Neap:

HSC Trial Examination 2012

Mathematics

This paper must be kept under strict security and may only be used on or after the morning of Monday 6 August, 2012 as specified in the Neap Examination Timetable.

General Instructions

Reading time – 5 minutes

Working time – 3 hours

Write using black or blue pen

Board-approved calculators may be used

A table of standard integrals is provided at the back of this paper

All necessary working should be shown in every question

Section I – 10 marks

10 multiple-choice questions

Section II – 90 marks

6 short-answer questions

Total marks – 100

Attempt questions 1–16

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2012 HSC Mathematics Examination.

Section I – 10 marks
Attempt Questions 1–10
All questions are of equal value

Answer each question on the multiple-choice answer sheet for Questions 1–10.

1. Solve $x^2 - 9 \geq 0$.

(A) $x \geq 3$
(B) $x \leq -3$
(C) $-3 \leq x \leq 3$
(D) $x \leq -3, x \geq 3$

2. Evaluate $\dfrac{\sqrt{3.84}}{2.65 + 7.7}$ correct to two decimal places.

(A) 0.19
(B) 0.61
(C) 5.28
(D) 8.44

3. What are the conditions for the expression $ax^2 + bx + c$ to be positive definite?

(A) $a > 0$ and $\Delta > 0$
(B) $c > 0$ and $\Delta > 0$
(C) $a > 0$ and $\Delta < 0$
(D) $c > 0$ and $\Delta < 0$

4. If $\sin\alpha = -\dfrac{3}{5}$ and $\cos\alpha < 0$, what is the exact value of $\tan\alpha$?

(A) $\dfrac{3}{4}$
(B) $\dfrac{4}{3}$
(C) $-\dfrac{3}{4}$
(D) $-\dfrac{4}{3}$

5. A parabola has equation $x^2 = 8(y + 2)$.

What are the coordinates of its vertex (V) and focus (S), respectively?

(A) $V(0, 2)$ and $S(0, 0)$
(B) $V(0, -2)$ and $S(0, 0)$
(C) $V(0, 2)$ and $S(0, -4)$
(D) $V(0, -2)$ and $S(0, -4)$

6. In solving the equation $\frac{x}{3} - \frac{2x+1}{4} = 5$, which of the following is correct?

(A) $x > 0$

(B) $x < 0$

(C) x is an integer.

(D) x is irrational.

7. Solve for n, $|2n+1| = -n+1$.

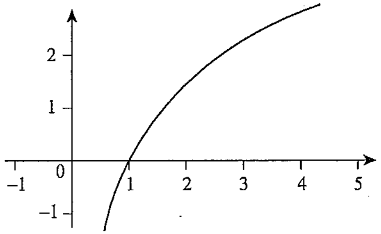
(A) There are no solutions.

(B) There is only one solution.

(C) There are two solutions.

(D) There are three solutions.

8. The graph below has which of the following properties?



(A) $f'(x) > 0$ and $f''(x) > 0$

(B) $f'(x) > 0$ and $f''(x) < 0$

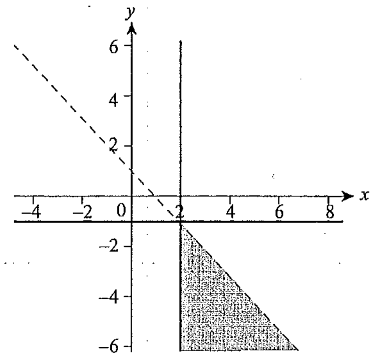
(C) $f'(x) < 0$ and $f''(x) > 0$

(D) $f'(x) < 0$ and $f''(x) < 0$

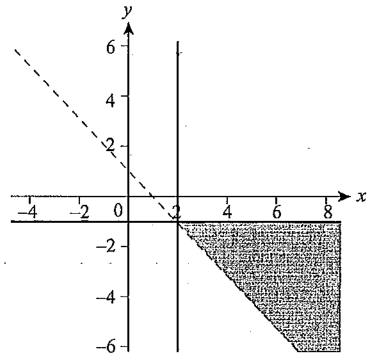
9. Which of the following diagrams show the region where the following inequalities hold simultaneously:

$$y + 1 \leq 0;\ x + y - 1 < 0;\ x \geq 2\ ?$$

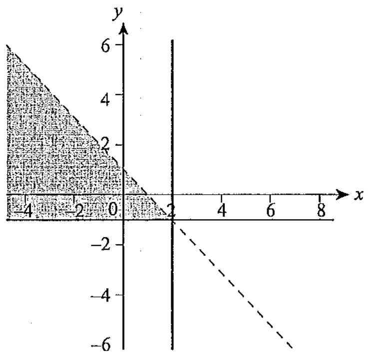
(A)



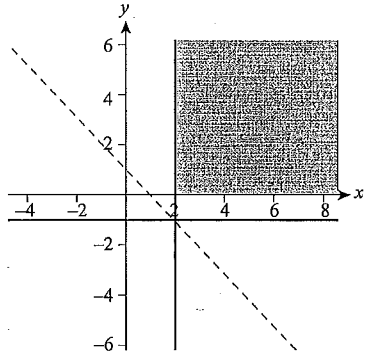
(B)



(C)



(D)



10. If α and β are the roots of $2x^2 + 3x - 4 = 0$, then the values of $\alpha + \beta$ and $\frac{1}{\alpha} + \frac{1}{\beta}$ are respectively:

(A) $\frac{3}{2}$ and $\frac{3}{4}$

(B) $\frac{3}{2}$ and $-\frac{3}{4}$

(C) $-\frac{3}{2}$ and $\frac{3}{4}$

(D) $-\frac{3}{2}$ and $-\frac{3}{4}$

Marks

Section II – 90 marks
Attempt Questions 11–16
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) An urn contains 3 white and 5 black balls. A second urn contains 1 white and 2 black balls. A ball is drawn at random from the first urn and placed in the second urn. **2**

If a ball is then drawn from the second urn, what is the probability that the ball is white?

(b) Evaluate $\lim_{x \to 5} \frac{x^3 - 125}{x - 5}$. **2**

(c) Differentiate:

(i) $\frac{1}{\sqrt{x}}$ **1**

(ii) xe^{2x} **1**

(iii) $\frac{\ln x}{x}$ **2**

(d) The three sides of a triangle are $x + y - 1 = 0$, $2x - y + 3 = 0$ and $x - 2y + 3 = 0$. **4**

Find the equation of the line through the intersection of the first two sides that is perpendicular to the third side.

(e) (i) Simplify $\frac{\cos^2\theta}{1 - \sin\theta} - \frac{\cos^2\theta}{1 + \sin\theta}$. **2**

(ii) Hence, solve $\frac{\cos^2\theta}{1 - \sin\theta} - \frac{\cos^2\theta}{1 + \sin\theta} = 1$, $0 \le \theta \le \frac{\pi}{2}$. **1**

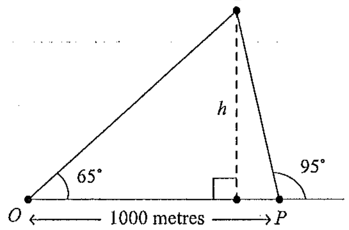
Marks

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Solve for x: $\left(x+\frac{1}{x}\right)^2-7\left(x+\frac{1}{x}\right)+6=0$. **3**

(b) Express $2x^2+x+1$ in the form $Ax(x-1)+B(x-1)+C$. **3**

(c) A balloon B is in the same vertical plane as two observers O and P. O and P are 1000 metres apart. OB makes an angle of 65° to the horizontal and PB an angle of 95° to the horizontal (see diagram).



(i) Using the Sine Rule, show that $OB = 2000\sin 85°$. **2**

(ii) Find the height of the balloon (h) to the nearest metre. **2**

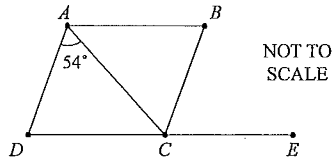
(d) Evaluate $\int_0^1 (1-x)^{\frac{1}{2}}\,dx$. **2**

(e) Find the volume when $y=\ln(x)$ is rotated about the y-axis between $y=1$ and $y=3$. Express your answer in exact form. **3**

Marks

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a)

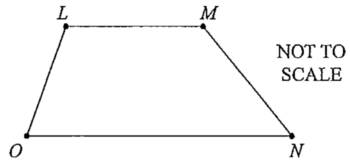


In the diagram, $ABCD$ is a rhombus where $\angle DAC = 54°$ and DC is produced to E. Copy the diagram into your writing booklet

(i) What is the value of $\angle DAB$? **1**

(ii) What is the value of $\angle BCE$? Give reasons. **2**

(b)



$LMNO$ is a quadrilateral. $\angle LON = \angle MNO$ and $LO = MN$. Copy this diagram into your writing booklet.

(i) Prove that the triangle OLN is congruent to triangle NMO. **2**

(ii) Why are $\angle LNO$ and $\angle MON$ equal? **1**

(iii) Prove that $\angle LOM = \angle LNM$. **2**

Question 13 continues on page 9

Marks

Question 13 (continued)

(c) Light passing through an absorbent material has its intensity I reduced according to the law

$$\frac{dI}{dx} = -kI$$

where x is the distance travelled by the light from the surface of the material.

(i) Show that $I = I_0 e^{-kx}$ is a solution to this equation. **1**

(ii) In water the intensity of red light is reduced by one half after travelling 2 metres. **2**

What is the value of the constant k?

(iii) After what distance travelled will its intensity be $\frac{1}{1000}$ of its initial value? **2**

(Answer correct to 2 decimal places.)

(d) Differentiate $\log_3 x^2$. **2**

End of Question 13

Marks

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) The acceleration of a particle moving in a straight line is $(t-4)$ cm/s^2. The particle starts at the origin with a velocity of 5 cm/s.

(i) Find the velocity and position of the particle at any time t. **2**

(ii) When does the particle come to rest? **1**

(iii) How often does the particle pass through the origin? **2**

(b) Consider the curve $y = x^3 + 3x^2 - 9x - 11$.

(i) Find $\frac{dy}{dx}$. **1**

(ii) Find the coordinates of the two stationary points. **2**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve for the domain $-4 \le x \le 3$. **2**

(v) By drawing an appropriate line on your graph, or otherwise, solve **1**

$x^3 + 3x^2 - 9x - 27 = 0$.

(c) The equation of a parabola is $(y-3)^2 = -12(x-1)$. **2**

Find the equation of its directrix.

Marks

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the function given by $y = \cos^2 x$.

(i) Copy and complete the following table in your writing booklet (note that x is measured in radians). **2**

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1				

(ii) Apply Simpson's rule with five function values to find an approximation to **2**

$$\int_0^{\pi} \cos^2 x \, dx.$$

(b) Nick would like to save \$80 000 for a deposit on his first home. He has decided to invest his net monthly salary of \$4 000 in a bank account at the beginning of the month. It pays 5.5% in interest per annum, compounded monthly. Nick intends to withdraw \$$W$ at the end of each month from his account for living expenses, immediately after the interest has been paid.

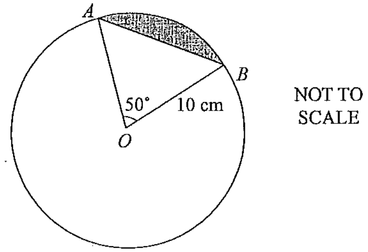
(i) Show that the amount of money in the account following the second withdrawal of \$$W$ is given by $\$4000(R^2 + R) - \$W(R+1)$, where $R = \left(1 + \frac{5.5}{1200}\right)$. **2**

(ii) Calculate the value of W if Nick is to reach his goal after 5 years. **3**

Question 15 continues on page 12

Question 15 (continued)

(c)



In the circle shown, a chord AB subtends an angle of 50° at the centre.

(i) Calculate the length of the arc AB. Leave your answer in terms of π. **1**

(ii) Calculate the area of the sector AOB. Leave your answer in terms of π. **1**

(iii) Calculate the perimeter of the shaded segment. (Answer to 1 decimal place). **1**

(d) The point $P(x, y)$ is equidistant from the lines $x = 3$ and $4x + 3y - 15 = 0$. **3**

Find the locus of point P.

End of Question 15

Marks

Question 16 Use a SEPARATE writing booklet.

(a) Solve the equation $2\ln(x\sqrt{3}) = \ln(6-7x)$. **3**

(b) When the tap is opened to fill an empty water tank, the volume (V) measured in litres of water in the tank increases at a rate $\frac{dV}{dt} = e^{t} + 3e^{-t}$, where t is measured in minutes from the time that the tap is opened.

(i) At what rate does the water initially fill the tank? **1**

(ii) Use integration to find an expression for V in terms of t. **1**

(iii) Show that $e^{t} - 3e^{-t} - 8 = 0$, when $V = 10$. **1**

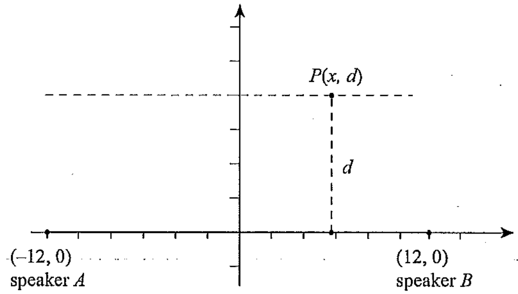
(iv) Find t when $V = 10$ to the nearest second. **3**

Question 16 continues on page 14

Marks

Question 16 (continued)

(c) At a music festival, the speakers at the front are placed on either side of the main stage and are 24 metres apart. The sound mixers are to be a distance of d metres from the stage (in a particular row).



It is known that the total sound level (S) from these speakers at point $P(x, d)$ is:

$$S = \frac{100}{d^2 + (x+12)^2} + \frac{100}{d^2 + (x-12)^2}$$

(i) Show that $\frac{dS}{dx} = -200\frac{M}{Q}$, where **2**

$M = (x+12)(d^2 + (x-12)^2)^2 + (x-12)(d^2 + (x+12)^2)^2$ and

$Q = (d^2 + (x+12)^2)^2(d^2 + (x-12)^2)^2$.

(ii) Noting that $M = 2x(x^2 + 144 + d^2 + 24\sqrt{144+d^2})(x^2 + 144 + d^2 - 24\sqrt{144+d^2})$, **2**

use $\frac{dS}{dx}$ to show that Mario the mixer, who moves along a row 20 metres from the stage, measures the sound to be at a maximum when in line with the centre of the stage.

(iii) Another sound mixer (Aaron) decides it may be better to be closer to the stage. Aaron moves along a row, which is 5 metres from the stage. **2**

Describe how the sound level changes for Aaron as he moves along the row. Give clear reasons for your answer.

End of paper

Neap:

HSC Trial Examination 2012

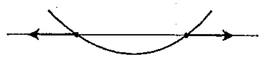
Mathematics

Solutions and marking guidelines

Section I

		Sample answer
Question 1	**D**	

$$x^2 - 9 \ge 0$$
$$(x-3)(x+3) = 0$$
$$x \le -3, x \ge 3$$

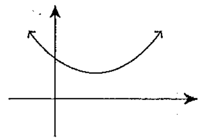


Question 2 **A**

$$\frac{\sqrt{3.84}}{2.65 + 7.7}$$
$$= 0.18(93325406\ldots)$$
$$= 0.19 \text{ (2 decimal places)}$$

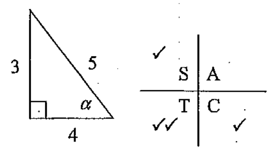
Question 3 **C**

Positive definite



$\therefore a > 0, \Delta < 0$

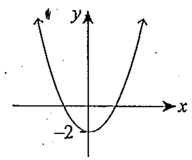
Question 4 **A**



- Complete triangle
- Determine quadrants
- $\tan\alpha = +\frac{3}{4}$

Question 5 **B**

$$x^2 = 8(y+2)$$
$$\therefore (x-0)^2 = 4(2)(y+2)$$
$$V(0,-2), S(0,0)$$



Sample answer

Question 6 **B**

$$\frac{x}{3} - \frac{2x+1}{4} = 5$$
$$4x - (6x+3) = 60$$
$$-2x - 3 = 60$$
$$-2x = 63$$
$$x = -31\frac{1}{2}$$
$$\therefore x < 0$$

Question 7 **C**

$$|2n+1| = -n+1$$
$$2n+1 = -n+1 \quad \text{or} \quad -2n-1 = -n+1$$
$$3n = 0 \qquad -n = 2$$
$$n = 0 \qquad n = -2$$

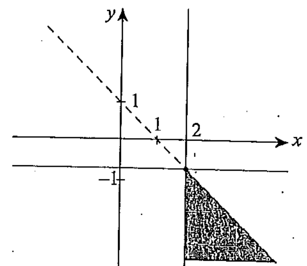
i.e. 2 solutions

Question 8 **B**

$f(x)$ is increasing, $\therefore f'(x) > 0$

$f(x)$ is concave down $\therefore f''(x) < 0$

Question 9 **A**



Question 10 **C**

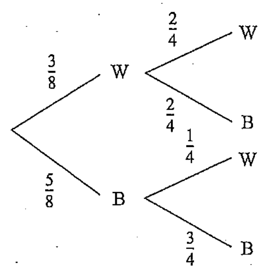
$$2x^2 + 3x - 4 = 0$$
$$\alpha + \beta = -\frac{3}{2}, \alpha\beta = -\frac{4}{2}$$
$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$$
$$= \frac{-\frac{3}{2}}{-\frac{4}{2}}$$
$$= \frac{3}{4}$$

Section II

Sample answer

Question 11

(a)



$$P(\text{white from second urn}) = \frac{3}{8}\times\frac{2}{4}+\frac{5}{8}\times\frac{1}{4} = \frac{11}{32}$$

(b) $\lim_{x\to 5}\frac{x^3-125}{x-5} = \lim_{x\to 5}\frac{(x-5)(x^2+5x+25)}{(x-5)}$

$= \lim_{x\to 5}(x^2+5x+25)$

$= 75$

(c) (i) $\frac{d}{dx}\left(x^{-\frac{1}{2}}\right) = -\frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{-1}{2\sqrt{x^3}}$ or $\frac{-1}{2x\sqrt{x}}$

(ii) $\frac{d}{dx}(xe^{2x}) = 1\cdot e^{2x}+x\cdot 2e^{2x}$

$= e^{2x}(1+2x)$

(iii) $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{x\cdot\frac{1}{x}-1\cdot\ln x}{x^2}$

$= \frac{1-\ln x}{x^2}$

OR

$\frac{d}{dx}(x^{-1})\ln x = -1x^{-2}\cdot\ln x + x^{-1}\cdot\frac{1}{x}$

$= -\frac{\ln x}{x^2}+\frac{1}{x^2}$

$= \frac{1-\ln x}{x^2}$

Sample answer

(d) $s_1: x+y-1=0$ Solving s_1 and s_2:

$s_2: 2x-y+3=0$ $\quad 3x+2=0$

$s_3: x-2y+3=0$ $\quad x=-\frac{2}{3}, y=1\frac{2}{3}$

$m_3=\frac{1}{2}$ $\quad \therefore y-y_1=m(x-x_1)$

$\therefore m_\gamma=-2$ $\quad y-\frac{5}{3}=-2\left(x+\frac{2}{3}\right)$

$6x+3y-1=0$

Alternative:

$m_\gamma=-2$

$l_1+kl_2=0$

$(x+y-1)+k(2x-y+3)=0$

$\therefore (1+2k)x+(1-k)y+(-1+3k)=0$

$m=\frac{-(1+2k)}{1-k}=-2$

$\therefore k=\frac{1}{4}$ and $6x+3y-1=0$

(e) (i) $\frac{\cos^2\theta}{1-\sin\theta}-\frac{\cos^2\theta}{1+\sin\theta}$

$= \frac{1-\sin^2\theta}{1-\sin\theta}-\frac{1-\sin^2\theta}{1+\sin\theta}$

$= (1+\sin\theta)-(1-\sin\theta)$

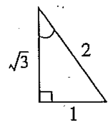
$= 2\sin\theta$

(ii) $\frac{\cos^2\theta}{1-\sin\theta}-\frac{\cos^2\theta}{1+\sin\theta}=1$

$\therefore 2\sin\theta = 1$

$\sin\theta=\frac{1}{2}$

$\theta=\frac{\pi}{6}$



Question 12

(a) $\left(x+\frac{1}{x}\right)^2-7\left(x+\frac{1}{x}\right)+6=0$

Put $M=x+\frac{1}{x}$

$M^2-7M+6=0$

$M=6$ or 1

$\therefore x+\frac{1}{x}=6$ or $x+\frac{1}{x}=1$

i.e. $x^2-6x+1=0$

using quadratic formula

$x=3\pm 2\sqrt{2}$

and $x^2-x+1=0$

$\therefore$ no solutions

Sample answer

(b) $2x^2+x+1 = Ax(x-1)+B(x-1)+C$

$= Ax^2-Ax+Bx-B+C$

$= Ax^2+(-A+B)x+(-B+C)$

$\therefore A=2, -2+B=1$ and $-3+C=1$

$B=3$ $\quad C=4$

(c) (i) By sine rule:

$\frac{\sin 85}{OB}=\frac{\sin 30}{1000}$ $\quad \angle OBP = 95-65 = 30°$

$\therefore OB=\frac{1000\sin 85}{\sin 30}$ $\quad \sin 30=\frac{1}{2}$

$OB=2000\sin 85$

(ii) By sine ratio:

$\sin 65=\frac{h}{OB}$

$\therefore h = OB\sin 65$

$h=2000\sin 85\times\sin 65$

$h=1805.718025$

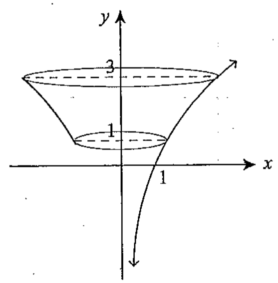
$h=1806$ metres (nearest metre)

(d) $\int_0^1 (1-x)^{\frac{1}{2}}dx = -\frac{2}{3}\left[(1-x)^{\frac{3}{2}}\right]_0^1$

$= -\frac{2}{3}[0-1]$

$= \frac{2}{3}$

(e)



$y=\ln x$

$\therefore x=e^y$

$\text{Vol}=\pi\int_1^3 x^2\,dy$

$=\pi\int_1^3 e^{2y}\,dy$

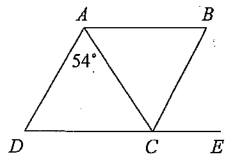
$=\frac{\pi}{2}[e^{2y}]_1^3$

$=\frac{\pi}{2}[e^6-e^2]u^3$

$=\frac{1}{2}\pi e^2(e^3-1)u^3$

Question 13

(a) (i)



$\angle DAB = 2\times 54°$

$= 108°$

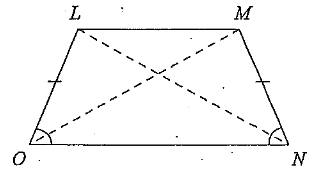
(diagonals in rhombus bisect angles through which they pass)

(ii) $\angle ADC = 72°$ (co-interior angles are equal $AB \parallel CD$)

$\angle BCE = 72°$

(corresponding angles are equal in parallel lines)

(b) (i)



ON is common to ΔOLN and ΔNMO.

$\angle LON = \angle MNO$ (given)

$LO = MN$ (given)

$\therefore \Delta OLN \equiv \Delta NMO$ (SAS)

(ii) $\angle LNO = \angle MON$

(corresponding angles in congruent triangles are equal)

(iii) Now, $\angle LOM + \angle MON = \angle MNL + \angle LNO$,

but from (ii) we see that $\angle LNO = \angle MON$.

$\therefore \angle LOM = \angle MNL$

(c) (i) $\frac{dI}{dx}=-kI$

$I=I_0e^{-kx}$

$\frac{dI}{dx}=I_0(-ke^{-kx})$

$=-k(I_0e^{-kx})$

$=-kI$, as required

Sample answer

(ii) $x = 2, I = \frac{1}{2}I_0$

Since $I = I_0 e^{-kx}$

$\frac{1}{2}I_0 = I_0 e^{-2k}$

$\ln\left(\frac{1}{2}\right) = -2k$

$-k = \frac{1}{2}\ln\left(\frac{1}{2}\right)$

(iii) $I = 10^{-3} I_0$

$\therefore 10^{-3} I_0 = I_0 e^{-kx}$

$\ln 10^{-3} = -k \cdot x$

$x = \dfrac{\ln 10^{-3}}{\frac{1}{2}\ln\left(\frac{1}{2}\right)}$

$= \dfrac{-6\ln 10}{-\ln 2} = 19.93\ldots$ metres

(d) $\frac{d}{dx}\log_3 x^2 = \frac{d}{dx}\left\{\frac{\ln x^2}{\ln 3}\right\}$ (change of base)

$= \frac{1}{\ln 3}\cdot\frac{2x}{x^2}$

$= \frac{2}{x\ln 3}$

Question 14

(a) (i) $\ddot{x} = t - 4 \quad t = 0, x = 0, \dot{x} = 5$

$\dot{x} = \frac{1}{2}t^2 - 4t + C_1$

$5 = 0 - 0 + C_1$

$\therefore \dot{x} = \frac{1}{2}t^2 - 4t + 5$

$x = \frac{1}{6}t^3 - 2t^2 + 5t + C_2$

$0 = 0 - 0 + 0 + C_2$

$\therefore x = \frac{1}{6}t^3 - 2t^2 + 5t$

(ii) $\dot{x} = 0$

$\therefore \frac{1}{2}t^2 - 4t + 5 = 0$

$t^2 - 8t + 10 = 0$

$t = \frac{8 \pm \sqrt{24}}{2}$

$t = 4 \pm \sqrt{6}$ seconds

(iii) $x = 0$

$\therefore \frac{1}{6}t^3 - 2t^2 + 5t = 0$

$t^3 - 12t^2 + 30t = 0$

$t(t^2 - 12t + 30) = 0$

$t = 0$ or $t = 6 \pm \sqrt{6}$

$\therefore$ passes through the origin twice when $t = 6 \pm \sqrt{6}$ (NB. starts at 0).

(b) (i) $y = x^3 + 3x^2 - 9x - 11$

$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$

(ii) $\frac{dy}{dx} = 0$

$\therefore 3x^2 + 6x - 9 = 0$

$3(x^2 + 2x - 3) = 0$

$3(x-1)(x+3) = 0$

$\therefore x = 1$ or $x = -3$

$(-3, 16)$ and $(1, -16)$

(iii) $\frac{d^2y}{dx^2} = 6x + 6 = 6(x+1)$

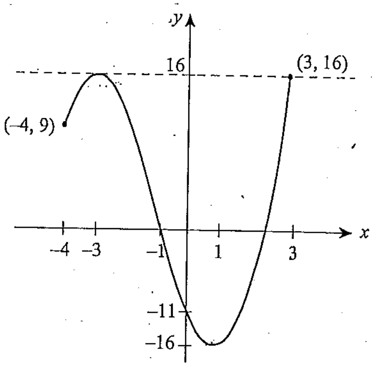
$\therefore f''(-3) = -12$

i.e. $(-3, 16)$ is a maximum

and $f''(1) = 12$

i.e. $(1, -16)$ is a minimum

(iv)



(v) $x^3 + 3x^2 - 9x - 27 = 0$

$\therefore x^3 + 3x^2 - 9x - 11 - 16 = 0$

$x^3 + 3x^2 - 9x - 11 = 16$

$\therefore$ drawing $y = 16$ will provide solutions

i.e. $x = -3, 3$

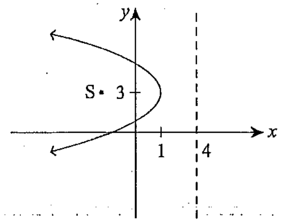
Sample answer

(c) $(y-3)^2 = -12(x-1)$

$\therefore 4a = 12$

$a = 3$

$\therefore$ directrix is $x = 4$



Question 15

(a) (i)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

(ii) $= \frac{h}{3}[y_0 + 4(y_1 + y_3) + 2y_2 + y_4]$

$= \frac{\frac{\pi}{4}}{3}\left[1 + 4\left(\frac{1}{2} + \frac{1}{2}\right) + 2 \times 0 + 1\right]$

$= \frac{\pi}{12} \times 6$

$= \frac{\pi}{2}$

(b) (i) Let $R = 1 + \frac{5.5}{1200} = 1.00458\dot{3}$

$A_1 = 4000 \times \left(1 + \frac{5.5}{1200}\right)^1 - W$

$\therefore A_1 = 4000R - W$

$A_2 = (4000R - W + 4000)R - W$

$= 4000R^2 + 4000R - RW - W$

$\therefore A_2 = 4000(R^2 + R) - W(R + 1)$

(ii) Continuing this pattern:

$A_3 = 4000(R^3 + R^2 + R) - W(R^2 + R + 1)$

$\ldots$

$A_{60} = 4000(R^{60} + \ldots + R) - W(R^{59} + \ldots + 1)$

Nick wants $A_{60} = \$80000$.

$\therefore 80000 = 4000R\frac{(R^{60} - 1)}{R - 1} - W \cdot 1\frac{(R^{60} - 1)}{R - 1}$

so, $W = 4000R - 80000\frac{(R - 1)}{(R^{60} - 1)}$

Giving $W = \$2856.91$ per month for Nick to reach his goal.

(c) (i) Arc length $= r\theta$

$= 10 \times \frac{50\pi}{180}$

$= \frac{50\pi}{18}$ cm

(ii) Sector area $= \frac{1}{2}r^2\theta^c$

$= \frac{1}{2} \times 10^2 \times \frac{50\pi}{180}$

$= \frac{250\pi}{18}$

$= \frac{125\pi}{9}$ cm^2

(iii) Perimeter of shaded segment:

= length of chord + length of arc

$= \frac{10\sin 50}{\sin 65} + \frac{50\pi}{18}$

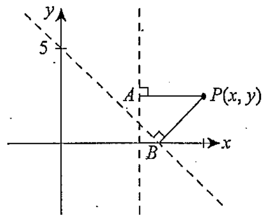
using Sine Rule and arc length

$= 17.17\ldots$

$= 17.2$ cm

Sample answer

(d)



$$AP = PB$$

$$\therefore (x-3) = \frac{|4x+3y-15|}{\sqrt{4^2+3^2}}$$

$$5x-15 = |4x+3y-15|$$

$$\therefore 5x-15 = 4x+3y-15$$

$$\text{or } 5x-15 = -4x-3y+15$$

$\therefore x-3y=0$ and $9x+3y-30=0$, i.e. $3x+y-10=0$ are loci for point P.

Question 16

(a)

$$2\ln(x\sqrt{3}) = \ln(6-7x)$$

$$\ln(x\sqrt{3})^2 = \ln(6-7x)$$

$$\therefore 3x^2 = 6-7x$$

$$3x^2+7x-6=0$$

$$(3x-2)(x+3)=0$$

$$\therefore x = \frac{2}{3} \text{ or } -3 \text{ but } x>0$$

$$\therefore x = \frac{2}{3} \text{ is the only solution}$$

(b) (i) $\frac{dV}{dt} = e^t + 3e^{-t}$

$t=0$, $\therefore \frac{dV}{dt} = e^0 + 3e^0 = 4$ L/min

(ii) $V = e^t - 3e^{-t} + C$

When $t=0$ and $V=0$, $\therefore C=2$

$\therefore V = e^t - 3e^{-t} + 2$

(iii) When $V=10$

$$10 = e^t - 3e^{-t} + 2$$

$$\therefore e^t - 3e^{-t} - 8 = 0$$

(iv) $e^t - 3e^{-t} - 8 = 0$

$$(e^t)^2 - 8(e^t) - 3 = 0$$

$$e^t = \frac{8 \pm \sqrt{64+12}}{2}$$

$$= \frac{8 \pm \sqrt{76}}{2}$$

$$= 4 \pm \sqrt{19}$$

Since $e^t > 0$, $t = \ln(4 \pm \sqrt{19})$

$\therefore t = 2.1233\ldots$ min

t is approximately equal to 177 seconds.

(c) (i)

$$S = \frac{100}{d^2+(x+12)^2} + \frac{100}{d^2+(x-12)^2}$$

$$S = 100[d^2+(x+12)^2]^{-1} + 100[d^2+(x-12)^2]^{-1}$$

$$\frac{dS}{dx} = -100[d^2+(x+12)^2]^{-2} \times 2(x+12)$$

$$-100[d^2+(x-12)^2]^{-2} \times 2(x-12)$$

$$= \frac{-200(x+12)}{[d^2+(x+12)^2]^2} + \frac{-200(x-12)}{[d^2+(x-12)^2]^2}$$

$$= \frac{-200[(x+12)[d^2+(x-12)^2]^2 + (x-12)[d^2+(x+12)^2]^2}{[d^2+(x+12)^2]^2[d^2+(x-12)^2]^2}$$

$= -200\frac{M}{Q}$, as required

Sample answer	Syllabus outcomes and marking guide
(ii) The maximum sound that Mario experiences occurs when $\frac{dS}{dx}=0$.	H5, H9, Band 6 • Correctly states $x=0$ and tests for maximum/minimum .... 2 • Correctly states $x=0$ as a solution 1

$\therefore -200\frac{M}{Q} = 0$

i.e. when $M=0$:

$$2x(x^2+144+d^2+24\sqrt{144+d^2})$$

$$\times(x^2+144+d^2-24\sqrt{144+d^2}) = 0$$

Putting $d=20$, $\therefore d^2=400$

i.e. $2x(x^2+544+24\sqrt{544})(x^2+544-24\sqrt{544})=0$

The only possibility is if $x=0$ as both second and third expressions are positive.

When $x=0$, the sound is minimum or maximum, however on checking:

x	$-$	0	$+$
S	$-$	0	$+$

$\therefore$ Sound level is maximum when $x=0$.

Sample answer	Syllabus outcomes and marking guide
(iii) For Aaron:	H5, H9, Band 6 • Correctly obtains $x=0$, $x=\pm\sqrt{143}$ as solutions and proves min/max .... 2 • Correctly states $x=0$, $x=\pm\sqrt{143}$ as solutions 1

$$2x(x^2+(144+25)+24\sqrt{169})(x^2+169-24\sqrt{169})=0$$

$$x=0,\ x^2+169+24\sqrt{169}=0$$

$$\text{or } x^2+169-24\sqrt{169}=0$$

$\therefore x=0$ or $x^2 = 24\times 13 - 169$

$x^2 = 143$

$x = \pm\sqrt{143}$

Checking each of these:

x	<	$-\sqrt{143}$	>
S^1	+	0	$-$

$\therefore$ maximum

x	$-$	0	+
S^1	$-$	0	+

$\therefore$ minimum

x	<	$+\sqrt{143}$	>
S^1	+	0	$-$

$\therefore$ maximum

Hence, for Aaron the sound level will peak at two locations, when $x=\pm\sqrt{143} \approx \pm 11.96$m