrt{3}\sin x + 1 = 1 - \cos x$ to determine the x-coordinates of points A and B. **2**

(ii) Determine the size of the shaded area. **3**

Marks

Question 10 (12 Marks) Use a SEPARATE writing booklet

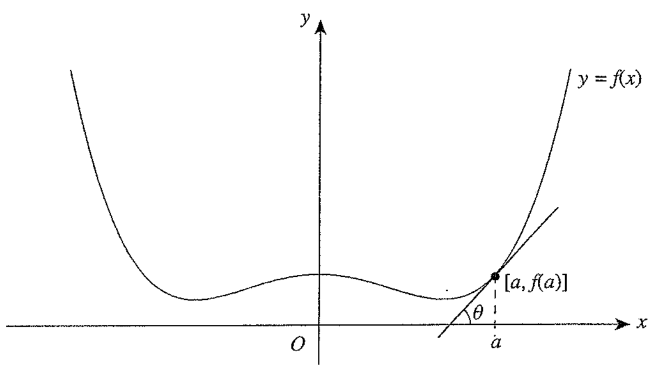
(a) Twenty-five patients who are waiting to see a doctor in a hospital casualty department have either broken bones, a sports injury or both. Of the patients waiting, 19 have a sports injury and 11 have a broken bone. A patient is selected at random. What is the probability that the patient has a sports injury but not a broken bone? **2**

(b) The acceleration of a particle moving in a straight line is given by $\frac{d^2x}{dt^2} = -\frac{1}{(t+1)^2}$. Initially the velocity of the particle is 1 m/s and it is 2 m to the left of the origin. Determine the velocity of the particle as it passes through the origin. **4**

(c) (i) Prove that the function $y = e^{-x^2}$ is an even function. **1**

(ii) Given that $\int_0^1 e^{-x^2}\,dx = 0.747$ to three decimal places and $\int_{0.5}^1 e^{-x^2}\,dx = 0.286$ to three decimal places, determine the value of $\int_{-0.5}^{0.5} e^{-x^2}\,dx$ correct to two decimal places. **2**

(iii) **3**



The diagram shows a representation of $y = f(x)$, an even, differentiable function. The tangent at $[a, f(a)]$ makes an angle of θ with the x-axis as shown on the diagram.

Justify geometrically, or otherwise, that the derivative of an even function is an odd function.

End of paper

Neap:

HSC Trial Examination 2008

Mathematics

Solutions and marking guidelines

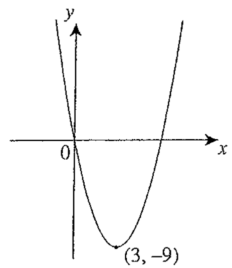
Abbreviations:

CFPA = correct from previous answer
IU = ignore units
IR = ignore rounding
CNE = correct numerical expression, i e. correct substitution into a correct formula
ISE = ignore subsequent errors

Question 1

	Sample answer	Syllabus outcomes and marking guide
(a)	$1.26 \times \frac{180}{\pi} = 72°12'$	H5 • Gives correct answer .... 2 • Obtains $1.26 \times \frac{180}{\pi}$ or 72° or 72°11′ .. 1
(b)	$8 - 2x > 17$ $-2x > 9$ $x < -4\frac{1}{2}$	P3 • Gives correct answer .... 2 • Obtains $-2x > 9$ 1
(c)	$16\sqrt{5} + 3\sqrt{5} = 19\sqrt{5}$	P3 • Gives correct answer .... 2 • Implies $\sqrt{45} = 3\sqrt{5}$ OR • Adds two like surds correctly 1
(d)	$\frac{(a-2)(a^2+2a+4)}{(a-2)} = a^2 + 2a + 4$	P4 • Gives correct answer .... 2 • Factorises the numerator correctly...... 1
(e)	(i) $\log_k 10 = \log_k 5 + \log_k 2$ $= 0.627 + 0.270$ $= 0.897$	H3 • Gives correct answer 1
	(ii) $\log_k 25 = \log_k(5^2)$ $= 2\log_k 5$ $= 2 \times 0.627$ $= 1.254$	H3 • Gives correct answer 1

Question 1 (Continued)

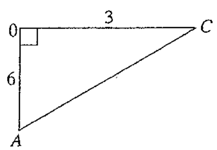
Sample answer	Syllabus outcomes and marking guide
(f) $y = x(x-k)$ One root is zero.	P4, P5, P6, P7, H6, H9 • Gives correct answer 2
 The other root is to the right of (3, –9) By symmetry the other root is 3 units to the right of $x = 3$ $\therefore$ the other root is $x = 6$ $\therefore k = 6$ Alternatively, $y = x^2 - kx$ $y' = 2x - k = 0$, when $x = 3$ $6 - k = 0$ $k = 6$ OR $x = 3$, when $y = -9$ $-9 = 3^2 - k \times 3$ $-18 = -3k$ $k = 3$	• Makes progress towards a solution e g uses an appropriate sketch OR • Determines $y' = 0$, when $x = 3$ OR • Similar merit 1

HSC Mathematics Trial Examination Solutions and marking guidelines

Question 2

Sample answer	Syllabus outcomes and marking guide
(a) (i) $y' = 12 - 3x^2$	P7 • Gives correct answer 1
(ii) $y' = \dfrac{(x+1)\cos x - \sin x}{(x+1)^2}$	P7, H5 • Gives correct answer 2
	• Uses the quotient rule OR • Implies $\dfrac{d}{dx}\sin x = \cos x$ 1
(iii) $y' = \log_e x + x \times \dfrac{1}{x}$ $= \log_e x + 1$	H5 • Gives correct solution 2
	• Uses the product rule OR • Implies $\dfrac{d}{dx}\log_e x = \dfrac{1}{x}$ 1
(b) (i) $\int 8x^{-2}\,dx = -8x^{-1} + C$	H5, H8 • Gives correct answer (ignore + C) 1
(ii) $\dfrac{1}{\pi}\tan \pi x + C$	H8 • Gives correct answer (ignore + C) 1
(c) $\left[\frac{1}{2}e^{2x} + e^{-x}\right]_0^1 = \frac{1}{2}e^2 + e^{-1} - \left(\frac{1}{2}e^0 + e^0\right)$ $= \frac{1}{2}e^2 + e^{-1} - 1\frac{1}{2}$	H3, H8 • Gives correct answer 3
	• Gives correct answer with + C OR • Correct substitution of limits of integration into a correct primitive function 2
	• Gives a correct primitive function OR • Correct substitution of limits of integration into an incorrect, non-trivial primitive function 1
(d) $y = 2x^3 - 4x + C$ $8 = 2 - 4 + C$ $C = 10$ $y = 2x^3 - 4x + 10$	H5 • Gives correct answer 2
	• Gives $y = 2x^3 - 4x + C$ OR • Gives an incorrect, non-trivial primitive function with + C evaluated correctly . . . 1

Question 3

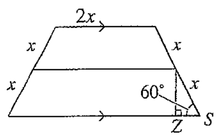
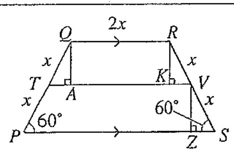
Sample answer	Syllabus outcomes and marking guide
(a) $T_{78} = 1+2+3+4+ .. +78$ $T_{78} = \frac{78}{2}\{2 \times 1 + (78-1) \times 1\}$ $= 3081$	H5 • Gives correct answer .... 2 • Uses the sum of an arithmetic sequence OR • Uses an arithmetic sequence with $a = 1$ and $d = 1$ 1
(b) x: 2, 4, 6, 8 y: 11, 10, 5, 4 $A \approx \frac{h}{2}\{f + l + 2 \times \text{others}\}$ $h = 2$ $A \approx \frac{2}{2}\{11 + 4 + 2 \times (10+5)\}$ $= 45\ u^2$	H5, H8 • Gives correct answer .... 3 • Substitutes correctly into a valid formula or valid method OR • Makes a minor error, e.g. uses $h = 1$ .... 2 • Obtains the correct value for h. 1
(c) (i) $m = \frac{-3}{3}$ $= -1$ The y-intercept = 3, so the equation is $y = -x + 3$.	P3 • Gives correct answer .... 2 • Obtains either the gradient or y-intercept correctly OR Attempts to use the 2-point formula correctly 1
(ii)  $AC^2 = 6^2 + 3^2$ $= 36 + 9$ $= \sqrt{45}$ $= 3\sqrt{5}$	P3, P4 • Gives correct answer in **any** form 1
(iii) $x > 0,\ y < -x + 3$ and $y > 2x - 6$	H9 • Gives three correct inequalities (CFPA), accept appropriate $\geq$ or $\leq$ .... 2 • Gives two correct inequalities (CFPA), accept appropriate $\geq$ or $\leq$ 1
(iv) Area $= \frac{1}{2}bh$ $= \frac{1}{2} \times (6+3) \times 3$ $= 13.5u^2$	P4, P3 • Gives correct answer .... 2 • Uses the correct length for base or height in $A = \frac{1}{2}bh$ 1

HSC Mathematics Trial Examination Solutions and marking guidelines

Question 4

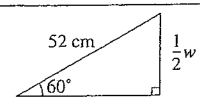
Sample answer	Syllabus outcomes and marking guide
(a) (i) $x^2 = y + 2$ $x^2 = 4a(y+2)$ vertex = (0, –2) and $a = \frac{1}{4}$ directrix: $y = -2\frac{1}{4}$	P4 • Gives correct answer .... 2 • Determines $a = \frac{1}{4}$ OR • Determines vertex = (0, –2) 1
(ii) when $x = 3$, and $y = 7$ $y' = 2x$ $m_T = 2 \times 3$ $= 6$ $m_N = -\frac{1}{6}$ $y - 7 = -\frac{1}{6}(x-3)$ $y = -\frac{1}{6}x + 7.5$	P4, P6, P7, H6 • Gives correct answer in any form ...... 3 • Gives the gradient of the normal as $\frac{1}{6}$ OR • Numerical error in gradient calculation but equation otherwise correct OR • Determines the equation of the tangent .. 2 • Gives $y' = 2x$ or the point (3, 7) 1
(b) (i) $\angle BDC = 70°$ The angles making a straight line add to 180°.	P4 • Gives correct answer with a correct reason 1
(ii) $\frac{\sin\theta}{8} = \frac{\sin 70°}{9}$ $\sin\theta = \frac{8\sin 70°}{9}$ $\theta = 56°39'$	P4 • Gives correct answer (CFPA). Ignore any error in expressing the answer to the nearest minute .... 2 • Correct substitution (CFPA) into the sine rule 1
(iii) $A = \frac{1}{2} \times 5 \times 8 \times \sin 110°$ $\approx 18.8\ \text{cm}^2$	P4 • Gives correct answer (IU, IR) 1
(c) $T = \frac{1}{60\ 000}(120m^2 - m^3)$ $\frac{dT}{dm} = \frac{1}{60\ 000}(240m - 3m^2)$ $\frac{dT}{dm} = 0$ $m = 0$ or 80 Test: $\frac{d^2T}{dm^2} = \frac{1}{60\ 000}(240 - 6m)$ When $m = 80$, $\frac{d^2T}{dm^2} = -\frac{240}{60\ 000}$. $\therefore$ max $\therefore$ An 80 mg dose will give the biggest change.	H6 • Gives correct answer .... 3 • Solution essentially correct but with minor errors or omissions, e.g. omitting the test for maximum .... 2 • Makes relevant progress towards a solution, e.g. equating a correct derivative to zero 1

Question 5

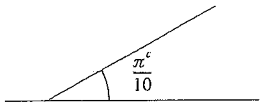
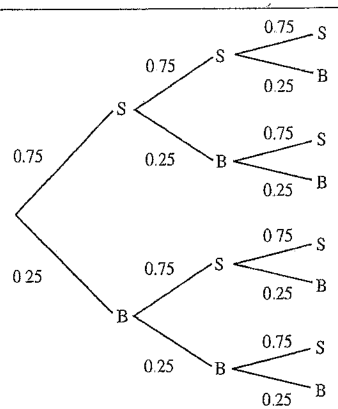
Sample answer	Syllabus outcomes and marking guide
(a) (i) $T_6 = 60 \times (0.75)^5$ $= 14.24$ cm	P4 • Gives correct answer (IU, IR) . . . . 2 • Indicates a geometric sequence, e.g. a = 60, $r = 0.75$. 1
(ii) $S_6 = \dfrac{60(0.75^6 - 1)}{0.75 - 1}$ $= 197.29$ cm	H4, H5 • Gives correct answer (IU, IR) . . . . 2 • Correct substitution into the formula. . . . 1
(iii) $S_\infty = \dfrac{60}{1 - 0.75}$ $= 240$ cm	H4, H5 • Gives correct answer (IU) 1
(b) Construct AE parallel to BC. Point E is on DC. $\because ABCE$ is a parallelogram (two pairs of parallel sides) $\therefore AE = BC$ (opposite sides of a parallelogram are equal) $\therefore AD = AE$ (both are equal to BC) $\therefore \Delta ADE$ is isosceles ($AD = AE$) $\angle ADE = \angle AED$ (base angles of an isosceles triangle are equal) $\angle AED = \angle BCE = \theta$ (corresponding, $AE \parallel BC$) $\therefore \angle ADE = \theta$ $\therefore \angle ADC = \angle BCD$ as required	P2, H2, H5 • Gives a correct proof . . . . 3 • Minor errors in otherwise correct proof OR • Correct proof without reasoning . . . . 2 • Two relevant facts deduced with supporting reasons 1
(c) (i)  $\dfrac{ZS}{x} = \cos 60°$ $ZS = x \times \dfrac{1}{2}$ $= \dfrac{x}{2}$	H6, H9 • Gives correct demonstration . . . . 2 • Makes some progress towards $ZS = \dfrac{x}{2}$, e.g. $\dfrac{ZS}{x} = \cos 60°$ 1
(ii)  $TV \parallel QR$ and $TV \parallel PS$ (ratios of intercepts are equal) $\therefore \Delta RKV \equiv \Delta VZS$ (ASA) $\therefore KV = ZS = \dfrac{x}{2}$ By symmetry $TA = \dfrac{x}{2}$. $AK = QR = 2x$ $TV = \dfrac{x}{2} + 2x + \dfrac{x}{2}$ $= 3x$	P4, H9 • Gives the correct expression . . . . 2 • Makes significant progress towards an expression, e.g. shows $KV = ZS$ 1

HSC Mathematics Trial Examination Solutions and marking guidelines

Question 6

Sample answer	Syllabus outcomes and marking guide
(a) (i)  $\dfrac{\frac{1}{2}w}{52} = \sin 60°$ $w = 52 \sin 60° \times 2$ ≈ 90 cm An alternate approach involves using the cosine rule.	P4 • Gives correct answer (IR) . . . . 2 • Determines either $\dfrac{\left(\frac{1}{2}\right)w}{52} = \sin 60°$ or $w^2 = 52^2 + 52^2 - 2 \times 52^2 \times \cos 60°$. . . . 1
(ii) Using area $= \dfrac{1}{2} r^2 \theta$, area of two wings $= 2 \times \dfrac{1}{2} \times 52^2 \times \dfrac{2\pi}{3}$ ≈ 5663 cm^2	H5 • Gives correct answer (IU, IR) . . . . 2 • Expresses 120° in radians OR • Correctly determines the area of one wing 1
(iii) Length required for one wing $= 52 + 52 + 52 \times \dfrac{2\pi}{3}$ ≈ 212.909 cm Length for both wings $= 2 \times 212.909$ cm $= 425.8$ cm	H5 • Gives correct answer (IR, IU) . . . . 2 • Gives the correct numerical expression for the length required for one wing OR • Correctly determines the length of the arc. 1
(b) (i) Speed $= -16 + 48$ $= 32$ m/s	H5 • Gives correct answer (IU) 1
(ii) $-t^2 + 12t = 20$ $t^2 - 12t + 20 = 0$ $(t - 2)(t - 10) = 0$ $0 \le t \le 6$ $\therefore t = 2$	H4, H5 • Gives correct answer. Ignore any mention of $t = 10$ 1
(iii) $\displaystyle\int_0^6 (-t^2 + 12t)\, dt = \left[-\frac{1}{3}t^3 + 6t^2\right]_0^6$ $= -\dfrac{1}{3} \times 6^3 + 6 \times 6^2 - (0)$ $= 144$ m	H4 • Gives correct answer (IU) . . . . 2 • Determines correct integral or equivalent merit 1
(iv) Acceleration is the gradient of the velocity. By inspection, the gradient, and hence the acceleration, is the greatest at the beginning. Alternatively, $\ddot{x} = -2t + 12$ $= 12 - 2t$ As t is always zero or positive, the maximum value for $\ddot{x}$ is 12 and this occurs when $t = 0$.	H2, H4, H5 • Gives correct answer . . . . 2 • Approaches the question via either the gradient of the tangent to the velocity curve, or by determining the equation of the acceleration 1

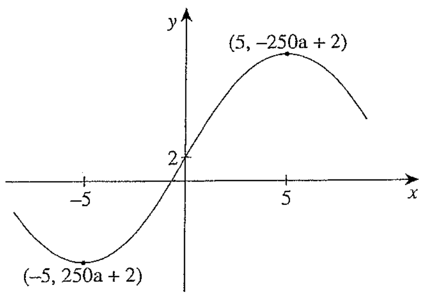
Question 7

Sample answer	Syllabus outcomes and marking guide
(a) (i)  Each exterior angle is $\frac{9\pi}{10}$ radians (angles making a straight line add to π radians).	P4, H5 • Gives the correct answer in terms of π . . 1
(ii) The sum of the exterior angles is 2π Let the number of sides be n. $\frac{\pi}{10}\times n = 2\pi$ $n = 20$ The polygon has 20 sides.	P4, H5 • Gives correct answer 1
(b) (i) 25%, 0.25 or $\frac{1}{4}$	H4, H5 • Gives correct answer 1
(ii) 	H4, H9 • Correct tree diagram showing the correct probabilities (CFPA) . . . . 2 • Draws a tree diagram with minor errors (CFPA). 1
(iii) $3\times 0.75\times 0.75\times 0.25 = 0.421875$ or $3\times\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}=\frac{27}{64}$	H4, H5 • Gives correct answer (CFPA, IR). . . . . . . 2 • Makes significant progress toward the answer, e.g. determines the probability that Sharyn wins the first two games but doesn't multiply by three 1
(iv) $1-(0.75)^3 = 0.578125$ or $1-\left(\frac{3}{4}\right)^3=\frac{37}{64}$	H4, H5 • Gives correct answer (IR) 1

Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) The equation of the circle is $x^2+y^2=R^2$ $y^2=R^2-x^2$ In the formula, $V=\pi\int_b^a y^2\,dx$ $a=R$ and $b=R-1$ Thus $V=\pi\int_{R-1}^{R}(R^2-x^2)\,dx$.	H8 • Gives correct demonstration 1
(ii) $V=\pi\left[R^2x-\frac{1}{3}x^3\right]_{R-1}^{R}$ $=\pi\left[R^3-\frac{1}{3}R^3-\left\{R^2(R-1)-\frac{1}{3}(R-1)^3\right\}\right]$ $=\frac{\pi}{3}[3R-1]$	H8 • Gives correct demonstration . . . . 3 • Correctly substitutes limits of integration into a correct integral, or equivalent . . . 2 • Determines the correct integral 1

Question 8

	Sample answer	Syllabus outcomes and marking guide
(a)	Let the roots be α and 2α. $\alpha + 2\alpha = -\frac{b}{a}$ $3\alpha = \frac{24}{4}$ $= 6$ $\alpha = 2$ The roots are 2 and 4. $\alpha \times 2\alpha = \frac{c}{a}$ $2 \times 4 = \frac{k}{4}$ $k = 32$	P4 • Gives correct answer .... 2 • Makes significant progress, e.g. determines that the roots are 2 and 4. 1
(b) (i)	$y' = 3ax^2 - 75a = 0$ $\therefore x^2 = \frac{75a}{3a}$ $= 25$ $x = \pm 5$ Stationary points are $(5, -250a + 2)$ and $(-5, 250a + 2)$. Testing $y'' = 6ax$ at $x = 5$, $y'' = 30a < 0$ as $a < 0$ $\therefore (5, -250a + 2)$ is the local maximum turning point.	P4 • Gives correct answer .... 3 • Determines the coordinates of the stationary points or equivalent merit .... 2 • Determines that the solution to $x^2 = 25$ is required 1
(ii)		P4 • Gives correct answer (CFPA). 1
(iii)	In $-1 \leq x \leq 1$, the maximum value will occur at $x = 1$. $\therefore$ the maximum value is $2 - 74a$.	P4 • Gives correct answer (CFPA). 1

HSC Mathematics Trial Examination Solutions and marking guidelines

Question 8 (Continued)

	Sample answer	Syllabus outcomes and marking guide
(c) (i)	\$600 for four years of monthly repayments. total = \$600 × 48 = \$28 800	P3 • Gives correct answer 1
(ii)	Value in two years $= 24\,000 \times (1 - 0.2)^2$ = \$15 360 Depreciation = \$24 000 − \$15 360 = \$8640	P3 • Gives correct answer 1
(iii)	Let A_n = value at the end of n months. $A_1 = 600$ $A_2 = 600 \times (1.005) + 600$ $A_3 = 600 \times (1.005)^2 + 600 \times (1.005) + 600$ $A_n = 600(1.005)^{n-1} + 600(1.005)^{n-2} \ldots + 600$ $= 600[1.005^{n-1} + 1.005^{n-2} + \ldots + 1]$ $= 600 \times 1 \times \frac{1.005^n - 1}{1.005 - 1}$ $= 120\,000(1.005^n - 1)$	H4, H5 • Gives correct demonstration .... 2 • Gives a demonstration containing A_1, A_2 and A_3. 1
(iv)	$A_{24} = 120\,000(1.005^{24} - 1)$ = \$15 259.17 John will save \$15 259.17. The ute will cost approximately \$15 360. John will be approximately \$100 short.	H2, H5 • Gives correct answer 1

Question 9

Sample answer	Syllabus outcomes and marking guide
(a) Let $u = \sin x$ $2u^2 = 1 + u$ $2u^2 - u - 1 = 0$ $(2u+1)(u-1) = 0$ $u = -\frac{1}{2}$ or 1 $\sin x = -\frac{1}{2}$ or 1 $x = \pi + \frac{\pi}{6}$ or $2\pi - \frac{\pi}{6}$ or $\frac{\pi}{2}$ $= \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ or $\frac{\pi}{2}$	P4, H5 • Gives correct answer .... 3 • Makes significant progress .... 2 • Obtains $\sin x = -\frac{1}{2}$ and $\sin x = 1$ 1
(b) (i) Use the formula $P = P_0e^{-kt}$. When $t = 5730$, $P = \frac{1}{2}P_0$. $\frac{1}{2} = e^{-5730k}$ $\log_e\left(\frac{1}{2}\right) = -5730k$ $k = \frac{-\log_e\left(\frac{1}{2}\right)}{5730}$ $= 1.2097 \times 10^{-4}$	H3, H4 • Gives correct answer. Accept answer in exact or decimal form (IR). .... 2 • Obtains the equation $\frac{1}{2} = e^{-5730k}$ 1
(ii) $0.35 = e^{-kt}$ $\log_e 0.35 = -kt$ $t = \frac{\log_e 0.35}{-k}$ $= \frac{\log_e 0.35}{\left(\frac{\log_e \frac{1}{2}}{5730}\right)}$ ≈ 8678.5 years The axe is 8680 years old, correct to the nearest ten years.	H3, H4 • Gives correct answer (IR) .... 2 • Attempts to solve the equation $0.35 = e^{-kt}$ 1

HSC Mathematics Trial Examination Solutions and marking guidelines

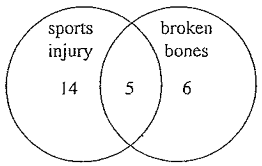
Question 9 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) $\sqrt{3}\sin x + 1 = 1 - \cos x$ $\sqrt{3}\sin x = -\cos x$ Divide both sides by $\sqrt{3}\cos x$ $\tan x = -\frac{1}{\sqrt{3}}$ $x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ $= \frac{5\pi}{6}, \frac{11\pi}{6}$	H5 • Gives correct answers .... 2 • Determines $\tan x = -\frac{1}{\sqrt{3}}$ 1
(ii) $A = \int_{\frac{5\pi}{6}}^{\frac{11\pi}{6}} [(1 - \cos x) - (\sqrt{3}\sin x + 1)]\, dx$ $= -\int_{\frac{5\pi}{6}}^{\frac{11\pi}{6}} (\cos x + \sqrt{3}\sin x)\, dx$ $= -[\sin x - \sqrt{3}\cos x]_{\frac{5\pi}{6}}^{\frac{11\pi}{6}}$ $= -\left[-\frac{1}{2} - \sqrt{3} \times \frac{\sqrt{3}}{2} - \left(\frac{1}{2} - \sqrt{3} \times -\frac{\sqrt{3}}{2}\right)\right]$ $= 4$	H8 • Gives correct answer (CFPA, IU) ...... 3 • Integrates correctly (CFPA) .... 2 • Determines the correct integral required to solve the problem, i.e. the first line of the solution (CFPA) 1

Question 10

Sample answer | **Syllabus outcomes and marking guide**

(a)



P(sports injury but not broken bones) $= \frac{14}{25}$

H4, H5
- Gives correct answer 2
- Determines that five patients have both broken bones and a sports injury 1

(b) $\frac{d^2x}{dt^2} = -(t+1)^{-2}$

When $t = 0$, $v = 1$ and $x = -2$

$\frac{dx}{dt} = (t+1)^{-1} + C_1$

$1 = 1 + C_1$

$C_1 = 0$

$\frac{dx}{dt} = \frac{1}{t+1}$

$x = \log_e(t+1) + C_2$

$-2 = \log_e 1 + C_2$

$C_2 = -2$

$x = \log_e(t+1) - 2$

The question requires the value of $\dot{x}$ when $x = 0$.

$\log_e(t+1) = 2$

$t + 1 = e^2$

$t = e^2 - 1$ and $\dot{x} = \frac{1}{t+1}$

$\dot{x} = \frac{1}{e^2 - 1 + 1}$

$= \frac{1}{e^2}$

H3, H4
- Gives correct answer 4
- Determines the correct expression for the position of the particle at time t and attempts to determine the time when the particle is at the origin, or equivalent 3
- Determines the correct expression for the position of the particle at time t........ 2
- Demonstrates that integration, with constants of integration evaluated, is required 1

Question 10 (Continued)

Sample answer | **Syllabus outcomes and marking guide**

(c) (i) $f(a) = e^{-a^2}$

$f(-a) = e^{-(-a)^2}$

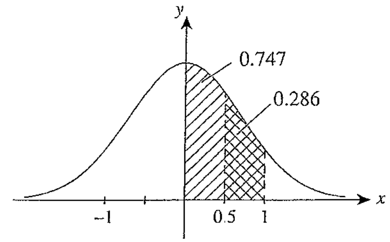
$= e^{-a^2}$

$f(a) = f(-a)$

$\therefore f(x)$ is an even function.

P2, H3
- Gives a correct proof 1

(ii)



$\int_0^{0.5} e^{-x^2}\,dx = 0.747 - 0.286$

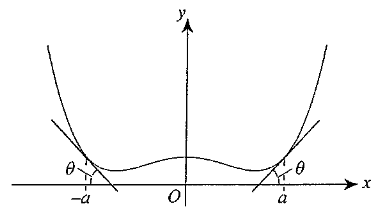
$= 0.461$

$\int_{-0.5}^{0.5} e^{-x^2}\,dx = 0.461 \times 2$

≈ 0.92

H3, H8
- Gives correct answer (IR) 2
- Makes progress towards the answer, e.g. determines the value of the integral between $x = 0$ and $x = 0.5$, or equivalent merit 1

(iii)



$f'(a) = \tan\theta$

Even functions are symmetrical about the y-axis. By symmetry,

$f'(-a) = \tan(\pi - \theta)$

$= -\tan\theta$

$f'(a) = -f'(-a)$

The derivative is odd.

Alternatively,

$f(x) = f(-x)$

Differentiating both sides,

$f'(x) = f'(-x) \times \frac{d}{dx}(-x)$

$= f'(-x) \times -1$

$= -f'(-x)$

The derivative is odd.

P2, H6
- Gives a correct proof 3
- Makes significant progress toward a proof........................ 2
- Makes a valid start to a proof......... 1