STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(x+\sqrt{x^2-a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln(x+\sqrt{x^2+a^2})$$

Note: $\ln x = \log_e x, \quad x > 0$

Neap. advancing education

HSC Trial Examination 2005

Mathematics

This paper must be kept under strict security and may only be used on or after the morning of Monday 8 August, 2005 as specified in the NEAP Examination Timetable.

General Instructions

Reading time 5 minutes

Working time 3 hours

Board-approved calculators may be used

Write using blue or black pen

A table of standard integrals is provided at the back of this paper

All necessary working should be shown in every question

Total marks – 120

Attempt questions 1–10

All questions are of equal value

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2005 HSC Mathematics Examination.

Total marks 120
Attempt Questions 1–10
All questions are of equal value
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) On average, Abdullah's heart beats approximately 74 times per minute. How many times will Abdullah's heart beat in 65 years (assuming 365 days in a year). Give your answer in scientific notation correct to three significant figures. **2**

(b) Solve for x: $|1-2x|=5$. **2**

(c) The radius, r, of a conical flask of height h and volume V is given by **2**

$$r=\sqrt{\frac{3V}{\pi h}}.$$

A manufacturer is required to produce a conical flask with a volume of 1000 m^3. Find the radius of this flask if the height and radius are to be of equal length. Answer correct to two decimal places.

(d) Rationalise the denominator and express in simplest terms **2**

$$\frac{\sqrt{2}-\sqrt{6}}{\sqrt{6}+\sqrt{2}}.$$

(e) A hat has 10 marbles in it. Four of the 10 marbles are red. Two marbles are chosen at random, the first **not** being replaced before the second is chosen. Find the probability that both of the marbles are red. **2**

(f) Simplify $\dfrac{125}{(5^n)^6\times 125^{1-2n}}$. **2**

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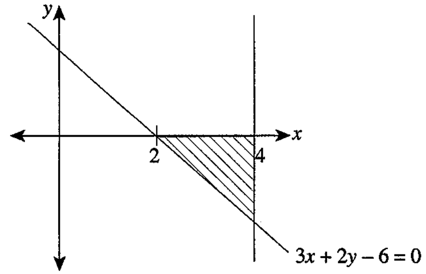
HSC Mathematics Trial Examination

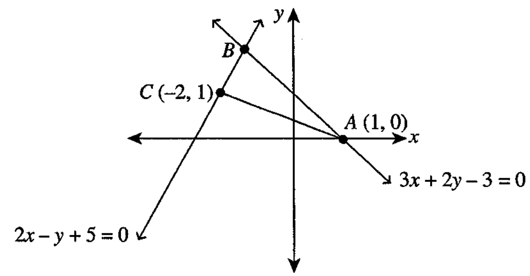
Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Write down three inequalities to describe the shaded region shown below. **2**



(b) In the diagram below, the lines $2x-y+5=0$ and $3x+2y-3=0$ intersect at the point B. The point A has coordinates (1, 0) and the point C has coordinates (–2, 1).



(i) Show that the line AC has the equation, $x+3y-1=0$. **2**

(ii) Show that point B has the coordinates (–1, 3). **2**

(iii) Find the perpendicular distance from point B to AC. Leave your answer as a surd. **2**

(iv) Find the length of AC. Leave your answer as a surd. **2**

(v) Hence find the area of ΔABC. **2**

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Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

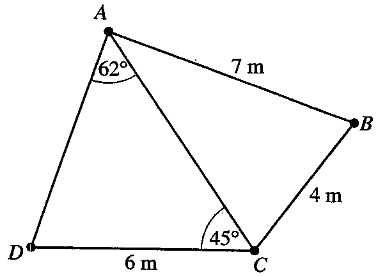
(a) Differentiate with respect to x:

(i) $3x^3 - \sqrt[3]{x} + \frac{2}{x}$ **1**

(ii) $(3 - x^2)^5$ **2**

(iii) $2x^2 \ln x$. **2**

(b) Find the size of the angle ABC in the following diagram correct to the nearest degree. **3**



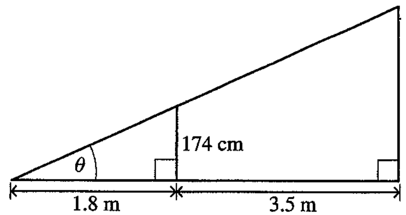
(c) Evaluate the following:

(i) $\int_0^2 e^{\frac{x}{2}-1}\,dx$. Leave your answer as an exact value. **2**

(ii) $\int_0^{\frac{\pi}{6}} (2\sin x - \sec^2 2x)\,dx$. Leave your answer as an exact value. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Graeme is 174 cm tall. He stands 3.5 m from a light pole which has a light on top of the pole. When the light is on, Graeme has a shadow 1.8 m long.



(i) How high is the top of the light pole above the ground? **1**

(ii) Calculate the angle, θ, correct to the nearest degree. **1**

(iii) If Graeme moves 50 cm closer to the pole, how much shorter will his shadow be? **2**

(b) If $f(x) = 2^{x-4}$, find:

(i) $f(1)$ **1**

(ii) x if $f(x) = 1$. **2**

(c) Rebecca borrows \$25 000 at 1% per month reducible interest. She pays the loan off over 5 years by paying equal monthly repayments of \$$R$. Let \$$A_n$ be the amount of money Rebecca still owes after the nth repayment is made.

(i) Write an expression for A_1. **1**

(ii) Show that $A_n = 25\ 000 \times 1.01^n - R(1 + 1.01 + 1.01^2 + \ldots + 1.01^{n-1})$. **2**

(iii) Hence find the value of R. **2**

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

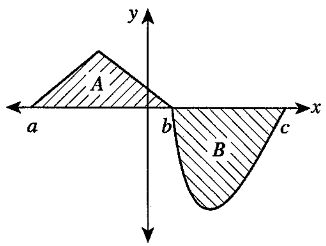
(a) In an opinion poll, 60% of people living in a town were in favour of the river being dredged. By drawing a tree diagram, or otherwise, find the probability that if three people from the town were chosen at random and interviewed:

(i) all three will be in favour of the dredging **1**

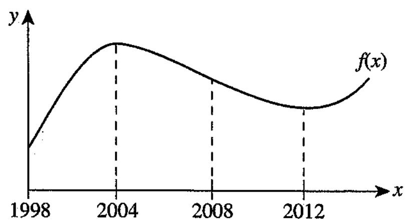
(ii) two or more would be in favour of the dredging **1**

(iii) not more than two would be in favour of the dredging. **1**

(b) The graph below represents the function $y = f(x)$. The area of shape A is 12 square units and the area of shape B is 2π square units. Find a value in terms of π for $\int_a^c f(x)\,dx$. **1**



(c) The graph below represents the median house price in a particular Sydney suburb since 1998 and its predicted price for the next 8 years.



(i) Draw a graph representing the first derivative of $f(x)$. **2**

(ii) What could you say about the sign of the second derivative at:

- 2004
- 2008. **2**

(d) Given the equation $3x^2 + 4x - 3 = 0$ has the roots α and β evaluate the following:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $2\alpha^2 + 2\beta^2$. **2**

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Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) The following table gives values for $f(x) = x^2\log x$. **2**

x	1	2	3	4	5
$f(x)$	0	1.20	4.29	9.63	17.47

Use Simpson's Rule with five function values to approximate $\int_1^5 (x^2\log x)\,dx$, correct to two decimal places.

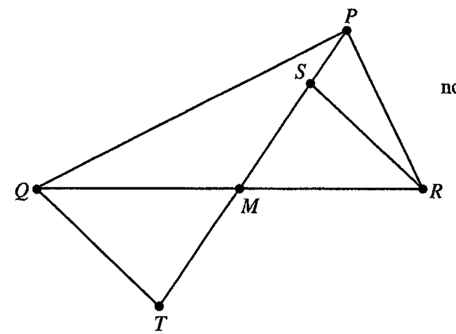
(b) Simplify $\dfrac{2\sec^2 A - 2}{4\tan A}$. **2**

(c) A boat builder needs 20 pieces of steel, all of differing lengths. The shortest rod is 5.5 m with each piece 0.8 m longer than the previous one.

(i) Find the total length of steel required for the 20 rods. **1**

(ii) After the 20th rod, the builder then needs another 10 rods. The first is 0.5 m shorter than the 20th and each successive rod is a further 0.5 m shorter than the previous. What is the length of the last rod? **2**

(d)



not to scale

In the figure above, QT and RS are both perpendicular to PT.
$TS = 12$ cm and M is the midpoint of QR.
Copy or trace this diagram into your booklet.

(i) Prove ΔQMT is congruent to ΔRMS. **3**

(ii) Find the length of TM. Provide reasons for your answer. **1**

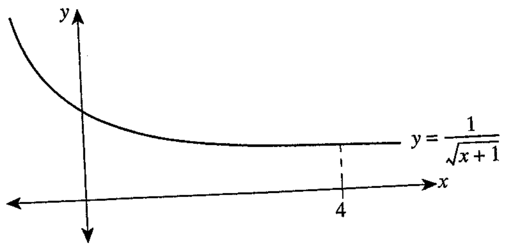
(iii) Prove, with reasons, that $QTRS$ is a parallelogram. **1**

Marks

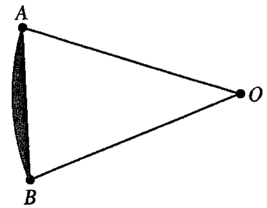
Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Show that the sequence log2, log4, log16, ... is a geometric sequence. **2**

(b) Find the volume generated when the area between the curve $y = \frac{1}{\sqrt{x+1}}$, the x-axis and the lines at $x = 0$ and $x = 4$ is rotated around the x-axis. Leave your answer in exact form. **3**



(c) In the figure OA and OB are radii of length 10 cm of a circle with centre O. The arc AB of the circle subtends an angle of $\frac{\pi}{3}$ radians at O. AB is a chord of the circle.



(i) Calculate the area of the sector AOB. **2**

(ii) Calculate the area of the triangle AOB. **2**

(iii) Find the area of the shaded segment of the circle. **1**

(d) Fully simplify $\log_{\frac{1}{a}} a^2 - \log_{\frac{1}{a}} a^3$. **2**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

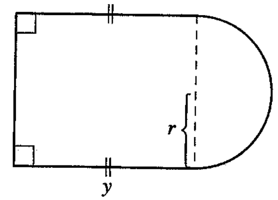
(a) A particle is moving in a straight line so that its distance, x m, from a fixed point, O, after t seconds is given by $x = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$.

(i) Find the equation for its velocity. **1**

(ii) Find its initial position and velocity. **2**

(iii) Find when the particle is at rest. **2**

(b) Miss Brown wants to build a garden bed in the shape shown in the diagram. The curved part is a semi-circle. She has 20 m of garden edging.



(i) Show that $y = 10 - r - \frac{\pi r}{2}$. **2**

(ii) Show that the area of the garden bed is given by $A = 20r - \frac{1}{2}\pi r^2 - 2r^2$. **2**

(iii) Hence, find the value of r which gives the maximum area. Justify your answer and leave your answer in terms of π. **3**

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the function $y = \frac{1}{x}e^{-x}$.

(i) For what values of x is this function defined? **1**

(ii) Describe the behaviour of the function as x:

- approaches zero from the positive side
- increases to positive infinity. **2**

(iii) Find any stationary point/s and determine their nature. **3**

(iv) Sketch the curve of this function. **2**

(b) When cane toads were first introduced into Queensland, their number, N, increased exponentially such that **4**

$$N = N_0 e^{kt}.$$

In one area, the number of cane toads increased from 50 to 267 in the first two years after introduction.
Assuming the same rate of growth, find out how many cane toads will be in that area 5 years after the toads were introduced.

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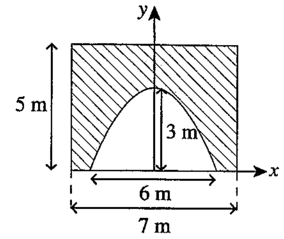
Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) The diagram represents an archway of a building that is 5 m high and 6 m wide. The curved part is in the shape of a parabola with vertex 3 m above the ground.



Use the axis shown in the diagram to:

(i) show that the equation of the parabola is $y = -\frac{1}{3}x^2 + 3$ **1**

(ii) find the shaded area. **3**

(b) For the function $y = 3\cos 4x - 5\sin x$, find the value of k if **3**

$$y + \frac{d^2y}{dx^2} = k\cos 4x.$$

(c) Two particles A and B are moving along a line. Their displacement at time t from the starting point is given as x_A and x_B respectively.

(i) Given that $\frac{d^2x_A}{dt^2} = 8 + e^{-t}$, and that $\frac{dx_A}{dt} = -1$ when $t = 0$, and $x_A = -1$ when $t = 0$, find an expression for the displacement x_A of particle A. **2**

(ii) Given that $x_B = 3\cos 4t + 4t^2 + 2$, prove that $x_B > x_A$ for $t \geq 0$. **2**

(iii) Explain the result proved in part (ii) in terms of the motions of particles A and B. **1**

End of paper

Question 1

Sample answer	Syllabus content and marking guide
(a) $74 \times 60 \times 24 \times 365 \times 65 = 2\,528\,136\,000$ $= 2.53 \times 10^9$ (3 significant figures)	P3 • Gives the correct answer to three significant figures and in scientific notation........ 2 • Gives the correct answer but not correct to 3 significant figures or not in scientific notation.......................... 1
(b) $\lvert 1-2x \rvert = 5$ $1-2x=5$ OR $-(1-2x)=-5$ $-2x=4$ $\quad -1+2x=5$ $x=-2$ $\quad 2x=6$ $x=3$	P3, P4 • Gives both correct values for x......... 2 • Demonstrates a correct method for solving the absolute value equation............ 1
(c) $r=\sqrt{\dfrac{3v}{\pi h}}$ $=\sqrt{\dfrac{3000}{\pi h}}$ $r^2=\dfrac{3000}{\pi h}$ $r^3=\dfrac{3000}{\pi}$ as $h=r$ $r=\sqrt[3]{\dfrac{3000}{\pi}}$ $= 9.85$ (to 2 decimal places)	P1, P4, H1 • Gives the correct answer............. 2 • Gives a substantially correct method of calculating the answer................ 1
(d) $\dfrac{\sqrt{2}-\sqrt{6}}{\sqrt{6}+\sqrt{2}}=\dfrac{\sqrt{2}-\sqrt{6}}{\sqrt{6}+\sqrt{2}}\times\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ $=\dfrac{\sqrt{12}-2-6+\sqrt{12}}{6-2}$ $=\dfrac{2\sqrt{12}-8}{4}$ $=\dfrac{4\sqrt{3}-8}{4}$ $=\sqrt{3}-2$	P3 • Gives the correct answer fully simplified. .... 2 • Correctly multiplies by the conjugate.... 1
(e) P(first marble is red) $=\dfrac{4}{10}$ P(second marble is red) $=\dfrac{3}{9}$ $\therefore$ P(both are red) $=\dfrac{4}{10}\times\dfrac{3}{9}=\dfrac{2}{15}$	H5 • Gives the correct answer............. 2 • Gives first probability correctly and multiplies. 1
(f) $\dfrac{125}{(5^n)^6 \times 125^{1-2n}}=\dfrac{5^3}{5^{6n}\times 5^{3-6n}}$ $=\dfrac{5^3}{5^3}$ $=1$	P3 • Gives the correct answer............. 2 • Gives a substantially correct method of calculating the answer................ 1

Question 2

Sample answer	Syllabus outcomes and marking guide
(a) $3x+2y-6\ge 0$ $x\le 4$ $y\le 0$	P5 • Gives the three correct inequalities...... 2 • Gives two correct inequalities. OR • Gives three inequalities correct except for a minor error. 1
(b) (i) $y-y_1=m(x-x_1)$ where $m=-\dfrac{1}{3}$ $y-0=-\dfrac{1}{3}(x-1)$ $\quad (x_1,y_1)=(1,0)$ $3y-0=-x+1$ $\therefore$ Equation of AC is $x+3y-1=0$.	P3, P4 • Gives correct substitution into correct formula and shows expansion. OR • Correctly substitutes both points into given equation.......................... 2 • Demonstrates a correct method for finding AC.............................. 1
(ii) $3x+2y-3=0$ ...(1) $2x-y+5=0$ ...(2) $(2)\times 2$ $\quad 4x-2y+10=0$ ...(3) $(3)+(1)$ $\quad 7x+7=0$ $x=-1$ Substitute into (2) $\quad (-2)-y+5=0$ $y=3$ $\therefore$ B is $(-1, 3)$. **Alternative answer:** $(-1, 3)$ lies on $3x+2y-3=0$ because $3\times -1+2\times 3-3=0$ $(-1, 3)$ lies on $\quad 2x-y+5=0$ as $2\times -1-3+5=0$ $\therefore$ The lines intersect at $(-1, 3)$. $\therefore B=(-1, 3)$	P4 • Uses a correct method to prove correct coordinates. .... 2 • Demonstrates a correct method for finding B. 1
(iii) $\perp d=\dfrac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $=\dfrac{\lvert -1.1+3.3+(-1)\rvert}{\sqrt{1^2+3^2}}$ $=\dfrac{7}{\sqrt{10}}$	P4 • Gives the correct answer............. 2 • Gives the correct substitution into the correct formula. 1
(iv) $AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(1+2)^2+(0-1)^2}$ $=\sqrt{10}$	P4 • Gives the correct answer............. 2 • Gives the correct substitution into the correct formula. 1
(v) $A=\dfrac{1}{2}\times\sqrt{10}\times\dfrac{7}{\sqrt{10}}$ $=3\dfrac{1}{2}$ square units	P4 • Gives the correct answer............. 2 • Gives the correct substitution into the correct formula. 1

Question 3

	Sample answer	Syllabus outcomes and marking guide
(a) (i)	$\frac{d}{dx}\left(3x^3-\sqrt[3]{x}+\frac{2}{x}\right)=\frac{d}{dx}\left(3x^3-x^{\frac{1}{3}}+2x^{-1}\right)$ $=9x^2-\frac{1}{3}x^{-\frac{2}{3}}-2x^{-2}$ $=9x^2-\frac{1}{3\sqrt[3]{x^2}}-\frac{2}{x^2}$	P7 • Gives the correct answer in any form. . . . 1
(ii)	$\frac{d}{dx}(3-x^2)^5=5(3-x^2)^4(-2x)$ $=-10x(3-x^2)^4$	P7, H5 • Gives the correct answer in any form. . . . 2 • Uses function of a function rule or chain rule. . . . 1
(iii)	$\frac{d}{dx}(2x^2\log x)=\log x\times 4x+2x^2\times\frac{1}{x}$ $=4x\log x+2x$	P7, H5 • Gives the correct answer in any form. . . . 2 • Uses the product rule. . . . 1
(b)	$\angle ADC=73°$ (Angle sum of ΔACD) Sine rule in triangle ACD: $\frac{AC}{\sin 73°}=\frac{6}{\sin 62°}$ $AC=\frac{6\sin 73°}{\sin 62°}$ Cosine rule in triangle ABC: $\cos B=\frac{7^2+4^2-AC^2}{2\times 7\times 4}$ $B=66°$ (to the nearest degree)	P4, H5 • Gives the correct answer for B ignoring rounding. . . . 3 • Correctly substitutes into the sine rule and cosine rule formulas. . . . 2 • Correctly substitutes into one of the formulas. . . . 1
(c) (i)	$\int_0^2 e^{\frac{x}{2}-1}\,dx$ $=\left[2e^{\frac{x}{2}-1}\right]_0^2$ $=2e^{\frac{2}{2}-1}-2e^{\frac{0}{2}-1}$ $=2e^0-2e^{-1}$ $=2-\frac{2}{e}$	H5 • Gives the correct answer. . . . 2 • Gives a correct integration. OR • Performs the correct working with a minor error. . . . 1
(ii)	$\int_0^{\frac{\pi}{6}}(2\sin x-\sec^2 2x)dx$ $=\left[-2\cos x-\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{6}}$ $=\left(-2\cos\frac{\pi}{6}-\frac{1}{2}\tan\left(2\times\frac{\pi}{6}\right)\right)-\left(-2\cos 0-\frac{1}{2}\tan 0\right)$ $=\left(-2\times\frac{\sqrt{3}}{2}-\frac{1}{2}\times\sqrt{3}\right)-(-2-0)$ $=-\frac{3}{2}\sqrt{3}+2$	H5 • Gives the correct answer. . . . 2 • Gives a correct integration. OR • Performs the correct working with a minor error. . . . 1

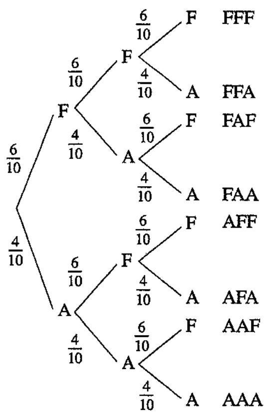
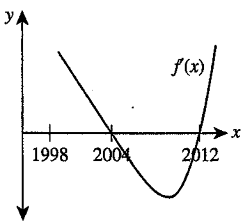
Question 4

	Sample answer	Syllabus outcomes and marking guide
(a) (i)	Triangles are similar. $\therefore\frac{h}{1.74}=\frac{1.8+3.5}{1.8}$ $h=5.12$ (to 2 decimal places)	H4, H5 • Gives the correct answer, ignoring rounding. . . . 1
(ii)	$\tan\theta=\frac{1.74}{1.8}$ $\theta=44°$ (correct to the nearest degree)	H4, H5 • Gives the correct answer, ignoring rounding. . . . 1
(iii)	Let x be his shadow, then by similar triangles: $\frac{x}{x+3}=\frac{1.74}{5.12}$ $5.12x=1.74(x+3)$ $=1.74x+5.22$ $3.38x=5.22$ $x=1.54$ (correct to 2 decimal places) $\therefore$ Shadow is $(1.8-1.54)=0.26$ m shorter.	H4, H5 • Gives the correct answer, ignoring rounding. . . . 2 • Uses similarity correctly. OR • Uses trigonometry correctly. . . . 1
(b) (i)	$f(1)=2^{1-4}$ $=2^{-3}$ $=\frac{1}{8}$	P3, H3 • Gives correct answer in any form. . . . 1
(ii)	$f(x)=1$ $2^{x-4}=1$ $2^{x-4}=2^0$ $x-4=0$ $x=4$	P3, H3 • Gives the correct solution. . . . 2 • Demonstrates correct technique but makes one error. . . . 1
(c) (i)	$A_1=25\,000\times 1.01-R$	H4, H5 • Gives correct answer. . . . 1
(ii)	$A_2=A_1\times 1.01-R$ $=(25\,000\times 1.01-R)\times 1.01-R$ $=25\,000\times 1.01^2-1.01R-R$ $=25\,000\times 1.01^2-R(1+1.01)$ $A_3=A_2\times 1.01-R$ $=(25\,000\times 1.01^2-R(1+1.01))\times 1.01-R$ $=25\,000\times 1.01^3-R(1+1.01+1.01^2)$ etc. $A_n=25\,000\times 1.01^n-R(1+1.01+1.01^2+\ldots+1.01^{n-1})$	H4, H5 • Demonstrates a full solution to find A_n. . . 2 • Gives a substantial solution. . . . 1

Question 4 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(iii) After 60 months: $A_{60} = 25\ 000 \times 1.01^{60} - R(1 + 1.01 + 1.01^2 + \ldots + 1.01^{59})$ But there is \$0 owing after 60 months. $225\ 000 \times 1.01^{60} - R(1 + 1.01 + 1.01^2 + \ldots + 1.01^{59}) = 0$ $R(1 + 1.01 + 1.01^2 + \ldots + 1.01^{59}) = 25\ 000 \times 1.01^{60}$ $R = \dfrac{25\ 000 \times 1.01^{60}}{1 + 1.01 + 1.01^2 + \ldots + 1.01^{59}}$ $= \dfrac{25\ 000 \times 1.01^{60}}{\frac{1(1.01^{60} - 1)}{1.01 - 1}}$ = \$556.11 (to the nearest cent)	H4, H5 • Gives correct answer (ignore rounding). . 2 • Equates $A_n = 0$ and uses the sum of a GP to find R. 1

Question 5

	Sample answer	Syllabus outcomes and marking guide
(a)	(i)  $\frac{6}{10}\times\frac{6}{10}\times\frac{6}{10}=\frac{216}{1000}=\frac{27}{125}$	H5 • Gives the correct answer in any form. . . . 1
	(ii) $\frac{27}{125}+3\times\left(\frac{6}{10}\times\frac{6}{10}\times\frac{4}{10}\right)=\frac{81}{125}$	H5 • Gives the correct answer in any form. . . . 1
	(iii) This is the complement to part (ii): $1-\frac{81}{125}=\frac{44}{125}$	H5 • Gives the correct answer in any form. . . . 1
(b)	$\int_a^b f(x)dx=12-2\pi$	H8 • Gives correct answer. 1
(c)	(i) 	P5, H7 • Gives a correct graph. . . . . 2 • Correctly plots both x-intercepts. OR • Gives the correct shape. 1
	(ii) 2004 – Second derivative is negative. 2008 – Second derivative is zero.	H7 • Gives correct statement for both. . . . . 2 • Gives correct statement for one. 1
(d)	(i) $\alpha+\beta=-\frac{b}{a}=-\frac{4}{3}$	P4 • Gives correct answer. 1
	(ii) $\alpha\beta=\frac{c}{a}=-1$	P4 • Gives correct answer. 1
	(iii) $2(\alpha^2+\beta^2)=2[(\alpha+\beta)^2-2\alpha\beta]$ $=2\left[\left(-\frac{4}{3}\right)^2-2\times-1\right]$ $=7\frac{5}{9}$	P4 • Calculates correct answer. . . . . 2 • Uses correct form for $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$. 1

Question 6

	Sample answer	Syllabus outcomes and marking guide
(a)	$\int_1^5 x^2(\log x)dx \doteqdot \frac{1}{3}\{0+17.47+2(4.29)+4(1.2+9.63)\}$ $=23.12$ (to 2 decimal places)	H5 • Gives correct answer. . . . . 2 • Substitutes correctly into Simpson's rule. 1
(b)	$\frac{2\sec^2A-2}{4\tan A}=\frac{2(1+\tan^2A)-2}{4\tan A}$ $=\frac{2+2\tan^2A-2}{4\tan A}$ $=\frac{2\tan^2A}{4\tan A}$ $=\frac{1}{2}\tan A$	P3 • Gives correct answer. . . . . 2 • Makes the correct substitution for $\sec^2A$. 1
(c)	(i) This makes the arithmetic series $5.5+6.3+7.1+\ldots$ for 20 terms. $a=5.5, d=0.8, n=20$ $S_n=\frac{n}{2}\{2a+(n-1)d\}$ $S_{20}=\frac{20}{2}\{2(5.5)+(20-1)0.8\}$ $=262$ m of steel	H4, H5 • Gives correct answer. 1
	(ii) The 20th term of the original series is: $T_{20}=5.5+(19)0.8$ $=20.7$ $\therefore$ The new series is 20.2, 19.7, 19.2, ... which is an A.P. with $a=20.2$ and $d=-0.5$. $T_{10}=20.2+(9)\times-0.5$ $=15.7m$	H4, H5 • Gives correct answer. . . . . 2 • Correctly finds 20th term and uses A.P. to find the new 10th term. 1
(d)	(i) To prove $\Delta QMT=\Delta RMS$ $QM=MR$ (as M is midpoint of QR) $\angle QTM=\angle MSR$ (both equal to 90°, given) $\angle QMT=\angle SMR$ (vertically opposite angles) $\therefore \Delta QMT\equiv\Delta RMS$ (AAS)	H5 • Gives a correct proof with reasons. . . . . . 3 • Gives a substantially correct proof with reasons. . . . . 2 • Gives a correct proof without reasons. OR • Makes two correct statements only. 1
	(ii) $TM=SM$ (corresponding sides in congruent triangles) $\therefore TM=6$ cm	H5 • Gives the correct answer with reasons. . . 1
	(iii) $TM=SM$ (corresponding sides of congruent triangles) $QM=MR$ (given) $\therefore$ $QSRT$ is a parallelogram as the diagonals bisect each other.	H5 • Gives a correct proof with reasons. 1

Question 7

	Sample answer	Syllabus outcomes and marking guide
(a)	$\log 2, \log 4, \log 16$ $\frac{T_2}{T_1} = \frac{\log 4}{\log 2}$ $= \frac{\log 2^2}{\log 2}$ $= \frac{2\log 2}{\log 2}$ $= 2$ $\frac{T_3}{T_2} = \frac{\log 16}{\log 4}$ $= \frac{\log 2^4}{\log 2^2}$ $= \frac{4\log 2}{2\log 2}$ $= 2$ $\therefore \frac{T_3}{T_2} = \frac{T_2}{T_1}$ and it is a geometric series.	H3, H5 • Correctly proves result with adequate working. . . . . 2 • Uses ratio test to prove result with errors. 1
(b)	$V = \pi\int_0^4 \left(\frac{1}{\sqrt{x+1}}\right)^2 dx$ $= \pi\int_0^4 \frac{1}{x+1} dx$ $= \pi[\ln(x+1)]_0^4$ $= \pi(\ln(5 - \ln 1))$ $= \pi\ln 5$ square units	H8 • Gives correct answer in any form. . . . . . . 3 • Gives a substantially correct solution. . . . 2 • Expresses the volume in terms of an integral. 1
(c) (i)	$A = \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 10^2 \times \frac{\pi}{3}$ $= \frac{50\pi}{3}$ cm^2	H5 • Gives correct answer (ignore rounding and units). . . . . 2 • Correctly substitutes into correct formula. 1
(ii)	$A = \frac{1}{2}ab\sin C$ $= \frac{1}{2} \times 10 \times 10 \times \sin\frac{\pi}{3}$ $= 25\sqrt{3}$ cm^2	H5 • Gives correct answer (ignore rounding and units). . . . . 2 • Correctly substitutes into correct formula. 1
(iii)	Shaded area $= \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$ cm^2	H5 • Gives correct answer. 1
(d)	$\log_{\frac{1}{a}} a^2 - \log_{\frac{1}{a}} a^3 = \log_{\frac{1}{a}} \frac{a^2}{a^3}$ $= \log_{\frac{1}{a}} \frac{1}{a}$ $= 1$	H3 • Gives correct answer . . . . . 2 • Uses one log law correctly. 1

Question 8

	Sample answer	Syllabus outcomes and marking guide
(a) (i)	$x = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$ velocity $= \frac{dx}{dt} = t^2 - 5t + 6$	H4, H5 • Shows correct differentiation. 1
(ii)	At $t = 0, x = 0, v = 6$ Therefore the initial position is 0 metres from fixed point O and the initial velocity is 6 m s^{-1}.	H4, H5 • Correctly gives both values. . . . . 2 • Find one value correctly. 1
(iii)	The particle is at rest when velocity is zero, i.e. $v = 0$. $t^2 - 5t + 6 = 0$ $(t-3)(t-2) = 0$ $t = 3$ or 2 Therefore the particle is at rest at 2 seconds and 3 seconds.	H4, H5 • Gives both correct answers. . . . . 2 • Uses $v = 0$ and attempts to solve resulting equation. 1
(b) (i)	$P = 2y + 2r + \frac{1}{2} \times 2 \times \pi \times r$ $= 2y + 2r + \pi r$ As the perimeter needs to be 20 m: $20 = 2y + 2r + \pi r$ $2y = 20 - 2r - \pi r$ $y = 10 - r - \frac{\pi r}{2}$	H4, H5 • Uses perimeter to correctly demonstrate result. . . . . 2 • Gives a substantially correct solution. . . . 1
(ii)	$A = 2ry + \frac{1}{2}\pi r^2$ $= 2r\left(10 - r - \frac{\pi r}{2}\right) + \frac{1}{2}\pi r^2$ $= 20r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$ $= 20r - 2r^2 - \frac{1}{2}\pi r^2$	H4, H5 • Uses area to correctly demonstrate result. 2 • Gives a substantially correct solution. . . . 1
(iii)	$A = 20r - \frac{1}{2}\pi r^2 - 2r^2$ $\frac{dA}{dr} = 20 - \pi r - 4r$ For maximum $\frac{dA}{dr} = 0$ $20 - \pi r - 4r = 0$ $\pi r + 4r = 20$ $r(\pi + 4) = 20$ $r = \frac{20}{4 + \pi}$ $\frac{d^2A}{dr^2} = -\pi - 4$ which is negative. $\therefore$ This will give a maximum area.	H4, H5 • Gives the correct value for r with justification. . . . . 3 • Gives the correct value without justification. OR • Makes substantial progress towards solution including justification. . . . . 2 • Differentiates and attempts to find r. 1

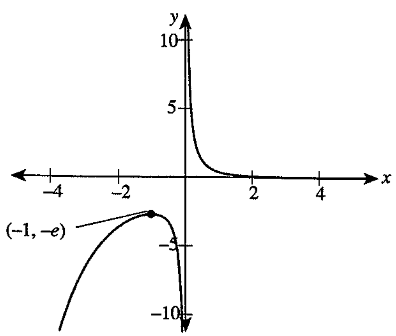
Question 9

Sample answer	Syllabus outcomes and marking guide
(a) (i) This function is defined for all real x except for $x = 0$.	P5 • Gives correct answer of $x \neq 0$ in any form. 1
(ii) $x \to 0, y \to \infty$ $x \to \infty, y \to 0$	P5, H9 • Gives correct answer, in any form, for both. . . . . 2 • Gives correct answer, in any form, for one. 1
(iii) $y = \frac{1}{x}e^{-x}$ $\frac{dy}{dx} = \frac{1}{x} \times (-e^{-x}) + e^{-x} \times -x^{-2}$ $= -e^{-x}\frac{1}{x} - e^{-x}\frac{1}{x^2}$ For stationary points: $\frac{dy}{dx} = 0$ $-e^{-x}\frac{1}{x} - e^{-x}\frac{1}{x^2} = 0$ $-e^{-x}\left(\frac{1}{x} + \frac{1}{x^2}\right) = 0$ $-e^{-x} = 0$ or $\frac{1}{x} + \frac{1}{x^2} = 0$ No solution. $1 + x = 0$ $x = -1$ At $x = -1$, $y = -e$. And	H5, H6 • Gives the correct answer. . . . . 3 • Gives a substantially correct solution with only minor errors. . . . . 2 • Differentiates correctly and equates this to zero. 1

x	−1.1	−1	−0.9
$\frac{dy}{dx}$	+ve	0	−ve

Therefore, there is a maximum turning point at $(-1, -e)$.

(iv)



P5, H6
- Draws a graph that is correct from their previous answers. 2
- Draws a graph that has the correct turning points from part (iii). 1

Question 9 (Continued)

Sample answer	Syllabus outcomes and marking guide
(b) $N = N_0e^{kt}$ At $t = 0$, $50 = A_0e^0$, $\therefore A_0 = 50$ At $t = 2$, $267 = 50e^{2k}$ $e^{2k} = \frac{267}{50}$ $\ln e^{2k} = \ln\left(\frac{267}{50}\right)$ $2k = \ln\left(\frac{267}{50}\right)$ $k = \frac{1}{2}\ln\left(\frac{267}{50}\right)$ At $t = 5$, $N = 50e^{5 \times k}$ $= 3295$ (to the nearest whole cane toad)	H3, H4 • Gives the correct solution ignoring rounding. . . . . 4 • Demonstrates a correct method with a small error. . . . . 3 • Demonstrates a correct method with a couple of minor errors or one major error. . . . . 2 • Equates $A_0 = 50$ and attempts to find k. . 1

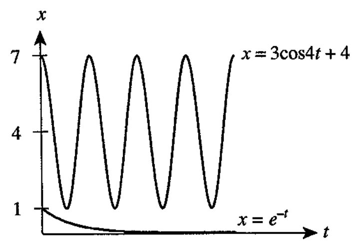
Question 10

Sample answer	Syllabus outcomes and marking guide
(a) (i) In the form $y = a(x-3)(x+3)$ and passing through (0, 3) $3 = a(0-3)(0+3)$ $3 = -9a$ $a = -\frac{1}{3}$ $\therefore y = -\frac{1}{3}(x-3)(x+3)$ $y = -\frac{1}{3}x^2 + 3$ **Alternative answer:** Sub (0, 3) into $y = -\frac{1}{3}x^2 + 3$, i.e. $3 = -\frac{1}{3} \times 0^2 + 3$. Sub (3, 0) into $y = -\frac{1}{3}x^2 + 3$, i.e. $0 = -\frac{1}{3} \times 9 + 3$. Sub (–3, 0) into $y = -\frac{1}{3}x^2 + 3$, i.e. $0 = -\frac{1}{3} \times 9 + 3$. $\therefore$ (0, 3)(3, 0) and (–3, 0) lie on $y = -\frac{1}{3}x^2 + 3$ as 3 points are sufficient to determine a parabola. Its equation is $y = -\frac{1}{3}x^2 + 3$.	P4 • Gives the correct answer in any form.... 1
(ii) $A = 5 \times 7 - 2\int_0^3 \left(-\frac{1}{3}x^2 + 3\right)dx$ $= 35 - 2\left[-\frac{1}{9}x^3 + 3x\right]_0^3$ $= 35 - 2\left(-\frac{1}{9}(27) + 9 - 0\right)$ = 23 square units	H4, H8 • Gives correct value in any form. .... 3 • Correctly integrates and gives substantially correct solution. .... 2 • Gives correct form of integration with correct limits. 1
(b) $y = 3\cos 4x - 5\sin x$ $\frac{dy}{dx} = -12\sin 4x - 5\cos x$ $\frac{d^2y}{dx^2} = -48\cos 4x + 5\sin x$ $y + \frac{d^2y}{dx^2} = k\cos 4x$ $(3\cos 4x - 5\sin x) + (-48\cos 4x + 5\sin x) = k\cos 4x$ $-45\cos 4x = k\cos 4x$ $\therefore k = -45$	H5 • Gives correct value for k. .... 3 • Finds second derivative and correctly substitutes into $y + \frac{d^2y}{dx^2} = k\cos 4x$. ..... 2 • Finds second derivative correctly. 1

Question 10 **(Continued)**

Sample answer	Syllabus outcomes and marking guide
(c) (i) $\frac{d^2x_A}{dt^2} = 8 + e^{-t}$ $\frac{dx_A}{dt} = 8t - e^{-t} + c_1$ on integrating $\frac{dx_A}{dt} = -1$ when $t = 0$ $-1 = 0 - e^0 + c_1$ $\therefore c_1 = 0$ $\therefore \frac{dx_A}{dt} = 8t - e^{-t}$ $x_A = 4t^2 + e^{-4} + c_2$ on integrating $x_A = -1$ when $t = 0$ $\therefore -1 = 0 + e^0 + c_2$ $\therefore c_2 = -2$ $\therefore x_A = 4t^2 + e^{-t} - 2$	H5 • Gives correct solution .... 2 • Correctly integrates twice............ 1

Question 10 **(Continued)**

	Sample answer	Syllabus outcomes and marking guide
(ii)	Knowing that $t \geq 0$, we need to prove $x_B > x_A$ or $x_B - x_A > 0$. i.e. Prove $(3\cos 4t + 4t^2 + 2) - (4t^2 + e^{-t} - 2) > 0$. $3\cos 4t - e^{-t} + 4 > 0$ For $t = 0$, $3\cos 0 - e^0 + 4 = 6 > 0$. For $t > 0$, we know that $-3 \leq 3\cos 4t \leq 3$ and $-1 \leq -e^{-4} < 0$. Adding these results $-4 < 3\cos 4t - e^{-t} < 3$. Adding 4 $\quad 0 < 3\cos 4t - e^{-t} + 4 < 7$. Therefore $3\cos 4t - e^{-t} + 4 > 0$ for all $t \geq 0$ and $x_B - x_A > 0$. **Alternative answer:**  When $t = 0$, $3\cos 4t + 4 - e^{-t}$ $= 7 - 1$ $= 6$ At all other times $3\cos 4t + 4 \geq 1$ and $0 < e^{-t} < 1$. $\therefore 3\cos 4t + 4 - e^{-t} > 0$ as required.	H5 • Gives correct solution .... 2 • Gives partially correct solution involving $-3 \leq 3\cos 4t \leq 3$ or similar........... 1
(iii)	*If* $x_B > x_A$ for $t \geq 0$, then particle B will always be to the right of particle A.	H1 • Gives correct answer 1

