National Educational Advancement Programs

HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

1998

MATHEMATICS

2/3 UNIT (COMMON)

Time Allowed: Three hours
(Plus 5 minutes' reading time)

This paper must be kept under strict security and may only be used on or after the morning of Wednesday 12 August, 1998, as specified in the NEAP Examination Timetable.

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 12.
- Approved calculators may be used.
- Each question is to be returned in a separate Writing Booklet clearly labelled, showing your Student Name or Number.
- You may ask for extra Writing Booklets if you need them.

Students are reminded that this is a trial examination only and cannot in any way guarantee the content or the format of the 1998 Mathematics 2/3 Unit (Common) Higher School Certificate Examination.

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QUESTION 1. Use a *separate* Writing Booklet.

Marks

(a) Find the value of $\frac{1}{(1.05)^{10}-1}$ correct to 3 decimal places. **2**

(b) Simplify $\frac{x^2+2x}{x+2}$. **2**

(c) Using the table of standard integrals, find $\int \sec^2\left(\frac{x}{2}\right)dx$. **1**

(d) Two boxes each contain 50 coloured lollies. One box contains seven blue lollies and the other contains eleven blue lollies. If one lolly is chosen at random from each box, what is the probability they are both blue? **2**

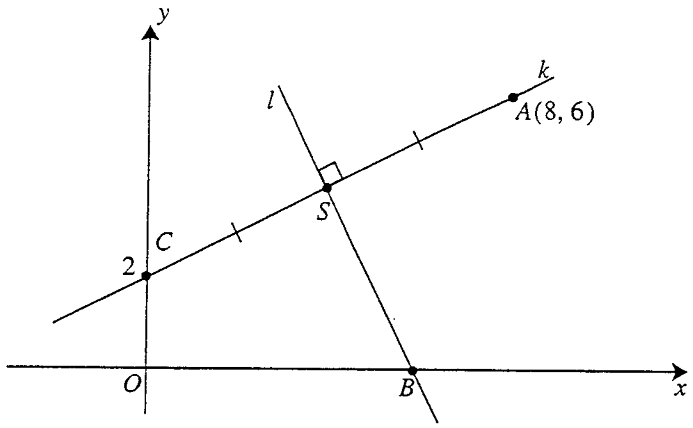
(e) Solve the equation $\frac{x-3}{2}-\frac{4-2x}{3}=1$. **3**

(f) In a triangle ABC the length of side AB is 12 cm and the length of side BC is 8 cm. Within what range of values does the length of side AC lie? **2**

QUESTION 2. Use a *separate* Writing Booklet.

Marks

(a) **9**



In the diagram the line k has equation $y = \frac{1}{2}x + 2$, the line l is perpendicular to k, and the lines k and l intersect at S. The line k cuts the y-axis at $C(0, 2)$ and the line l cuts the x-axis at B. Point A lies on k and has coordinates $(8, 6)$. S is the midpoint of AC.

(i) Show the coordinates of S are $(4, 4)$.

(ii) Find the equation of l.

(iii) Find the coordinates of B.

(iv) If S is the midpoint of BD, determine the coordinates of D.

(v) Explain, with reference to the above information and results, why $ABCD$ is a rhombus.

(vi) Explain which additional calculations you would need to perform to prove that $ABCD$ is also a square. Do not perform these calculations.

(b) A particle moves in a straight line so that its displacement x in metres, at time t in seconds, is given by **3**

$$x = t^3 - 3t^2$$

(i) At what times is the particle at rest?

(ii) How far does the particle travel between these times?

QUESTION 3. Use a *separate* Writing Booklet.

Marks

(a) Differentiate: **5**

(i) $2\sqrt{x}$.

(ii) $e^x \ln x$.

(iii) $\dfrac{x}{\sin x}$.

(b) Consider the function $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. **7**

The graph of $y = f(x)$ has a minimum turning point, P, at $x = 0$, a point of inflexion, Q, at $(1, 1)$, and a maximum turning point, R, at $(2, 3)$.

(i) Find $f'(x)$ and show that $c = 0$.

(ii) Find $f''(x)$ and hence show that $b = -3a$.

(iii) By considering the coordinates of Q and R, show that $d - 2a = 1$ and $d - 4a = 3$.

(iv) Hence find the values of the constants a and d.

TENM2KIM

QUESTION 4. Use a *separate* Writing Booklet.

Marks

(a) Solve the inequality $|3-x| \geq 4$. **2**

(b) By using the substitution $U = x^2$, or otherwise, show that the only real solutions of the equation $x^4 + x^2 - 20 = 0$ are $x = -2$ and $x = 2$. **2**

(c) Consider the function $y = \sqrt{20-x^2}$. **3**

(i) Copy and complete the table below in your Writing Booklet, giving the values of y correct to two decimal places.

x	0	1	2
y			

(ii) Use Simpson's rule with three function values to estimate

$$\int_0^2 \sqrt{20-x^2}\,dx.$$

(d) (i) Sketch the graphs of the semi-circle $y = \sqrt{20-x^2}$ and the parabola $y = x^2$ on the same set of axes. **5**

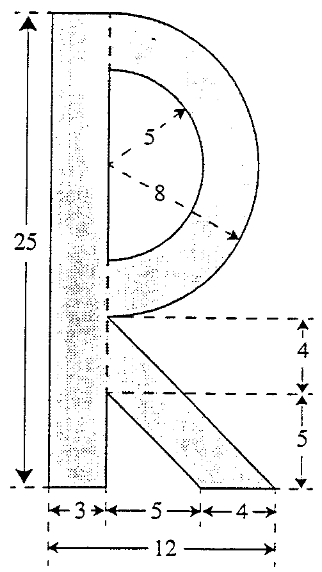
(ii) By considering the results of part (b), or otherwise, write on your diagram the x-coordinates of the points of intersection of the two curves.

(iii) Find the approximate area of the region enclosed by the curves $y = \sqrt{20-x^2}$ and $y = x^2$. The result of part (c) (ii) may be used.

QUESTION 5. Use a *separate* Writing Booklet.

Marks

(a) The letter **R** shown in the diagram has been formed by joining, without overlapping, a rectangle, an isosceles trapezium, and a figure formed with two concentric semi-circles. **6**



The rectangle is 25 cm high and 3 cm wide. The isosceles trapezium has its non-parallel ends each 4 cm long. The radius of the inner semi-circle is 5 cm, and the outer is 8 cm.

(i) Show that the lengths of the parallel sides of the trapezium are $5\sqrt{2}$ and $9\sqrt{2}$ cm.

(ii) Calculate the lengths of each semi-circle. Leave the answers in terms of π.

(iii) Find the exact total perimeter of the letter **R**.

(iv) Find the total area of the letter **R**.

(b) (i) Sketch using the same set of axes, on a number plane in the first quadrant, the graphs of $y = x$ and $y = x^2$. **6**

(ii) Find the points of intersection of these two graphs.

(iii) Find the volume formed when the region between these two curves is rotated about the y-axis.

QUESTION 6. Use a *separate* Writing Booklet.

Marks

(a) A sum of \$30 000 was invested in a bank account at a fixed rate of interest, compounded annually. At the end of seven years, after the seventh interest payment has been made, the sum of money has grown to \$50 000. What was the rate of interest, to the nearest one tenth of a percent? **3**

(b) The volume of water, V cubic metres, in a tank is increasing at time t minutes according to the relation **3**

$$\frac{dV}{dt} = 1 + \frac{2}{t+1}.$$

(i) Find an expression for the volume of water in the tank at time t if initially it was empty.

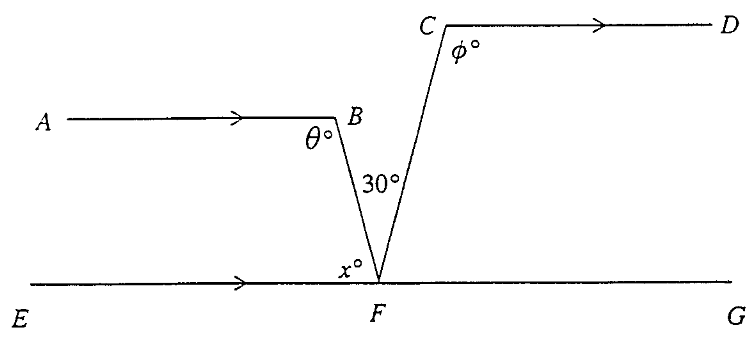
(ii) Calculate the volume of water in the tank at $t = 5$. Give your answer correct to one decimal place.

(c) A pair of dice is rolled, and the numbers showing on the uppermost faces are noted. **3**

(i) In how many ways can a difference of two be obtained from the numbers showing on the dice?

(ii) Find the probability that when a pair of dice is rolled, a difference of two is obtained.

(d) **3**



NOT TO SCALE

In the diagram, line segments AB, CD and EG are parallel, and F lies on EG. $\angle ABF = \theta°$, $\angle BFE = x°$, $\angle BFC = 30°$ and $\angle FCD = \phi°$.

(i) Explain why $x = 180 - \theta$.

(ii) Find the value of $\theta + \phi$.

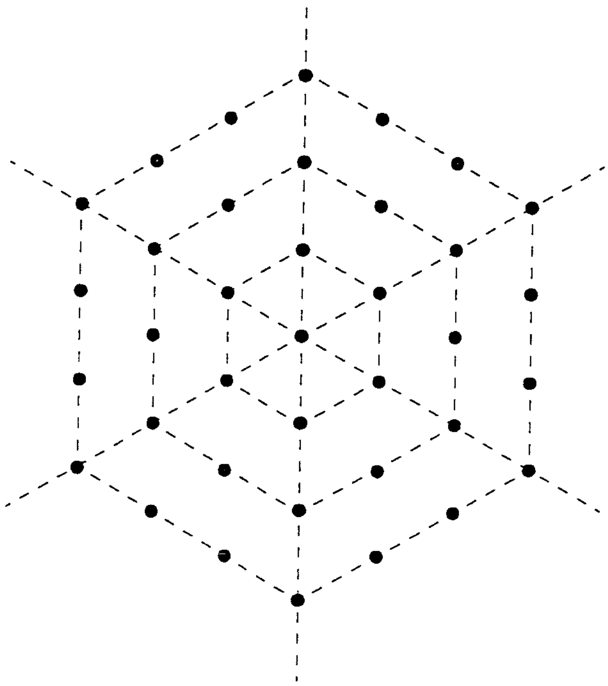
 TTSM2&FM

QUESTION 7. Use a *separate* Writing Booklet.

Marks

(a) Find all values of θ, where $0 \leq \theta \leq 2\pi$, for which $\tan\theta = \sqrt{3}$. Give answers in terms of π. **2**

(b) The board used for a game consists of a centre peg, and then pegs arranged to lie on the vertices and sides of hexagons. The diagram shows the centre peg, and the pegs on the first three hexagons. **6**



The numbers of pegs lying on successive hexagons form the terms of an arithmetic series.

(i) Write down the common difference of this arithmetic series.

(ii) If the outermost hexagon contains 114 pegs, how many hexagons are there?

(iii) Find the total number of pegs on the board.

(c) Assume that once aspirin has been absorbed into the bloodstream, its concentration, C milligrams per litre, decreases exponentially according to the formula $C = Ae^{-kt}$, where t is time elapsed, in hours, and A and k are positive constants. **4**

(i) Evaluate A if at $t = 0$, $C = 120$.

(ii) Find k if after four hours the value of C has fallen to half its original value.

(iii) What is the value of C after six hours?

TENM2R.FM

QUESTION 9. Use a *separate* Writing Booklet.

Marks

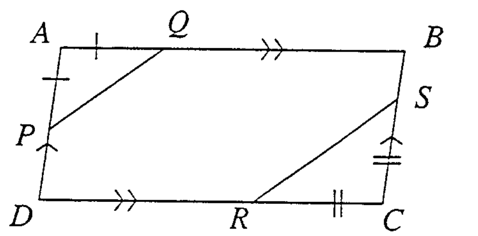
(a) (i) By "completing the square" or otherwise, express **2**

$$x^2 + y^2 + 6x - 10y - 2 = 0$$

in the form $(x-a)^2 + (y-b)^2 = c^2$, where a, b and c are integers.

(ii) Describe fully the locus of points whose coordinates satisfy the equation in part (a) (i).

(b) **5**



In the diagram $ABCD$ is a parallelogram. Points P and Q are on sides AD and AB respectively such that $AP = AQ$. Similarly points R and S are on side CD and CB, such that $CR = CS$.

Copy or trace the diagram into your Writing Booklet.

(i) Prove that $\angle AQP = \angle CRS$.

(ii) On the diagram in your Writing Booklet, produce QP and CD to meet at point T.

Prove that PQ is parallel to RS.

(c) \$2 000 is invested into a credit union account at the beginning of each year and interest is paid at the end of each year, at a rate of 6.5% per annum, on the whole amount in the account at that time. **5**

(i) What is the value of the investment at the end of the three years?

(ii) At the end of how many years, before the next \$2 000 is invested, would the accumulated amount in the account first exceed \$100 000?

QUESTION 10. Use a *separate* Writing Booklet.

Marks

(a) (i) State the *amplitude* and the *period* of the function $y = -\frac{1}{2}\cos 2x$. **5**

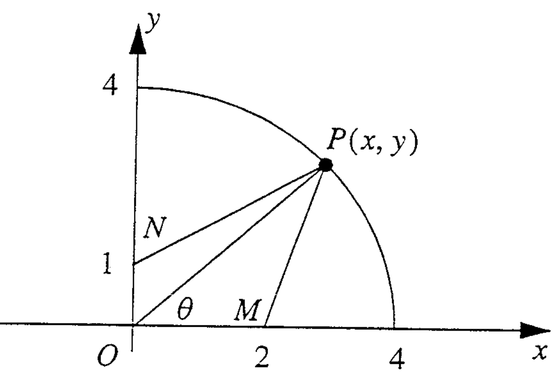
(ii) Sketch the graph of $y = -\frac{1}{2}\cos 2x$, for $0 \le x \le 2\pi$.

(iii) On the same axes, sketch the graph of

$$y = \frac{1}{2} - \frac{1}{2}\cos 2x, \text{ for } 0 \le x \le 2\pi.$$

Clearly distinguish between the two graphs.

(b) **7**



The diagram shows the part of the circle $x^2 + y^2 = 16$ that lies in the first quadrant. The point $P(x,y)$ is on the circle, O is the origin, M is on the x-axis at $x = 2$ and N is on the y-axis at $y = 1$. The size of angle MOP is θ radians.

(i) Show that the area, A, of the quadrilateral $OMPN$ is given by

$$A = 4\sin\theta + 2\cos\theta.$$

(ii) Find the value of $\tan\theta$ for which A is a maximum.

(iii) Hence determine in surd form the coordinates of P for which A is a maximum.

The following list of standard integrals may be used:

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq 1; \; x \neq 0, \; \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right).$$

Note: $\ln x = \log_e x, \quad x > 0$

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(1)(a) 1.590 (to 3 d.p)

(b) x

(c) $2\tan\left(\frac{x}{2}\right)+c$

(d) $\frac{77}{2500}$

(e) $x=\frac{23}{7}$

(f) $4<AC<20$

(2)(a)(i) $(4,4)$ (ii) $2x+y-12=0$

(iii) $B(6,0)$ (iv) $D(2,8)$

(v) Diagonals bisect each other at right angles.

(vi) Either show that adjacent sides are equal or at right angles.

(b)(i) $t=0$ or 2 sec (ii) 4 m

(3)(a)(i) $\frac{1}{\sqrt{x}}$ (ii) $e^x\left(\frac{1}{x}+\ln x\right)$

(iii) $\frac{\sin x - x\cos x}{\sin^2 x}$

(b)(i),(ii),(iii) Proofs

(iv) $a=-1, d=-1$

(4)(a) $x\le -1$ or $x\ge 7$

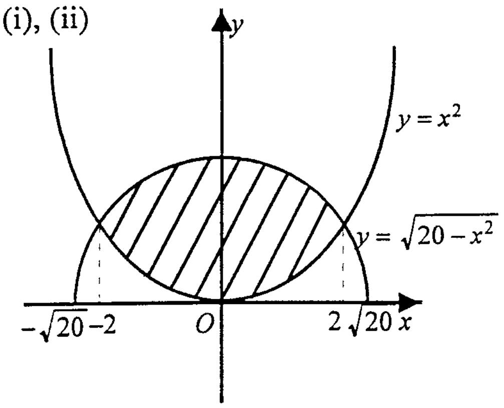
(b) Proof

(c)(i)

x	0	1	2
y	4.47	4.36	4.00

(ii) 8.64 (to 2 d.p)

(d) (i), (ii)



(iii) Area ≈ 11.95 units^2

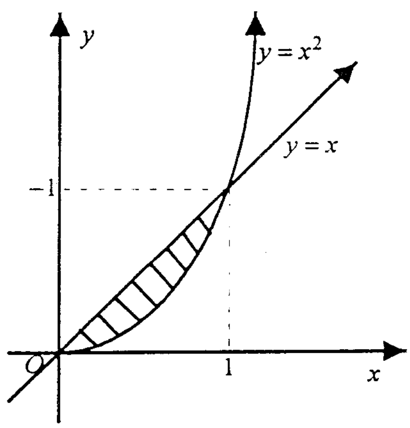
(5)(a)(i) Proof

(ii) 5π and 8π units

(iii) Perimeter $=50+13\pi+14\sqrt{2}$ units

(iv) Area $=103+\frac{39\pi}{2}$ sq. units

(b)(i)



(ii) $(0,0)$ and $(1,1)$

(iii) $\frac{\pi}{6}$ units^3

(6)(a) $r=7.6$

(b)(i) $V=t+2\ln(t+1)$

(ii) 8.6 m^3

(c)(i) 8 ways (ii) $\frac{2}{9}$

(d)(i) $x = 180 - \theta$ (ii) $\theta + \phi = 210$

(7)(a) $\frac{\pi}{3}$ or $\frac{4\pi}{3}$

(b)(i) $d = 6$ (ii) $n = 19$ (iii) 1141

(c)(i) $A = 120$ (ii) $k = 0.1733$ (to 4 d.p)

(iii) $c = 42.4$ mg/L

(8)(a)(i) $\frac{49}{512}$ (ii) $\frac{1}{8}$

(b)(i) At B, E and H, $f'(x) = 0$

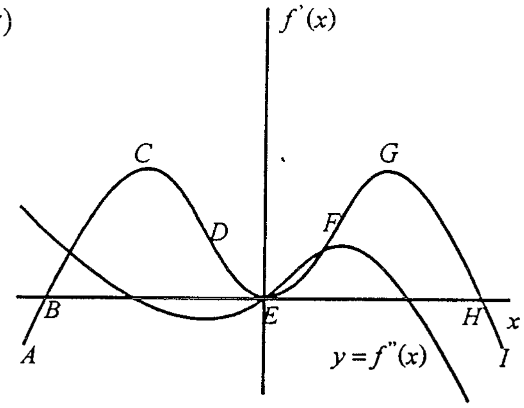
(ii) At B rel. min; at E pt. of inflexio

at H rel.max

(iii) Decreasing from between A and B and also between H and I.

(iv) C,E,G are points of inflexion.

(v)



(vi) Concave up between A and C, and also between E and G.

(9)(a)(i) $(x+3)^2 + (y-5)^2 = 36$

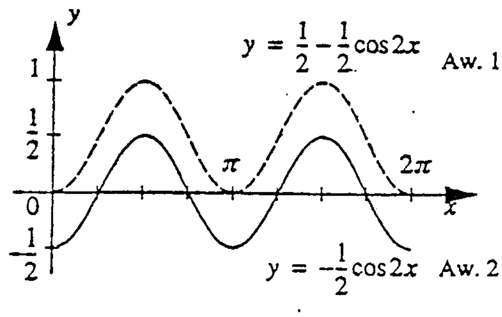
(ii) The locus is a circle with centre $(-3, 5)$ and radius 6 units.

(b) (i), (ii) Proofs

(c)(i) \$6814.35 (ii) 23 years

(10)(a)(i) Amplitude = $\frac{1}{2}$; Period = π units

(ii),(iii)



(b)(i) Proof (ii) $\tan\theta = 2$

(iii) $P\left(\frac{4\sqrt{5}}{5}, \frac{8\sqrt{5}}{5}\right)$