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HSC Trial Examination 2000

Mathematics

3 Unit (Additional)
and
3/4 Unit (Common)

Time Allowed – Two hours
(Plus 5 minutes reading time)

This paper must be kept under strict security and may only be used on or after the afternoon of Thursday 3 August, 2000, as specified in the NEAP Examination Timetable.

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 9.
- Board-approved calculators may be used.
- Answer each question in a SEPARATE Writing Booklet.
- You may ask for extra Writing Booklets if you need them.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2000 Mathematics 3 Unit (Additional) and 3/4 Unit (Common) Higher School Certificate Examination.

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QUESTION 1. Use a SEPARATE writing booklet. **Marks**

(a) Let A (–3, 6) and B (1, 10) be points on the number plane. Find the coordinates of the point C, which divides the interval AB externally in the ratio 5 : 3. **2**

(b) Find the obtuse angle between the lines $3y = 2x + 1$ and $y = -3x + 5$, correct to the nearest degree. **3**

(c) Use the substitution $u = 2x - 1$ to evaluate $\int_0^1 x(2x-1)^4 dx$. **4**

(d) Solve the inequality $\frac{x}{x-3} < 4$. **3**

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QUESTION 2. Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\int_{-3}^{3} \frac{1}{9+x^2} dx$. **3**

(b) Consider the function $y = \cos^{-1}(2x) - \frac{\pi}{2}$. **3**

(i) State the domain of this function.

(ii) State the range of this function.

(iii) Sketch the graph of this function.

(c) Find $\lim_{\theta \to 0} \frac{\theta + \sin 2\theta}{3\theta}$. **2**

(d) Use the table of standard integrals to find $\int \frac{dx}{\sqrt{x^2 - 4}}$. **1**

(e) Consider the polynomial $P(x) = x^3 - 5x + c$. **3**

(i) Find the value of c if $x + 2$ is a factor of $P(x)$.

(ii) For this value of c, find $Q(x)$ such that $P(x) = (x+2)Q(x)$.

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QUESTION 3. Use a SEPARATE writing booklet.

Marks

(a) If α, β, γ are the roots of the equation $x^3 + 2x^2 - x - 5 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **2**

(b) Find the coefficient of x^2 in the expansion of $(3 - 2x)(2 + x)^4$. **3**

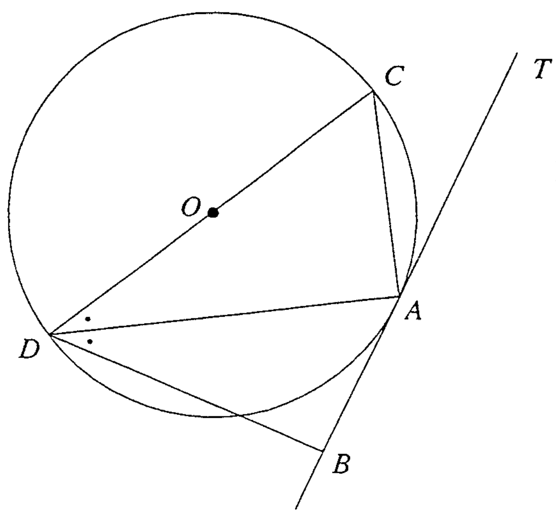
(c) A tennis club consists of 20 members, 12 men and 8 women. A committee of four people is to be chosen randomly. How many committees are possible if **4**

(i) there is to be equal numbers of men and women?

(ii) there is to be a majority of women on the committee?

(iii) the youngest member of the club must be on the committee?

(d) **3**



NOT TO SCALE

O is the centre of a circle. TAB is a tangent to the circle at A. AD bisects the angle CDB.

Copy or trace the diagram into your Writing Booklet.

Prove that the angle ABD is a right angle.

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QUESTION 4. Use a SEPARATE writing booklet. **Marks**

(a) Due to the general ageing of the community, the numbers in the local high school were declining at a rate proportional to the amount by which the numbers in the school exceeded 600. This is expressed by the equation **4**

$$\frac{dN}{dt} = k(N - 600),$$

where N is the number of students enrolled t years after 1990.

There were 1100 students enrolled at the beginning of 1990 and 900 students enrolled at the beginning of year 2000.

(i) Prove that $N = 600 + Ae^{kt}$ satisfies this equation.

(ii) Find the value of A.

(iii) Find the value of k correct to 4 significant figures.

(iv) How many students would you expect to be enrolled at the beginning of the year 2010 if the decline continued under the same conditions?

(b) Prove, using mathematical induction, that $7^n - 4^n$ is divisible by 3, where n is a positive integer. **4**

(c) (i) Using the identities for the expansions of $\sin(A + B)$, $\sin 2A$ and $\cos 2A$, prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **4**

(ii) Hence solve the equation $3\sin\theta - 4\sin^3\theta = -1$ for $0 \le \theta \le 2\pi$.

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QUESTION 5. Use a SEPARATE writing booklet.

Marks

(a) A particle P moves in a straight line in simple harmonic motion. The acceleration in metres per second per second is given by **6**

$$\ddot{x} = 2 - 3x$$

where x metres is the displacement of the particle from the origin.

Initially the particle is at $x = 1$ moving with a velocity of $\sqrt{5}$ m s^{-1}.

(i) Using integration show that the velocity v m s^{-1} of the particle is given by

$$v^2 = 4 + 4x - 3x^2.$$

(ii) Find the amplitude of motion.

(iii) Find the centre of motion.

(iv) Find the maximum speed of the particle.

(v) Find the period of the motion.

(b) (i) Prove that $e^{2x} - e^x = 56$ has a root between 2 and 3. **6**

(ii) Taking $x = 2$ as an approximation, use one application of Newton's method to find a better approximation correct to three significant figures.

(iii) By considering $e^{2x} - e^x = 56$ as a quadratic equation in e^x, solve the equation, giving your answer correct to three significant figures.

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QUESTION 6. Use a SEPARATE writing booklet.

Marks

(a) Twelve students, six boys and six girls, sat for the HSC French examination. After the exam, they sat randomly in a circle to discuss the exam. Find the probability that: 3

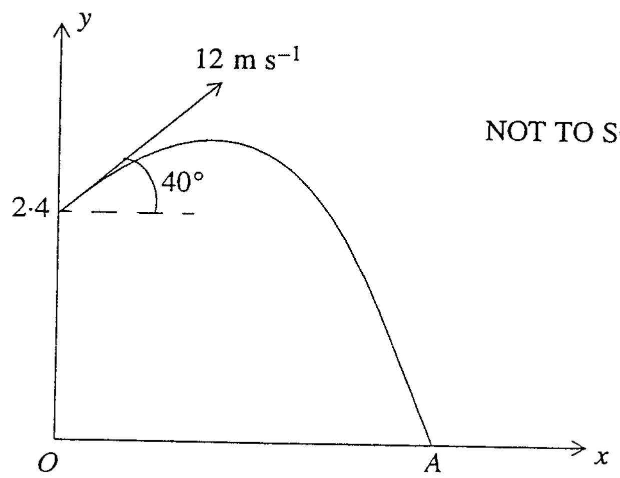
(i) no two boys are sitting next to each other.

(ii) the two top students (based on their school assessment) are sitting next to each other.

(b) If r is a positive integer and $1 \le r \le 10$, find the largest value of r which satisfies 3

$$\binom{10}{r} 3^{10-r} \times 2^{r} > \binom{10}{r-1} 3^{11-r} \times 2^{r-1}.$$

(c) 6



NOT TO SCALE

In an Olympic trial, a shot putter releases the shot from a height of 2·4 metres above ground level at an angle of 40° to the horizontal, and with a speed of 12 metres per second.

Take the origin O at a point on the ground directly under the point of release of the shot.

The equations of motion of the shot are

$$\ddot{x} = 0, \quad \ddot{y} = -g.$$

(i) Using calculus, show that the position of the shot at time t is given by

$$x = 12\cos 40°t, \quad y = 2{\cdot}4 + 12\sin 40°t - \frac{1}{2}gt^2.$$

(ii) The shot lands at a point A on the ground. Find the length of OA to the nearest centimetre. (Take $g = 9{\cdot}8$).

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QUESTION 7. Use a SEPARATE writing booklet.

Marks

(a) A new car, value \$35 000, is bought on a lease arrangement. The interest is 13% per annum reducible, calculated fortnightly (assume 26 fortnights in a year). Repayments are made every fortnight. At the end of three years, there is still 40% of the original value of the car to be repaid. **6**

(i) If the fortnightly repayments are \$$M$, show that the amount owing after the first repayment is \$$(35\,000 \times 1{\cdot}005 - M)$.

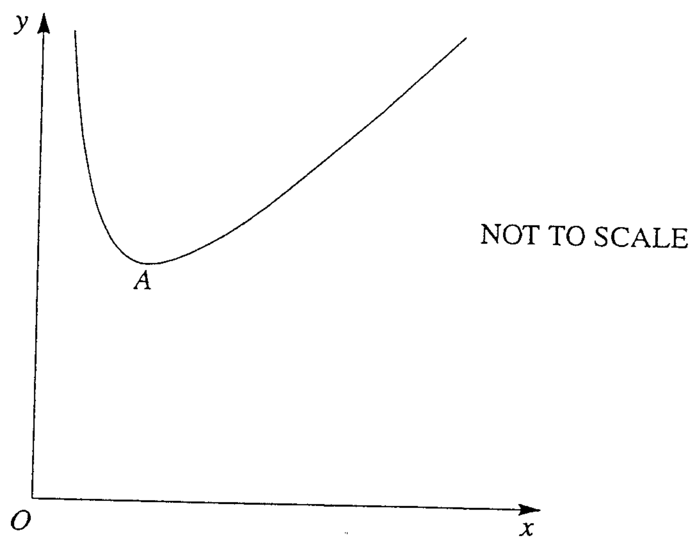
(ii) Show that the amount owing at the end of three years is

$$35\,000 \times 1{\cdot}005^{78} - 200M(1{\cdot}005^{78} - 1) \text{ dollars.}$$

(iii) Hence find the fortnightly repayments correct to the nearest cent.

(b) Consider the function $f(x) = 4x + \frac{1}{x}$ for $x > 0$. **6**

The diagram shows the graph of the function and $A\left(\frac{1}{2}, 4\right)$ is the stationary point.



(i) What is the largest domain for which the function $f(x)$ has an inverse function $f^{-1}(x)$?

(ii) Copy or trace the graph of $y = f(x)$ into your Writing Booklet.

On the same set of axes, draw the graph of $y = f^{-1}(x)$.

(iii) Find the inverse function $f^{-1}(x)$.

End of paper

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STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right).$$

NOTE: $\ln x = \log_e x, \quad x > 0$

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Solutions and suggested marking scheme

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QUESTION 1.

(a) $A(-3, 6) \quad B(1, 10) \quad 5{:}-3$

$$C\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\left(\frac{5 \times 1 + (-3) \times (-3)}{5 + (-3)}, \frac{5 \times 10 + (-3) \times 6}{5 + (-3)}\right) \quad ✓$$

$\therefore$ coordinates of C are $(7, 16)$ ✓

(b) If θ is obtuse

$$\tan\theta = -\left|\frac{m_1 - m_2}{1 + m_1m_2}\right| \qquad m_1 = \frac{2}{3},\ m_2 = -3$$

$$= -\left|\frac{\frac{2}{3} - (-3)}{1 + \frac{2}{3} \times (-3)}\right| \quad ✓$$

$$= -\frac{11}{3} \quad ✓$$

$\theta = 105°$ (nearest degree) ✓

(c) $$\int_0^1 x(2x - 1)^4 dx$$

$u = 2x - 1$

$du = 2dx$

$dx = \frac{1}{2}du$

$x = 0,\ u = -1$

$x = 1,\ u = 1$

$$= \int_{-1}^{1} \frac{u + 1}{2} \times u^4 \times \frac{1}{2}du \quad ✓$$

$$= \frac{1}{4}\int_{-1}^{1} (u^5 + u^4)du \quad ✓$$

$$= \frac{1}{4}\left[\frac{1}{6}u^6 + \frac{1}{5}u^5\right]_{-1}^{1} \quad ✓$$

$$= \frac{1}{4}\left[\left(\frac{1}{6} \times 1^6 + \frac{1}{5} \times 1^5\right) - \left(\frac{1}{6} \times (-1)^6 + \frac{1}{5} \times (-1)^5\right)\right]$$

$$= \frac{1}{4}\left[\frac{1}{6} + \frac{1}{5} - \frac{1}{6} + \frac{1}{5}\right]$$

$$= \frac{1}{10} \quad ✓$$

(d) $\frac{x}{x - 3} < 4 \qquad x \neq 3$

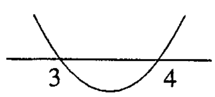
$\times (x - 3)^2$:

$$x(x - 3) < 4(x - 3)^2 \quad ✓$$

$4(x - 3)^2 - x(x - 3) > 0$

$(x - 3)[4(x - 3) - x] > 0$

$(x - 3)(3x - 12) > 0$ ✓



$x < 3,\ x > 4$ ✓

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QUESTION 2.

(a) $\displaystyle\int_{-3}^{3} \frac{1}{9+x^2}dx = \left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_{-3}^{3}$ ✓

$= \frac{1}{3}[\tan^{-1}1 - \tan^{-1}(-1)]$

$= \frac{1}{3}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right]$ ✓

$= \frac{\pi}{6}$ ✓

(b) (i) $y = \cos^{-1}(2x) - \frac{\pi}{2}$

$-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$

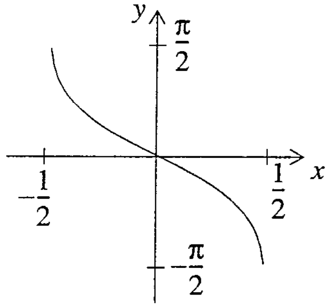
$\therefore$ domain is $-\frac{1}{2} \le x \le \frac{1}{2}$. ✓

(ii) $0 \le \cos^{-1}(2x) \le \pi$

$-\frac{\pi}{2} \le \cos^{-1}(2x) - \frac{\pi}{2} \le \frac{\pi}{2}$

$\therefore$ range is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ ✓

(iii)



✓

(c) $\displaystyle\lim_{\theta\to 0}\frac{\theta + \sin 2\theta}{3\theta} = \lim_{\theta\to 0}\left(\frac{\theta}{3\theta} + \frac{2}{3}\times\frac{\sin 2\theta}{2\theta}\right)$ ✓

$= \frac{1}{3} + \frac{2}{3}\times 1$

$= 1$ ✓

(d) $\displaystyle\int \frac{dx}{\sqrt{x^2-4}} = \ln(x + \sqrt{x^2-4}) + c$ ✓

(e) $P(x) = x^3 - 5x + c$

(i) $P(-2) = 0$: $(-2)^3 - 5 \times (-2) + c = 0$

$c = -2$ ✓

(ii) $P(x) = x^3 - 5x - 2$

$$\begin{array}{r|l} & x^2 - 2x - 1 \\ \hline x + 2 & x^3 + 0x^2 - 5x - 2 \\ & \underline{x^3 + 2x^2} \\ & -2x^2 - 5x \\ & \underline{-2x^2 - 4x} \\ & -x - 2 \\ & \underline{-x - 2} \end{array}$$

$P(x) = (x + 2)(x^2 - 2x - 1)$

$\therefore Q(x) = x^2 - 2x - 1$ ✓✓

QUESTION 3.

(a) $x^3 + 2x^2 - x - 5 = 0$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} \quad ✓$$

$$= \frac{-1}{5} \quad ✓$$

$$= -\frac{1}{5}$$

(b) $(3 - 2x)(2 + x)^4$.

Term in $x^2 = (3) \times (\text{term in } x^2) - (2x) \times (\text{term in } x)$

$$= 3 \times \binom{4}{2} 2^2 x^2 - 2x \times \binom{4}{1} 2^3 x \quad ✓✓$$

$$= 3 \times 6 \times 4x^2 - 2 \times 4 \times 8x^2$$

$$= 8x^2$$

Coefficient of x^2 is 8. ✓

(c) (i) Equal numbers of men and women: $\binom{12}{2}\binom{8}{2} = 66 \times 28$

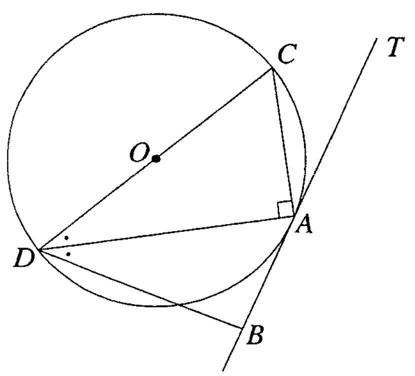
$$= 1848 \quad ✓$$

(ii) Majority of women: $3W$, $1M$ or $4W$: $\binom{8}{3}\binom{12}{1} + \binom{8}{4} = 56 \times 12 + 70$

$$= 742 \quad ✓✓$$

(iii) Youngest on committee: $\binom{19}{3} = 969$ ✓

(d)



$\angle CAD = 90°$ (angle in a semi circle) ✓

If $\angle CDA = \angle ADB = \theta$ (given)

$\angle ACD = 90° - \theta$ (angle sum of ΔACD)

$\angle BAD = \angle ACD$ (alternate segment theorem)

$= 90° - \theta$ ✓

$\angle ADB + \angle DAB = \theta + (90° - \theta)$

$= 90°$

$\therefore \angle ABD = 90°$ (angle sum of ΔABD) ✓

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QUESTION 4.

(a) $\frac{dN}{dt} = k(N - 600)$

(i) $N = 600 + Ae^{kt}$

$\frac{dN}{dt} = kAe^{kt}$

But $Ae^{kt} = N - 600$

$\therefore \frac{dN}{dt} = k(N - 600)$ ✓

(ii) When $t = 0, N = 1100$ $\therefore 1100 = 600 + Ae^0$

$\therefore A = 500$ ✓

(iii) $N = 600 + 500e^{kt}$

When $t = 10, N = 900$ $\therefore 900 = 600 + 500e^{10k}$

$e^{10k} = 0.6$

$10k = \ln 0.6$

$k = -0.05108$ (4 S.F.) ✓

(iv) $N = 600 + 500e^{-0.05108t}$

At the beginning of 2010, $t = 20$.

When t = 20, $N = 600 + 500 \times e^{-0.05108 \times 20}$

$= 780$

There will be 780 students at the start of 2010. ✓

(b) Prove $7^n - 4^n$ is divisible by 3.

When $n = 1$, $7^n - 4^n = 7 - 4 = 3$ which is divisible by 3

$\therefore$ it is true for $n = 1$. ✓

Assume $7^n - 4^n$ is divisible by 3 when $n = k$.

i.e. assume $7^k - 4^k = 3m$ where m is an integer. ✓

When $n = k + 1$, $7^n - 4^n = 7^{k+1} - 4^{k+1}$

$= 7 \times 7^k - 4 \times 4^k$

$= 7(3m + 4^k) - 4 \times 4^k$ from assumption

$= 21m + 3 \times 4^k$

$= 3(7m + 4^k)$

$= 3I$ where I is an integer. ✓

$\therefore$ if $7^n - 4^n$ is divisible by 3 when $n = k$, then $7^n - 4^n$ is divisible by 3 when $n = k + 1$.

Since $7^n - 4^n$ is divisible by 3 when $n = 1$, then $7^n - 4^n$ is divisible by 3 when $n = 2, 3, \ldots$

i.e. $7^n - 4^n$ is divisible by 3 for all positive integers n. ✓

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(c) (i) $\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ ✓

$= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$ ✓

(ii) $3\sin\theta - 4\sin^3\theta = -1$

$\therefore \sin 3\theta = -1$

$0 \le \theta \le 2\pi \quad \therefore 0 \le 3\theta \le 6\pi$

$3\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

$\therefore \theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ✓✓

QUESTION 5.

(a) $\ddot{x} = 2 - 3x \qquad x = 1,\ v = \sqrt{5}$

(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2 - 3x$

$$\frac{1}{2}v^2 = 2x - \frac{3}{2}x^2 + c \quad ✓$$

When $x = 1,\ v = \sqrt{5}$

$$\frac{1}{2} \times 5 = 2 - \frac{3}{2} + c$$

$$c = 2$$

$$\therefore \frac{1}{2}v^2 = 2x - \frac{3}{2}x^2 + 2$$

$$v^2 = 4x - 3x^2 + 4$$

$$v^2 = 4 + 4x - 3x^2 \quad ✓$$

(ii) When $v = 0$, $\quad 4 + 4x - 3x^2 = 0$

$$3x^2 - 4x - 4 = 0$$

$$(3x + 2)(x - 2) = 0$$

$$x = -\frac{2}{3},\ 2$$

Amplitude $= \frac{1}{2} \times 2\frac{2}{3}$

$= 1\frac{1}{3}m. \quad ✓$

(iii) $a = -3\left(x - \frac{2}{3}\right)$

Centre of motion is $x = \frac{2}{3} \quad ✓$

(iv) Maximum speed occurs at centre of motion.

When $x = \frac{2}{3},\ v^2 = 4 + 4 \times \frac{2}{3} - 3 \times \frac{4}{9}$

$$= 5\frac{1}{3}$$

$$v = \pm\frac{4}{\sqrt{3}}$$

Max. speed is $\frac{4}{\sqrt{3}}$ or $\frac{4\sqrt{3}}{3}$ m/s. $\quad ✓$

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(v) $n^2 = 3,\ n = \sqrt{3}\ \ (n > 0)$

Period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{\sqrt{3}}$ seconds ✓

(b) (i) $e^{2x} - e^x = 56$

i.e. $e^{2x} - e^x - 56 = 0$

Let $P(x) = e^{2x} - e^x - 56$

$P(2) = e^4 - e^2 - 56 = -8.79\ldots$

$P(3) = e^6 - e^3 - 56 = 327.3\ldots$ ✓

Since $P(2)$ and $P(3)$ have opposite signs, and $P(x)$ is continuous for $2 \le x \le 3$, there is a root between 2 and 3. ✓

(ii) $x_2 = x_1 - \dfrac{P(x_1)}{P'(x_1)}$

$= 2 - \dfrac{P(2)}{P'(2)}$ ✓

$= 2 - \dfrac{(-8.79)}{101.8}$

$= 2.09$ (3 S.F.) ✓

$P'(x) = 2e^{2x} - e^x$

$P'(2) = 2e^4 - e^2$

$= 101.8$

(iii) $e^{2x} - e^x - 56 = 0$

Let $u = e^x$:

$u^2 - u - 56 = 0$

$(u - 8)(u + 7) = 0$

$u = 8$ or $u = -7$ ✓

$e^x = 8$ or $e^x = -7$

$x = \ln 8$ [$e^x = -7$ has no solution]

$x = 2.08$ (3 S.F.) ✓

QUESTION 6.

(a) (i) No. of possible arrangements = 11! ✓

$$\text{Prob (no boys together)} = \frac{5! \times 6!}{11!} = \frac{1}{462} \quad ✓$$

(ii) $$\text{Prob (top 2 students together)} = \frac{2 \times 10!}{11!} = \frac{2}{11} \quad ✓$$

(b) $$\binom{10}{r} 3^{10-r} 2^r > \binom{10}{r-1} 3^{11-r} 2^{r-1}$$

$$2 \times \frac{10!}{r!(10-r)!} > 3 \times \frac{10!}{(r-1)!(11-r)!} \quad ✓$$

$$2 \times \frac{(11-r)!}{(10-r)!} > 3 \times \frac{r!}{(r-1)!}$$

$$2 \times (11-r) > 3r \quad ✓$$

$$22 > 5r$$

$$r < 4\tfrac{2}{5}$$

Largest value of r is 4. ✓

(c) (i)

$$\ddot{x} = 0$$
$$\dot{x} = c_1$$
When $t = 0$, $\dot{x} = 12\cos 40°$
$$\therefore c_1 = 12\cos 40°$$
$$\therefore \dot{x} = 12\cos 40°$$
$$x = 12\cos 40° t + c_2$$
When $t = 0$, $x = 0 \therefore c_2 = 0$
$$\therefore x = 12\cos 40° t$$

$$\ddot{y} = -g$$
$$\dot{y} = -gt + c_3$$
When $t = 0$, $\dot{y} = 12\sin 40°$
$$\therefore c_3 = 12\sin 40°$$
$$\therefore \dot{y} = 12\sin 40° - gt$$
$$y = 12\sin 40° t - \frac{1}{2}gt^2 + c_4$$
When $t = 0$, $y = 2\cdot4 \therefore c_4 = 2\cdot4$
$$\therefore y = 2\cdot4 + 12\sin 40° t - \frac{1}{2}gt^2 \quad ✓✓✓$$

(ii) When $y = 0$, $2\cdot4 + 12\sin 40° t - 4\cdot9t^2 = 0 \quad (g = 9\cdot8)$

$$t = \frac{-12\sin 40° \pm \sqrt{144\sin^2 40° - 4 \times (-4\cdot9) \times 2\cdot4}}{-9\cdot8} \quad ✓$$

$$= 1\cdot84032\ldots,\ t > 0 \quad ✓$$

When $t = 1\cdot84$, $x = 12\cos 40° \times 1\cdot84$

$$= 16\cdot9172\ldots$$

$\therefore OA = 16\cdot92$ metres. (nearest centimetre) ✓

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QUESTION 7.

(a) (i) 13% p.a. $= \dfrac{13}{26}\%$ per fortnight

$= 0{\cdot}5\%$ per fortnight.

$\therefore$ amount owing after one fortnight $= 35\,000 + 0{\cdot}5\%$ of $35\,000$

$= 35\,000 \times 1{\cdot}005$

Amount owing after 1st repayment $= 35\,000 \times 1{\cdot}005 - M$. ✓

(ii) Amt. owing after 2 repayments $= [35\,000 \times 1{\cdot}005 - M] \times 1{\cdot}005 - M$

$= 35\,000 \times 1{\cdot}005^2 - M \times 1{\cdot}005 - M$. ✓

Amt. owing after 78 repayments $= 35\,000 \times 1{\cdot}005^{78} - M(1 + 1{\cdot}005 + 1{\cdot}005^2 + \ldots)[78 \text{ terms}]$ ✓

$$= 35\,000 \times 1{\cdot}005^{78} - \frac{M[1{\cdot}005^{78} - 1]}{1{\cdot}005 - 1}$$

$$= 35\,000 \times 1{\cdot}005^{78} - \frac{M[1{\cdot}005^{78} - 1]}{0{\cdot}005}$$

$= 35\,000 \times 1{\cdot}005^{78} - 200M[1{\cdot}005^{78} - 1]$ ✓

(iii) Value after 3 years $= 40\%$ of $\$35\,000$

$= \$14\,000$

$\therefore 35\,000 \times 1{\cdot}005^{78} - 200M[1{\cdot}005^{78} - 1] = 14\,000$ ✓

$35\,000 \times 1{\cdot}005^{78} - 14\,000 = 200M[1{\cdot}005^{78} - 1]$

$$M = \frac{35\,000 \times 1{\cdot}005^{78} - 14\,000}{200[1{\cdot}005^{78} - 1]}$$

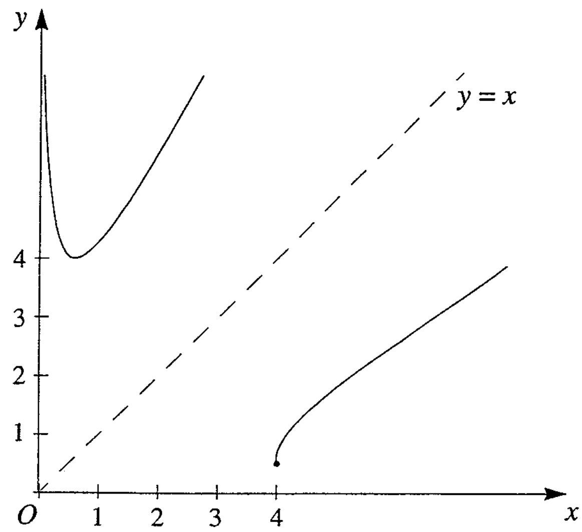
$M = 395{\cdot}80$ ✓

Fortnightly repayments are $\$395{\cdot}80$ (nearest cent).

(b) $f(x) = 4x + \dfrac{1}{x}, \quad x > 0$

(i) Largest domain: $x \geq \dfrac{1}{2}$. ✓

(ii)



✓✓

(iii) $y = 4x + \frac{1}{x}, \quad \text{for } x \ge \frac{1}{2}$

Inverse: $x = 4y + \frac{1}{y}, \quad \text{for } y \ge \frac{1}{2}$ ✓

$$xy = 4y^2 + 1$$

$$4y^2 - xy + 1 = 0$$

$$y = \frac{x \pm \sqrt{x^2 - 16}}{8}$$

$$y = \frac{x + \sqrt{x^2 - 16}}{8} \text{ or } \frac{x - \sqrt{x^2 - 16}}{8}$$ ✓

For the appropriate range, $y \ge \frac{1}{2}$

$$f^{-1}(x) = \frac{x + \sqrt{x^2 - 16}}{8}.$$ ✓