Neap:

HSC Trial Examination 2013

Mathematics

This paper must be kept under strict security and may only be used on or after the morning of Monday 5 August 2013 as specified in the Neap Examination Timetable.

General Instructions

Reading time – 5 minutes

Working time – 3 hours

Write using black or blue pen

Board-approved calculators may be used

A table of standard integrals is provided at the back of this paper

All necessary working should be shown in every question

Section I – 10 marks

10 multiple-choice questions

Section II – 90 marks

6 short-answer questions

Total marks – 100

Attempt Questions 1–16

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2013 HSC Mathematics Examination.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

Section I – 10 marks
Attempt Questions 1 – 10
All questions are of equal value

Answer each question on the multiple-choice answer sheet for Questions 1–10.

1. Evaluate $\frac{3.57\times 0.089}{0.0029+\sqrt{0.036}}$, giving your answer correct to 2 decimal places.

(A) 109.75
(B) 1.65
(C) 0.32
(D) 109.56

2. Given that $6x+2y=13x-3y$. Find the ratio $x:y$.

(A) 5:7
(B) 6:2
(C) 13: –3
(D) 7:5

3. Selling a television for $209.10, a retailer makes a loss of 15% on the cost price. How much must he sell it for to make a 25% profit?

(A) $307.50
(B) $261.38
(C) $258.30
(D) $230.01

4. There are 12 girls and x boys in a group. The probability of selecting boy is $\frac{3}{7}$.
How many additional boys will have to join the group so that the probability of selecting a boy at random from the group is $\frac{4}{5}$?

(A) 9
(B) 12
(C) 30
(D) 39

5. Solve for x, $|2x+1|=7$

(A) $x=-3, x=4$
(B) $x=-3, x=-4$
(C) $x=3, x=-4$
(D) $x=3, x=4$

6. What is the value of $\int_0^{\ln 5} -e^{-x}dx$?

(A) $-\ln 5$
(B) $e^{-\ln 5}$
(C) $-\frac{4}{5}$
(D) $-e^{-\ln 5}$

7. Factorise m^3-5m^2-36m completely.

(A) $m(m^2-5m-36)$
(B) $m(m-6)(m+6)$
(C) $m(m-8)(m+3)$
(D) $m(m-9)(m+4)$

8. Solve the trigonometric equation $\sin 2\alpha=\frac{-\sqrt{3}}{2}$, $-\pi\le\alpha\le 0$

(A) $\frac{-7\pi}{6},\frac{-11\pi}{6}$
(B) $\frac{7\pi}{6},\frac{11\pi}{6}$
(C) $\frac{\pi}{6},\frac{5\pi}{6}$
(D) $\frac{-\pi}{6},\frac{-5\pi}{6}$

9. In ΔABC, $AB = 6$ cm, $BC = 8$ cm and $B = 60°$.
What is the perimeter of ΔABC (answer correct to one decimal place)?

(A) 7.2 cm
(B) 14.0 cm
(C) 21.2 cm
(D) 24.0 cm

10. What are the co-ordinates for the vertex of the quadratic $y=x^2+6x+10$?

(A) (–3,1)
(B) (3,–1)
(C) (–1,3)
(D) (1,–3)

Section II – 90 marks
Attempt Questions 11–16
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) The straight line $y = mx + b$ has the same gradient as the line $2x + 3y = 17$ and passes through the point (6,–2).
Find the values of m and b. **2**

(b) Solve the following equations:

(i) $2^{x^2-4x} - \frac{1}{8} = 0$ **2**

(ii) $e^{2x} - 7e^{x} = -6$ **2**

(iii) $2^{x+1} = 3^{x-2}$ **2**

(c) A game involves randomly selecting two marbles from a bag without replacement. The bag contains 6 red marbles and 9 blue marbles. Three teams A, B and C play the game and the team that select two red marbles first gets 100 points. If a team selects two different-coloured marbles, the marbles are replaced and the team gets a second chance to play. If two red marbles are obtained in the second attempt, the team receives 75 points.
Find the probability that:

(i) team A gets 100 points, **1**

(ii) team B does not get a second chance, **1**

(iii) team C gets 75 points. **1**

(d) Write the exact value of $\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{2\pi}{3}\right)$ with a rational denominator. **2**

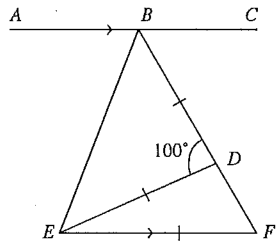
(e) Determine the exact area of the minor segment in a circle of radius 5 cm if the angle subtended at the centre by the arc is $\frac{5}{6}\pi$. **2**

Marks

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Simplify $\frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta}$ **2**

(b) In the diagram, BDF is a straight line and $BD = DE = EF$.
The line ABC is parallel to EF and $\angle BDE = 100°$.



NOT TO SCALE

Calculate, stating reasons;

(i) $\angle EBD$ **1**

(ii) $\angle DEF$ **1**

(iii) $\angle ABE$ **1**

(c) Differentiate the following with respect to x:

(i) $x^2 \cos 2x$ **2**

(ii) $\log_2 3x$ **2**

(d) Evaluate $\int_1^5 \frac{3x}{2x^2 - 1}\,dx$. **2**

(e) A radioactive material is decaying exponentially, $M = M_0 e^{-kt}$.
It takes two years for 100 grams of the material to reduce to 60 grams.

(i) Find an expression for $-k$. **2**

(ii) How long would it take the same mass to reduce to 30 grams? (Answer correct to two decimal places.) **2**

Marks

Question 13 (15 marks) Use a SEPARATE writing booklet.

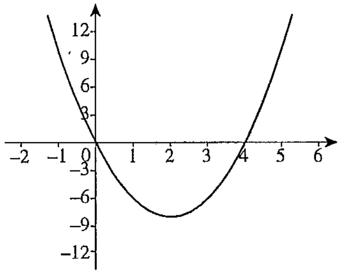
(a) (i) Prove that $\frac{1}{x+2}-\frac{1}{x+3}=\frac{1}{x^2+5x+6}$. **1**

(ii) Hence show that $\int\frac{dx}{x^2+5x+6}=\log_e\left[\frac{x+2}{x+3}\right]+C$ **2**

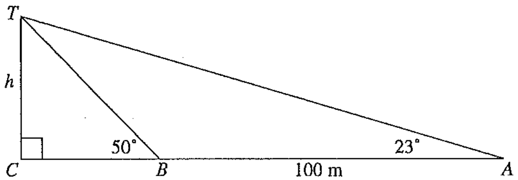
(b) (i) Find the nature of the stationary points of the curve $y=2x^3-3x^2-12x+2$. **2**

(ii) Draw a neat sketch of the curve $y=2x^3-3x^2-12x+2$ showing the stationary points and the point of inflection for $-2\le x\le 3$. **4**

(c) Given the graph of $\frac{dy}{dx}$ below, draw a sketch of a possibility for the graph of y. **2**



(d) The elevation of a tower from point A is 23°. From a point B, 100 metres closer to the tower and on the same horizontal level as A the elevation is 50°.



(i) Show that $\angle BTA = 27°$. **1**

(ii) Show that $BT=\frac{100\sin 23}{\sin 27}$. **1**

(iii) Find the height (h) of the tower. (Answer correct to one decimal place.) **2**

Marks

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) If $y=e^{3x}$, prove that y satisfies the equation **2**

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y=0.$$

(b) Using integration, find the volume of the sphere generated by the rotation of the semi-circle $y=\sqrt{a^2-x^2}$ about the x-axis. **3**

(c) Find the area enclosed between the curve $y=\ln x$ and the lines $y=1$ and $y=3$ using Simpson's rule with one application. Write your answer to two decimal places. **3**

(d) Given that $f''(x)=2$, find the equation of a curve $y=f(x)$ which has a stationary point at the point (1,3). **3**

(e) A given curve has a point of inflection at (1,1) and also passes through (2,3). Find its equation given that $f''(x)=6x+k$. **4**

Marks

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) A particle travels such that its acceleration a cm/s^2 after t seconds is given by $a \frac{dx}{da}$

Marks

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the function $h(x) = \frac{-2x}{x^2 - 1}$.

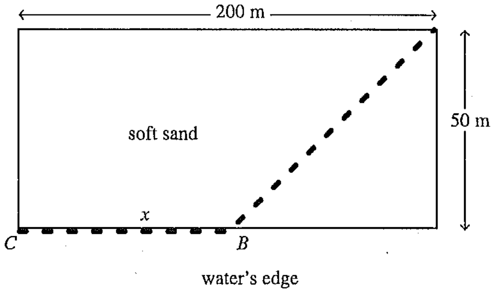
(i) Show that $h(x)$ is an odd function. **1**

(ii) State the domain of $y = h(x)$. **1**

(b) When Cait walks on hard sand near the water's edge she maintains an average speed of 1.4 m/s. On soft sand her average speed is 0.8 m/s.

The beach she is walking on is 50 m wide and 200 m long in the shape of a rectangle.

Cait (C) is at one corner of the beach at the water's edge and she plans to walk to the diagonally opposite corner (A) partly on the hard sand (to B) and the remaining distance on the soft sand.



(i) Show that the time taken (T) for Cait to do her walk is given by the function: **2**

$$T = \frac{x}{1.4} + \frac{\sqrt{50^2 + (200 - x)^2}}{0.8}$$

(ii) Find the value of x that gives the minimum time taken for the walk. **3**

(iii) Find the time taken for the walk. (Answer to nearest second.) **1**

(c) When Archie started his new job \$600 was deposited into a superannuation fund at the beginning of each month. This money was invested at $\frac{1}{2}$% per month, compounding monthly.

Let \$$S$ be the value of his superannuation fund after 20 years, when he retires.

(i) Show that $S = 278\,611$ (to the nearest dollar) **2**

(ii) After retirement, Archie withdraws \$2500 from his account at the end of each month, without making any further deposits. The account continues to earn interest at $\frac{1}{2}$% per month.

Let \$M be the amount remaining in his account n months after his retirement.

Show that $M = (S - 500000) \times 1.005^n + 500000$ **3**

(iii) For how long can he continue to withdraw \$2500 each month from his superannuation account? **2**

Neap:

HSC Trial Examination 2013

Mathematics

Solutions and marking guidelines

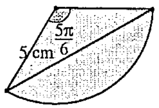
Section I

Sample answer	Syllabus outcomes and marking guide
Question 1 A Using calculator to evaluate the expression you obtain 1.64937453… which is 1.65 to 2 decimal places.	1.1, P2, Band 2-3
Question 2 B $6x+2y=13x-3y$ $-7x=-5y$ $\frac{x}{y}=\frac{5}{7}$ $x:y=5:7$	1.3, P2, Band 2-3
Question 3 A Let Cost Price be $\$C$ $0.85\times C=209.10$ $C=\frac{209.10}{0.85}$ $C=246$ New Price is 1.25×246 $=\$307.50$	1.1, P3, Band 3-4
Question 4 D $\frac{x}{x+12}=\frac{3}{7}$ $7x=3x+36$ $4x=36$ $x=9$ Now, $\frac{x}{x+12}=\frac{4}{5}$ $5x=4x+48$ $x=48$ Therefore, $Extra=48-9=39$	3.1, H5, Band 4-5
Question 5 C $2x+1=\pm 7$ $2x=-1\pm 7$ $x=\frac{-1\pm 7}{2}=3,-4$	1.4, P4, Band 3-4
Question 6 C $\int_0^{\ln 5} -e^{-x}dx=\left[e^{-x}\right]_0^{\ln 5}=e^{-\ln 5}-e^{-0}$ $e^{-\ln 5}-e^{-0}=\frac{1}{e^{\ln 5}}-1=\frac{1}{5}-1=-\frac{4}{5}$	12.5, H3, Band 3-4
Question 7 D m^3-5m^2-36m $=m(m^2-5m-36)$ $=m(m-9)(m+4)$	1.3, P3, Band 2-3

Sample answer	Syllabus outcomes and marking guide
Question 8 D $-\pi \le \alpha \le 0$ $-2\pi \le 2\alpha \le 0$ $\sin 2\alpha = \frac{-\sqrt{3}}{2}$ basic angle $= 60°$ $2\alpha = 60°$ $\alpha = 30°$ $\therefore \alpha = \frac{-\pi}{6}, \frac{-5\pi}{6}$	7.3, H5, Band 2-3
Question 9 C Using cosine rule $AC^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 60°$ $AC^2 = 100 - 48$ $AC = \sqrt{52}$ Therefore Perimeter of $\Delta ABC = 6 + 8 + \sqrt{52} = 21.2111...$	5.5, P4, Band 2-3
Question 10 A $x^2 + 6x = y - 10$ $x^2 + 6x + 9 = y - 1$ $(x+3)^2 = (y-1)$ $V(-3, 1)$	9.3, P5, Band 3-4

Section II

Sample answer	Syllabus outcomes and marking guide
Question 11	
(a) Gradient is $m = -\frac{2}{3}$ $\therefore y - y_1 = m(x - x_1)$ $y + 2 = -\frac{2}{3}(x - 6)$ $3y + 6 = -2x + 12$ $3y = -2x + 6$ $y = -\frac{2}{3}x + 2$	6.2, H5, Band 3-4 • Gives correct answer. . . . . 2 • Correctly finds gradient 1
(b) (i) $2^{x^2-4x} - \frac{1}{8} = 0$ $2^{x^2-4x} = 2^{-3}$ $\therefore x^2 - 4x = -3$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $x = 1, 3$	9.4, H3, Band 4-5 • Gives correct answer. . . . . 2 • Recognises a quadratic equation. 1
(ii) $e^{2x} - 7e^x + 6 = 0$ $(e^x - 6)(e^x - 1) = 0$ $e^x = 1, 6$ $x = \ln 1, \ln 6$ $x = 0, \ln 6$	9.4, H3, Band 4-5 • Gives correct answer. . . . . 2 • Factors quadratic. 1
(iii) $2^{x+1} = 3^{x+2}$ $\ln 2^{x+1} = \ln 3^{x+2}$ $(x+1)\ln 2 = (x+2)\ln 3$ $x(\ln 2 - \ln 3) = 2\ln 3 - \ln 2$ $x = \frac{2\ln 3 - \ln 2}{\ln 2 - \ln 3} = -3.7095...$	9.4, H3, Band 4-5 • Gives correct answer. . . . . 2 • Correctly uses log laws to simplify. . . . 1
(c) (i) P(Team A gets 100 points) $= \frac{6}{15} \times \frac{5}{14} = \frac{1}{7}$	3.3, P4, Band 3-4 • Gives correct answer. 1
(ii) P(Team B no second chance) $= \frac{6}{15} \times \frac{9}{14} + \frac{9}{15} \times \frac{6}{14} = \frac{18}{35}$	3.3, H5, Band 3-4 • Gives correct answer. 1
(iii) P(Team C gets 75 points) $= \frac{18}{35} \times \frac{1}{7} = \frac{18}{245}$	3.3, H5, Band 3-4 • Gives correct answer. 1

	Sample answer	Syllabus outcomes and marking guide
(d)	$\cos\frac{\pi}{4}-\sin\frac{2\pi}{3}$ $=\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}$ $=\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}$ $=\frac{\sqrt{2}-\sqrt{3}}{2}$	13.1, H5, Band 3-4 • Gives correct answer with rational denominator .... 2 • Uses exact trigonometric ratios....... 1
(e)	Area of minor segment: $A=\frac{1}{2}r^2(\theta-\sin\theta)$ $A=\frac{1}{2}\times 5^2(\frac{5\pi}{6}-\sin\frac{5\pi}{6})$ $A=\frac{1}{2}\times 25(\frac{5\pi}{6}-\frac{1}{2})$ $A=\frac{25(5\pi-3)}{12}$ 	13.1, H5, Band 3-4 • Gives correct answer to nearest integer.................... 2 • Uses the formula for the area of the minor segment.................... 1

	Sample answer	Syllabus outcomes and marking guide
Question 12		
(a)	$\frac{1}{\sin^2\theta}-\frac{1}{\tan^2\theta}$ $=\frac{1}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}$ $=\frac{1-\cos^2\theta}{\sin^2\theta}$ $=\frac{\sin^2\theta}{\sin^2\theta}=1$	5.2, P4, Band 3-4 • Correctly simplifies to give result..... 2 • Uses correct trigonometric ratio...... 1
(b) (i)	$\angle EBD=\frac{1}{2}(180-100)=40°$, Angle sum of an isosceles triangle.	2.3, P4, Band 2-3 • Gives correct answer............... 1
(ii)	$\angle DEF=180-2(180-100)=20°$, Angle sum of an isosceles triangle.	2.3, P4, Band 2-3 • Gives correct answer............... 1
(iii)	$\angle ABE=\angle FEB=60°$, Alternate angle in parallel lines are equal.	2.3, P4, Band 2-3 • Gives correct answer............... 1
(c) (i)	$\frac{d}{dx}\left[x^2\cos 2x\right]$ $=2x\cos 2x+x^2\times 2\sin 2x$ $=2x\left[\cos 2x-x\sin 2x\right]$	13.6, H5, Band 4-5 • Gives correct answer............... 2 • Correctly applies product rule 1
(ii)	$\frac{d}{dx}\left[\log_2 3x\right]$ $=\frac{d}{dx}\left[\frac{\ln 3x}{\ln 2}\right]$ $=\frac{1}{\ln 2}\times\frac{3}{3x}$ $=\frac{1}{x\ln 2}$	12.5, H3, Band 4-5 • Gives correct answer............... 2 • Correctly applies change of base theorem..................... 1
(d)	$\int_1^5\frac{3x}{2x^2-1}dx$ $=\frac{3}{4}\int_1^5\frac{4x}{2x^2-1}dx$ $=\frac{3}{4}\left[\ln\left(2x^2-1\right)\right]_1^5$ $=\frac{3}{4}\left[\ln(49)-\ln(1)\right]$ $=\frac{3}{4}\ln 49\ or\ \frac{3}{2}\ln 7$	12.5, H3, Band 4-5 • Gives correct answer............... 2 • Integrates to give a log function 1

Sample answer	Syllabus outcomes and marking guide
(e) (i) $M = M_0 e^{-kt}$ $60 = 100e^{-2k}$ $\frac{3}{5} = e^{-2k}$ $-2k = \ln\left(\frac{3}{5}\right)$ $-k = \frac{1}{2}\ln\left(\frac{3}{5}\right)$	14.2, H4, Band 4-5 • Gives correct expression for $-k$. . . . . . . 2 • Correctly substitutes into decay function. 1
(ii) $M = M_0 e^{-kt}$ $30 = 100e^{-kt}$ $\frac{3}{10} = e^{-kt}$ $-kt = \ln\left(\frac{3}{10}\right)$ $t = \frac{\ln\left(\frac{3}{10}\right)}{-k} = \frac{\ln\left(\frac{3}{10}\right)}{\frac{1}{2}\ln\left(\frac{3}{5}\right)} = \frac{2\ln\left(\frac{3}{10}\right)}{\ln\left(\frac{3}{5}\right)} = 4.7138...$ Hence, it will take approx. 4.7 years to decay to just 30 grams.	14.2, H4, Band 4-5 • Gives correct value for time to decay. . . . . 2 • Correctly substitutes into decay function. 1

Sample answer	Syllabus outcomes and marking guide
Question 13	
(a) (i) $\text{LHS} = \frac{1}{x+2} - \frac{1}{x+3}$ $= \frac{(x+3)-(x+2)}{x^2+5x+6}$ $= \frac{1}{x^2+5x+6}$ $= \text{RHS}$	1.3, P3, Band 2-3 • Gives correct answer. 1
(ii) $\int \frac{dx}{x^2+5x+6}$ $= \int \frac{1}{x+2} - \frac{1}{x+3}dx$ $= \int \frac{1}{x+2}dx - \int \frac{1}{x+3}dx$ $= \ln(x+2) - \ln(x+3) + C$ $= \ln\frac{(x+2)}{(x+3)} + C$	12.5, H5, Band 4-5 • Correctly shows result . . . . 2 • Makes some progress in showing the integral 1
(b) (i) $y = 2x^3 - 3x^2 - 12x + 2$ $y' = 6x^2 - 6x - 12$ $y' = 6(x^2 - x - 2)$ $y' = 6(x-2)(x+1)$ also, $y'' = 12x - 6$ Stationary points at (2,–18) and (–1,9) When $x = 2, y'' = 18 > 0$ therefore a minimum When $x = -1, y'' = -18 < 0$ therefore a maximum.	10.2, H5, Band 4-5 • Correctly shows minimum and maximum points . . . . 2 • Finds coordinates of stationary points. 1

Sample answer | **Syllabus outcomes and marking guide**

(ii) Inflections:

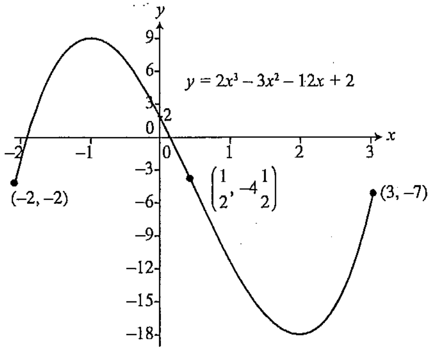
$y'' = 12x - 6 = 0$

$\therefore x = \frac{1}{2}, y = -4\frac{1}{2}$

x	0	0.5	1
y''	−6	0	+6

Since sign change in 2[nd] derivative exists then inflection occurs at $\left(\frac{1}{2}, -4\frac{1}{2}\right)$.

Intercepts:
When $x = 0$, $y = 2$, y-intercept is (0,2).



Note; must label axes, show minimum and maximum points and inflection as well as intercept.

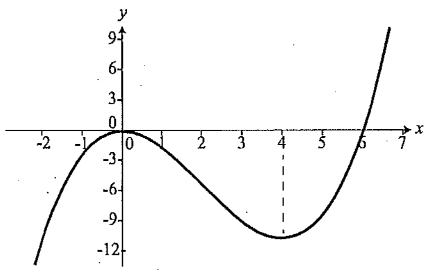
10.4, H5, Band 4-5

- Gives correct curve, showing all features as required 4
- Gives correct sketch but not showing all features 3
- Makes significant progress towards the solution 2
- Draws a sketch that resembles the required curve 1

(c) $\frac{dy}{dx} = (x - 0)(x - 4)$

$y' = x^2 - 4x$

$\therefore y = \frac{1}{3}x^3 - 2x^2 + C$



Note: this is just one possibility.

10.8, H6, Band 4-5

- Draws correct graph clearly showing stationary points when $x = 0$, $x = 4$. . . . 2
- Gives an incorrect cubic function, $a > 0$ 1

Sample answer | **Syllabus outcomes and marking guide**

(d) (i) $\angle ATB + \angle TAB = 50°$, external angle of triangle is sum of internal opposite angles.

$\angle ATB + 23° = 50°$

$\angle ATB = 50 - 23$

$\angle ATB = 27°$

5.5, H5, Band 2-3

- Gives correct answer. 1

(ii) Using the Sine Rule,

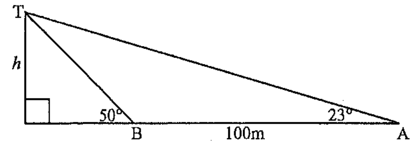
$\frac{BT}{\sin 23} = \frac{100}{\sin 27}$

$BT = \frac{100 \sin 23}{\sin 27}$

5.5, H5, Band 3-4

- Correctly shows BT. 1

(iii) To find the height of Tower:



$\sin 50 = \frac{h}{BT}$

$h = BT \sin 50$

$h = \frac{100 \sin 23}{\sin 27} \times \sin 50$

$h = 65.93...$

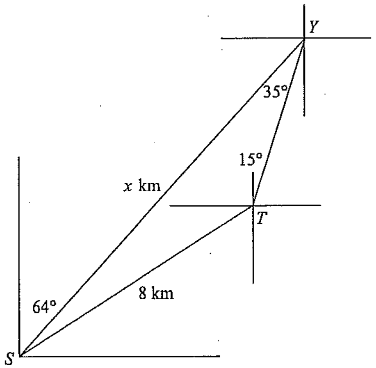
The height of the tower is 65.9 metres

5.5, H5, Band 4-5

- Correctly finds h. 2
- Uses Sine ratio to link h 1

	Sample answer	Syllabus outcomes and marking guide
Question 14		
(a)	$y=e^{3x}, y'=3e^{3x}, y''=9e^{3x}$ $\text{LHS}=\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y$ $=9e^{3x}-2\times3e^{3x}-3\times e^{3x}$ $=9e^{3x}-6e^{3x}-3e^{3x}$ $=0$ $=\text{RHS}$	12.4, H3, Band 3-4 • Shows result as required . . . . 2 • Correctly finds derivatives 1
(b)	$V=\pi\int_{-a}^{a}y^2\,dx$ $V=\pi\int_{-a}^{a}\left(\sqrt{a^2-x^2}\right)^2dx$ $V=\pi\int_{-a}^{a}\left(a^2-x^2\right)dx$ $V=\pi\left[a^2x-\frac{x^3}{3}\right]_{-a}^{a}$ $V=\pi\left[a^2(a)-\frac{a^3}{3}\right]-\pi\left[a^2(-a)-\frac{(-a)^3}{3}\right]$ $V=\pi\left[\frac{2}{3}a^3\right]-\pi\left[-\frac{2}{3}a^3\right]$ $V=\frac{4}{3}\pi a^3$	11.4, H8, Band 4-5 • Correctly substitutes and simplifies . . . 3 • Correctly integrates. . . . . 2 • Correctly substitutes into volume formula 1
(c)	To find area to y-axis need a $f(y)$ Now, $y=\ln x$ so $x=e^y$, that is $f(y)=e^y$ Simpson's rule: $A\approx\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$ $A\approx\frac{3-1}{6}\left[f(1)+4f(2)+f(3)\right]$ $A\approx\frac{1}{3}\left[e^1+4e^2+e^3\right]$ $A\approx17.45$	11.3, H4, Band 4-5 • Gives correct answer to 2 decimal places . . . . 3 • Correctly applies Simpson's rule . . . . . 2 • Changes to $f(y)$ 1
(d)	$f''(x)=2$ $f(x)=2x+C_1$ We know that when $x=1, f'(x)=0$ Hence, $C_1=-2$ $f'(x)=2x-2$ and $f(x)=x^2-2x+C_2$ We know that when $x=1, f(x)=3$ Hence, $C_2=4$ and so $f(x)=x^2-2x+4$	11.2, H5, Band 4-5 • Correctly integrates and finds the second constant. . . . . 3 • Correctly integrates and finds the first constant. . . . . 2 • Correctly integrates once 1

	Sample answer	Syllabus outcomes and marking guide
(e)	$f''(x)=6x+k$ Since there is an inflection at (1,1), then $0=6(1)+k$ $k=-6$ $f''(x)=6x-6$ $f'(x)=3x^2-6x+C_1$ $f(x)=x^3-3x^2+C_1x+C_2$ Since the function passes through (1,1) and (2,3), then $1=1-3+C_1+C_2$ and $3=8-12+2C_1+C_2$ $C_1+C_2=3$ and $2C_1+C_2=7$ Therefore, $C_1=4$ and $C_2=-1$ Hence, $f(x)=x^3-3x^2+4x-1$	10.8, H5, Band 4-5 • Writes the correct expression for $f(x)$ . . 4 • Solves to find C_1 and C_2. . . . . 3 • Correctly integrates to establish $f(x)$ with constants . . . . 2 • Finds the value of k. 1

Sample answer	Syllabus outcomes and marking guide
Question 15	
(a) $\frac{d^2x}{dt^2}=2t$ $\frac{ds}{dt}=t^2+C_1$ When $t=0, \frac{ds}{dt}=0 \therefore C_1=0$ $\frac{ds}{dt}=t^2$ $s=\frac{1}{3}t^3+C_2$ When $t=1, s=10 \therefore C_1=9\frac{2}{3}$ $s=\frac{1}{3}t^3+9\frac{2}{3}$ When $t=3$, $s=\frac{1}{3}\times 27+9\frac{2}{3}=18\frac{2}{3}$ Therefore, particle moves $8\frac{2}{3}$ metres in the next 2 seconds.	14.3, H5, Band 4-5 • Correctly finds how far the particle travels in the next 2 seconds .... 4 • Finds the displacement after 3 seconds .... 3 • Writes a correct expression for $\frac{d^2s}{dt^2}$ .... 2 • Writes a correct expression for $\frac{ds}{dt}$ 1
(b)  $\angle STY=15+90+(90-64)$ $\angle STY=131°$ $\angle TYS=35-15$ $\angle TYS=20°$ $\frac{8}{\sin 20}=\frac{x}{\sin 131}$ $x=\frac{8\sin 131}{\sin 20}$ $x=17.65$ Yacht is ~ 17.7 km from Sydney	5.4, H5, Band 4-5 • Correctly applies Sine Rule to obtain answer .... 4 • States need to use Sine Rule .... 3 • Finds $\angle TYS$ and $\angle STY$ .... 2 • Finds $\angle STY$ 1

Sample answer	Syllabus outcomes and marking guide
(c) Distance travelled by pendulum $=3+2+\frac{4}{3}+\cdots$ $S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{3}{1-\frac{2}{3}}$ $S_\infty=\frac{9}{3-2}=9$ The pendulum swings a total distance of 9 metres.	7.3, H5, Band 3-4 • Gives the correct answer .... 2 • Expresses series or identifies infinite sum 1
(d) (i) $\frac{2}{x-a}+\frac{k}{x}+\frac{8}{x+a}=0$ $\frac{2x(x+a)+k(x-a)(x+a)+8x(x-a)}{x(x-a)(x+a)}=0$ $\frac{2x^2+2ax+kx^2-ka^2+8x^2-8ax}{x(x-a)(x+a)}=0$ $\frac{(10+k)x^2+(-6a)x-(ka^2)}{x(x-a)(x+a)}=0$ $(10+k)x^2+(-6a)x-(ka^2)=0$	9.2, P4, Band 4-5 • Correctly writes the given expression as a quadratic .... 2 • Correctly writes expression with common denominator 1
(ii) For the roots to be equal, $\Delta=0$, i.e., $36a^2+4(10+k)(ka^2)=0 \qquad \div 4a^2$ $9+(10+k)k=0$ $k^2+10k+9=0$ $(k+9)(k+1)=0$ $k=-1,-9$	9.3, P4, Band 5-6 • Correctly finds the values for k .... 2 • Writes an expression for $\Delta=0$ 1
(iii) Using, $k=-1$, the larger value: $(10+k)x^2+(-6a)x-(ka^2)=0$ $(10-1)x^2+(-6a)x-(-1a^2)=0$ $9x^2-6ax+a^2=0$ $(3x-a)(3x-a)=0$ $\therefore x=\frac{a}{3},\frac{a}{3}$	9.3, P4, Band 5-6 • Using $k=-1$ to find values for x 1

Sample answer	Syllabus outcomes and marking guide
Question 16	
(a) (i) $h(x)=\dfrac{-2x}{x^2-1}$ $h(-x)=\dfrac{-2(-x)}{(-x)^2-1}$ $h(-x)=\dfrac{2x}{x^2-1}=-\dfrac{-2x}{x^2-1}=-h(x)$ Therefore, $h(x)$ is odd.	4.2, P5, Band 3-4 • Shows $h(x)$ is an odd function. 1
(ii) Domain of $h(x)$ is: $x\in R,\ x\neq\pm1$	4.1, P5, Band 3-4 • Correctly writes the domain for $h(x)$ 1
(b) (i) $T=\dfrac{d_1}{s_1}+\dfrac{d_2}{s_2}$ $d_1=x,\ s_1=1.4,\ d_2^2=50^2+(200-x)^2,\ s_2=0.8$ $T=\dfrac{x}{1.4}+\dfrac{\sqrt{50^2+(200-x)^2}}{0.8}$	10.6, H5, Band 4-5 • Shows required result . . . . 2 • Uses distance-speed relationship to establish total time 1
(ii) $T=\dfrac{x}{1.4}+\dfrac{\sqrt{50^2+(200-x)^2}}{0.8}$ $T=\dfrac{10x}{14}+\dfrac{10\sqrt{x^2-400x+42500}}{8}$ $\dfrac{dT}{dx}=\dfrac{10}{14}+\dfrac{5(2x-400)}{8\sqrt{x^2-400x+42500}}$ For a stationary point $\dfrac{dT}{dx}=0$ $\dfrac{10}{14}=\dfrac{-10(x-200)}{8\sqrt{x^2-400x+42500}}$ $8\sqrt{x^2-400x+42500}=14(200-x)$ $33x^2-13200x+1280000=0$ $x\approx165.18$ (since $x<200$) when $x=160$ $\dfrac{dT}{dx}=-0.0665\ldots<0$ when $x=170$ $\dfrac{dT}{dx}=+0.071\ldots>0$ Hence, $x=165.18$ is a minimum.	10.6, H5, Band 5-6 • Writes the correct result . . . . 3 • Establishes quadratic to solve. . . . . 2 • Correctly differentiates 1
(iii) At $x=165.18$, $T=194.1$ seconds (or 3 minutes and 14 seconds)	10.6, H5, Band 3-4 • Gives the correct value for time 1

Sample answer	Syllabus outcomes and marking guide
(c) (i) $S=600\times1.005^{240}+600\times1.005^{239}+\ldots+600\times1.005^{1}$ $S=600\left(1.005+1.005^2+1.005^3+\ldots+1.005^{240}\right)$ Using sum of a geometric series, $S=600\times\dfrac{1.005\left(1.005^{240}-1\right)}{1.005-1}$ The amount in the superannuation fund after 20 years is \$278 611.	7.5, H5, Band 5-6 • Correctly shows the value of superannuation . . . . 2 • Writes the amount of superannuation as a geometric series. 1
(ii) $A_1=S\times1.005-2500$ $A_2=S\times1.005^2-2500\times1.005-2500$ $A_3=S\times1.005^3-2500\times1.005^2-2500\times1.005-2500$ $\vdots$ $A_n=S\times1.005^n-2500\times1.005^{n-1}-\cdots-2500\times1.005-2500$ $A_n=S\times1.005^n-2500\left[1.005^{n-1}+\cdots+1.005+1\right]$ $A_n=S\times1.005^n-2500\times\dfrac{1\left(1.005^n-1\right)}{1.005-1}$ $A_n=S\times1.005^n-500000\left(1.005^n-1\right)$ $A_n=(S-500000)\times1.005^n+500000$	7.5, H5, Band 5-6 • Shows the result required . . . . 3 • Correctly uses the sum of geometric series . . . . 2 • Makes some progress towards writing out the series showing interest and withdrawals. 1
(iii) Putting $A_n=0$ $1.005^n=\dfrac{-500000}{(S-500000)}$ $1.005^n=\dfrac{500000}{(500000-278611)}$ $n\log1.005=\log\left[\dfrac{500000}{(500000-278611)}\right]$ $n=\dfrac{\log\left[\dfrac{500000}{(500000-278611)}\right]}{\log1.005}$ $n=163.344\ldots$ That is, Archie can withdraw money at a rate of \$2500 per month for 163 months or 13.6 years	7.5, H5, Band 5-6 • Writes the correct answer. . . . . 2 • Makes progress towards solving the log equation. 1