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CRANBROOK SCHOOL

+SOLNS

Year 12 Mathematics Extension 1

HSC Trial Examination- July, 2010

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

②

Total Marks – 84

Attempt questions 1 – 7

All questions are of equal value

Question 1 (12 Marks) **Start a NEW writing booklet** **Marks**

(a) The interval AB, where A is $(1, -3)$ and B is $(8, 11)$ is divided internally in the ratio 3:4 by the point $P(x, y)$. Find the values of x and y. **2**

$x_1\ y_1$ $x_2\ y_2$ $m\ n$

(b) Use the substitution $u = 3t + 1$ to evaluate $\int_0^1 \frac{dt}{\sqrt{3t+1}}$ **3**

(c) Solve the inequality; $\frac{2}{3x-2} \geq 1$ **3**

(d) Differentiate $y = x\cos^{-1}\frac{x}{4}$ **2**

(e) Evaluate $\lim_{x\to 0}\left(\frac{\sin 2x}{5x}\right)$ **2**

3

Question 2 (12 Marks) **Start a NEW writing booklet** **Marks**

(a) Oil is leaking into the Gulf of Mexico at a rate of 330 m^3 per hour. The oil rises to the surface of the sea and creates a circular slick of thickness 0.1mm.

(i) Show that the area of the slick is increasing at a constant rate and determine that rate in m^2 per hour. **2**

(ii) Find the radius of the slick, to the nearest kilometre, at the point when the rate of increase of the radius is 130 metres per hour. **2**

(b) Prove that: $\dfrac{\cos 2A}{\cos A}+\dfrac{\sin 2A}{\sin A}\equiv\dfrac{4\cos^2 A-1}{\cos A}$ **3**

(c) For the function, $y=3\sin^{-1}2x$

(i) State the domain. **1**

(ii) State the range. **1**

(d) α, β and γ are the roots of the equation $2x^3-6x^2+4x+2=0$.
Calculate:

(i) $\alpha+\beta+\gamma$ **1**

(ii) $\alpha\beta+\gamma\alpha+\beta\gamma$ **1**

(iii) $\alpha^2+\beta^2+\gamma^2$ **1**

4

Question 3 (12 Marks) **Start a NEW writing booklet** **Marks**

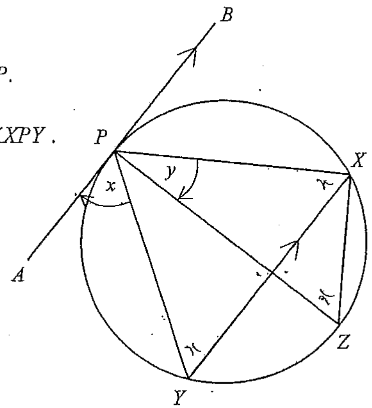
(a) Find the equation of the normal to the curve $y=5+2e^{3x}$ at $x=0$ **3**

(b) The roots of the equation $x-2+\ln x=0$ can be found by using the two functions $f(x)=\ln x$ and $g(x)=2-x$, and solving the equation $f(x)=g(x)$.

(i) Draw the graphs of $f(x)=\ln x$ and $g(x)=2-x$ on the same set of axes. Show that a root of the equation lies between $x=1$ and $x=2$. **2**

(ii) By taking $x=1.5$ as a first estimate of this root, use one application of Newton's Method to find a better approximation for the root, correct to 3 decimal places. **2**

(c) AB is tangent to a circle, touching the circle at P.
YX is a chord of the circle parallel to AB.
Z is a point on the circle such that PZ bisects $\angle XPY$.
Let $x=\angle APY$ and $y=\angle XPZ$.



Show, giving reasons, that:

(i) $\angle APY=\angle PXY$ **1**

(ii) $\angle YPZ=\angle YXZ$ **1**

(iii) $x+y=90°$ **2**

(iv) PZ is a diameter of the circle. **1**

Question 4 (12 Marks) **Start a NEW writing booklet** **Marks**

(a) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ **3**

(b) Newton's law of cooling states that for an object placed in surroundings at constant temperature, the rate of change of the temperature of the object is proportional to the difference between the temperature of the object and its surroundings.

i.e. $\frac{dT}{dt} = k(T - A)$

where A is the temperature in $°C$ of the surroundings, and T is the temperature in $°C$ of the object at a time t in minutes.

(i) Show that $T = A + Ce^{kt}$ satisfies Newton's law of cooling, where C and k are constants. **1**

Mr Lipton places a cup of tea that has a temperature of $95°C$ on a bench, after 6 minutes it falls in temperature to $80°C$. The air temperature is constant at $18°C$.

(ii) Find the values of C and k. (Write k to 3 decimal places.) **2**

(iii) Mr Lipton can't drink tea that has fallen below $40°C$. How long does he have to drink his tea? Write your answer to the nearest second. **2**

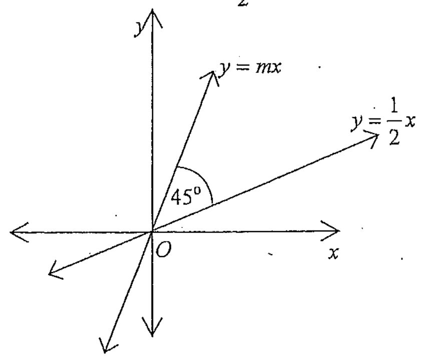
(c) Prove by mathematical induction that; **4**

$$\frac{1}{1\times 4} + \frac{1}{4\times 7} + \frac{1}{7\times 10} + \ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

Question 5 (12 Marks) **Start a NEW writing booklet** **Marks**

(a) The angle between the lines $y = mx$ and $y = \frac{1}{2}x$ is $45°$, as shown in the diagram. **3**

Find the value of m.



(b) The point $P(10t, 5t^2)$ lies on a parabola with equation $x^2 = 20y$.

(i) Derive the equation of the tangent at the point P. **2**

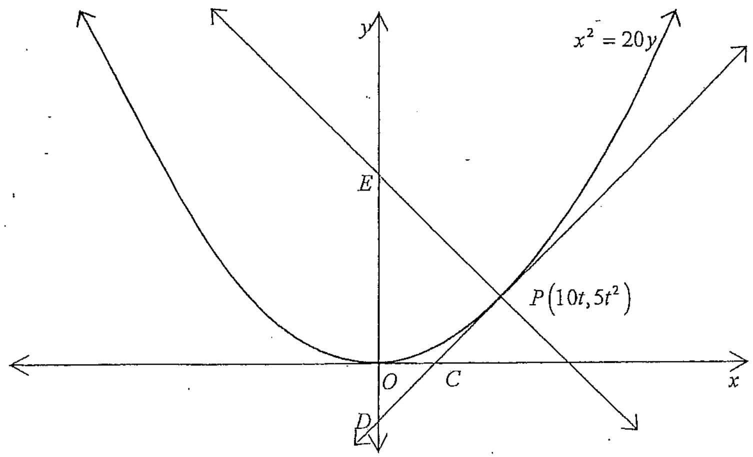
(ii) Given that the equation of the normal at the point P is $ty - 5t^3 = -x + 10t$, find the point E, where the normal cuts the y-axis. (Note: $t \neq 0$ for the intercept to be a single point.) **1**

(iii) Find the coordinates of S, the focus. **1**

(iv) Given that the tangent cuts the y-axis at the point $D(0, -5t^2)$, show that $ES = SD$. **2**

(v) Show the midpoint of PE is $M(5t, 5 + 5t^2)$. (Note: $t \neq 0$) **1**

(vi) Find the locus of the midpoint M as the point P moves along the parabola. (Note: $t \neq 0$) **2**

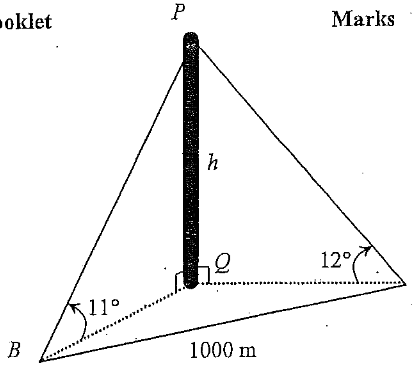


Question 6 (12 Marks) **Start a NEW writing booklet** **Marks**

(a) The angle of elevation of a tower PQ of height h metres at a point A due east of Q is $12°$.

From another point B, the bearing of the tower is 051°T and the angle of elevation is $11°$.

The points A and B are 1000 metres apart and on the same level as the base Q of the tower.



(i) Show that $\angle AQB = 141°$. 1

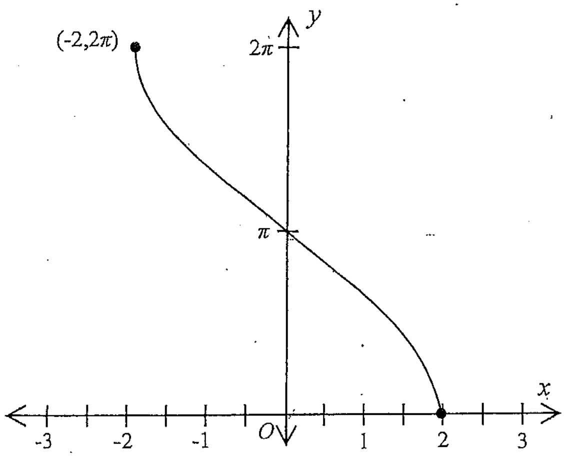
(ii) Consider the triangle APQ and show that $AQ = h\tan 78°$. 1

(iii) Find a similar expression for BQ. 1

(iv) Use the cosine rule in the triangle AQB to calculate h to the nearest metre. 3

(b) Find the exact value of $\sec 15°$. 2

(c)



(i) Write down the equation of the inverse trigonometric graph shown. 2

(ii) Calculate the area bounded by the curve and the x and y axes between $0 \le y \le \pi$. 2

Question 7 (12 Marks) **Start a NEW writing booklet** **Marks**

(a) The displacement, x metres, of a particle moving in a straight line is given by

$x = \sqrt{3}\sin 2t - \cos 2t$, where t is measured in seconds.

(i) Rewrite the equation of motion in the form $x = a\sin(2t-\alpha)$. 2

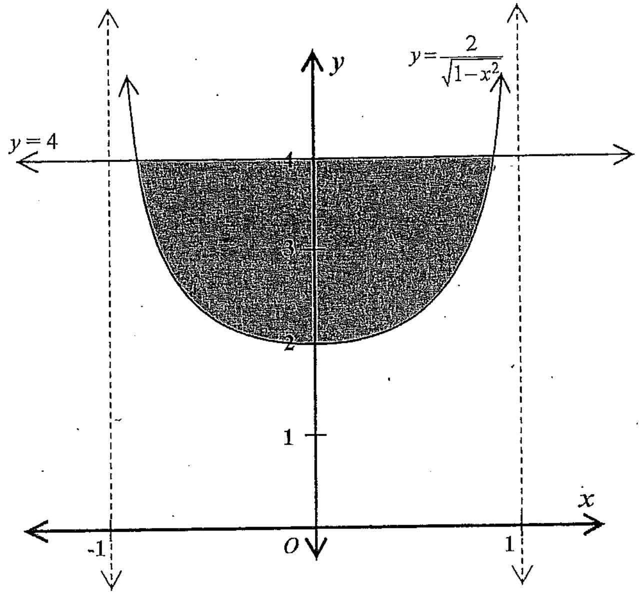
(ii) Hence, or otherwise, show that the particle is moving in Simple Harmonic Motion, write its period and the amplitude of the motion. 2

(iii) Find the maximum speed of the particle. 1

(iv) Sketch a velocity-time graph of the particle over one period of its motion. 2

(v) Calculate how far the particle travelled in the first 2π seconds. 2

(b) The following diagram shows the functions: $y = 4$ and $y = \dfrac{2}{\sqrt{1-x^2}}$. 3



Calculate the exact area between the two functions, as shaded.

END OF TEST

EXT 1 MATHEMATICS TRIAL SOLUTIONS

✓ = 1 MARK   CRANBROOK 2010

1 a) (2)

$$(x,y) = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right)$$

Well done

$$= \left(\left(\frac{4\times 1 + 3\times 8}{7}\right), \frac{4\times -3 + 3\times 11}{7}\right)$$

$$= (4, 3) \quad ✓✓$$

b) $u = 3t+1 \quad \therefore \frac{du}{dt} = 3$

$dt = \frac{du}{3}$

when $t=1, u=4$; $t=0, u=1$ ✓

$$\int_0^1 \frac{dt}{\sqrt{3t+1}} = \int_1^4 \frac{du}{3\sqrt{u}}$$

$$= \left[\frac{1}{3}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4 \quad ✓$$

Sides

$$= \left[\frac{2}{3}\sqrt{u}\right]_1^4$$

$$= \frac{2}{3}(2-1) = \frac{2}{3} \quad ✓$$

c) (3)

$$\frac{2}{3x-2} \geq 1 \quad \text{NOTE } x \neq \frac{2}{3}$$

→ NEEDED when giving final answer !!

1ST MAKE RHS ZERO

$$\frac{2}{3x-2} - 1 \geq 0$$

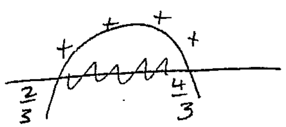
NOW PUT ON COMMON DENOMINATOR

$$\frac{2-(3x-2)}{(3x-2)} \geq 0$$

ONLY NOW multiply by $(3x-2)^2$

$$(3x-2)^{\cancel{2}}\frac{(4-3x)}{\cancel{(3x-2)}} \geq 0 \quad ✓$$

NOTE IT IS ALREADY FACTORISED!
The majority of those who expanded earlier made mistakes.



$$\frac{2}{3} < x \leq \frac{4}{3} \quad ✓ \quad ✓$$

d)

$$\frac{d}{dx}(x)(\cos^{-1}\frac{x}{4}) = x\left(\frac{-1}{\sqrt{4^2-x^2}}\right) + \cos^{-1}\frac{x}{4}(1)$$

$$= \frac{-x}{\sqrt{16-x^2}} + \cos^{-1}\left(\frac{x}{4}\right)$$

Done V. well although a few did not know the rule

either learn $\frac{d}{dx}\cos^{-1}\frac{x}{a} = \frac{-1}{\sqrt{a^2-x^2}}$ OR use chain rule and simplify

e)

$$\lim_{x\to 0}\left(\frac{\sin 2x}{5x} \times \frac{2}{2}\right) = \lim_{x\to 0}\frac{2}{5}\left(\frac{\sin 2x}{2x}\right)$$

$$= \frac{2}{5}\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\right)$$

$$= \left(\frac{2}{5}\right)\times 1 = \frac{2}{5}$$

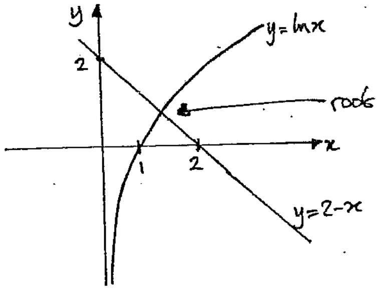
This working should be shown.

3. (a) $y = 5 + 2e^{3x}$ so $\frac{dy}{dx} = 6e^{3x}$ and at $x=0$, $\frac{dy}{dx} = 6$ (11)

ie. eqn. of normal has gradient $-\frac{1}{6}$ and passes through $(0,7)$ [3]

$y - 7 = -\frac{1}{6}(x - 0)$ ie. $y = -\frac{1}{6}x + 7$ or $x + 6y - 42 = 0$

(b) (i)



[2]

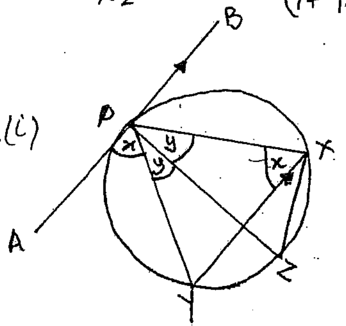
(ii) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ where $x_1 = 1.5$, $f(x) = x - 2 + \ln x$

$f'(x) = 1 + \frac{1}{x}$

$\therefore f(x_1) = (1.5 - 2 + \ln 1.5)$ & $f'(x_1) = (1 + \frac{1}{1.5})$

$x_2 = 1.5 - \frac{(1.5 - 2 + \ln 1.5)}{(1 + \frac{1}{1.5})} = 1.556720935 \approx 1.557$ (3.d.p.) [2]

(c) (i)



Alternate segment theorem states that the angle $\angle APY$ between a tangent and a chord is equal to the angle $\angle PXY$ subtended by the chord at the circumference in the alternate segment. Hence $\angle APY = \angle PXY$. [1]

(ii) $\angle YPZ = \angle YXZ$ since angles at the circumference, standing on the same arc, are equal.
(or angles subtended by a chord YZ at the diameter, in the same segment, are equal). [1]

(iii) $\angle XYP = \angle APY = x$ (alternate angles on parallel lines are equal)
Hence interior angles in $\Delta XYP = 2x + 2y = 180°$ (angle sum of Δ is 180°)
$\therefore x + y = 90°$ as required. [2]
(alternatively $\angle BPX = \angle PXY = x$)
and angles on $APB = 2x + 2y = 180°$)

(iv) $\angle APZ = x + y = 90°$ from part (iii). Hence PZ is a diameter, since a tangent meets a diameter at 90° [1]

[12 marks]

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

4. (a) $\int_0^{\pi/2} \sin^2 x \, dx = \frac{1}{2}\int_0^{\pi/2}(1 - \cos 2x)\,dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2}$ (12)

$= \frac{1}{2}\left(\frac{\pi}{2} - 0 - (0 - 0)\right) = \frac{\pi}{4}$ [3]

(b)(i) Either: $T = A + Ce^{kt}$

$\therefore \frac{dT}{dt} = kCe^{kt} = k(T - A)$ as required

OR: $\frac{dT}{dt} = k(T - A)$

$\therefore \int \frac{dT}{T-A} = k\int dt$ [1]

$\ln|T - A| = kt + C_1$

$T - A = e^{kt}e^{C_1}$

$\therefore T = A + Ce^{kt}$ where $C = e^{C_1}$

(ii) at $t=0$, $T=95$ $\therefore 95 = 18 + C$ $\therefore C = 77$

at $t=6$, $T=80$ $\therefore 80 = 18 + Ce^{6k}$

$62 = 77e^{6k}$

$6k = \ln\frac{62}{77}$ $\therefore k = \frac{1}{6}\ln\frac{62}{77} \approx -0.036$, $C = 77$ [2]

(iii) $40 = 18 + 77e^{kt}$ $\therefore e^{kt} = \frac{22}{77}$ $kt = \ln\frac{22}{77}$

$\therefore t = \frac{1}{k}\ln\frac{22}{77} \approx 34.69119$ mins $\approx$ 34 mins 41 secs [2]

(c) for $n=1$, $\frac{1}{1\times4} = \frac{1}{3+1} = \frac{1}{4}$, so it works for $n=1$

Let it be true for some $n=k$, ie. $\frac{1}{1\times4} + \frac{1}{4\times7} + \frac{1}{7\times10} + \cdots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1}$

then for $n=k+1$, $\frac{1}{1\times4} + \frac{1}{4\times7} + \cdots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3(k+1)-2)(3(k+1)+1)} = \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)}$

$= \frac{k(3k+4)+1}{(3k+1)(3k+4)} = \frac{3k^2+4k+1}{(3k+1)(3k+4)} = \frac{\cancel{(3k+1)}(k+1)}{\cancel{(3k+1)}(3k+4)}$

$= \frac{(k+1)}{3(k+1)+1}$ as expected. Hence it works for $n=k+1$

Thus, since it is true for $n=1$ and if it is true for $n=k$ then it is also true for $n=k+1$, hence it is true for $n=2,3,4$ and so on for all $n \in \mathbb{Z}^+$ by induction. [4]

3-Unit Trial HSC 2010 – Cranbrook School
Marker's Notes for Question 3 and 4

3 (a) Mostly well done.

3 (b) **NEWTON'S METHOD**

3 (b) (i) Full marks were not awarded only for a correct graph. A brief comment about the root being between 1 and 2 was all that was necessary for the 2nd mark.

3 (b) (ii) 3 decimal places MEANS 3 decimal places!

3 (c) **GEOMETRY**

In this question, I was quite hard on the correct phrasing and writing of reasons, even though the HSC markers are actually more strict.

In general, it is not sufficient to state a summary of the result, like "*angle in alternate segment*" or "*alternate segment theorem*". HSC markers require a **sentence**, which includes a verb (e.g. "**is equal to**").

3 (c) (i) The mark was awarded for: "*The angle between a tangent and a chord is equal to the angle in the alternate segment*" even though the full statement should be "*The angle between a tangent and a chord is equal to the angle subtended on the chord in the alternate segment*"

"*opposite angle*" was not awarded the mark

3 (c) (ii) Similar

3 (c) (iii) You cannot assume PZ is a diameter in this section. You are working towards proving this proposition.

Geometric reasoning requires you to express your working with clarity and organisation. In general, if you label a point with a new letter, you should redraw the diagram for the marker to follow.

3 (c) (iv) Similar

4 (a) **TRIG INTEGRATION**

Mostly well done.

4 (b) **EXPONENTIAL DECAY**

4 (b) (i) Mostly well done.

4 (b) (ii) 3 decimal places MEANS 3 decimal places!

4 (b) (iii) It is usually better to use exact values form a previous question rather than to retype a decimal approximation into your calculator (although in this instance you lost no marks). Easy marks were lost by those who didn't round to the nearest second.

4 (c) **MATHEMATICAL INDUCTION**

See my comments on your proof. You MUST phrase the working out in a logical manner. Statements like "Therefore $n = k + 1$" are nonsensical.

5.

(a) $\tan\theta = \tan 45° = 1 = \left|\frac{m-\frac{1}{2}}{1+\frac{1}{2}m}\right| = \frac{2m-1}{2+m}$

$\therefore 2m-1 = 2+m$ So $m-1=2$ $\therefore \underline{m=3}$ [3]

(b) (i) $x^2=20y$

$2x = 20\frac{dy}{dx}$ $\therefore \frac{dy}{dx} = \frac{x}{10}$ at P, $\frac{dy}{dx} = \frac{10t}{10} = t$

$\therefore$ Tangent at P: $y-5t^2 = t(x-10t)$ or $\underline{y = xt-5t^2}$ [2]

(ii) at $x=0$, $ty-5t^3=10t$

$ty = t(10+5t^2)$ $\therefore y = 10+5t^2$ $(t\neq 0)$

Hence $\underline{E(0, 10+5t^2)}$ [1]

(iii) $x^2=4ay=20y$ $\therefore a=5$ Hence focus is $\underline{S(0,5)}$ [1]

(iv) $ES = 5+5t^2$ from (ii) & (iii).

$SD = 5-(-5t^2) = 5+5t^2 = ES$ as required. [2]

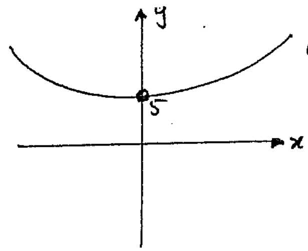
(v) $P(10t, 5t^2)$ $E(0, 10+5t^2)$ $\therefore M\left(\frac{10t+0}{2}, \frac{10+5t^2+5t^2}{2}\right)$

So $M(5t, 5+5t^2)$ as required. [1]

(vi) $x=5t$ $y=5+5t^2$

$\frac{x}{5}=t$ $y=5+5\left(\frac{x}{5}\right)^2 = 5+\frac{x^2}{5}$ $\Big\}$ $t\neq 0$

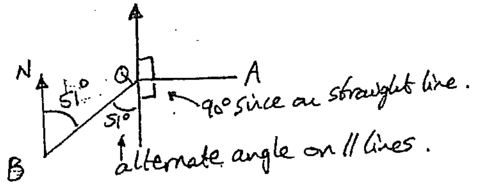
$\therefore$ locus is $\underline{y = \frac{x^2+25}{5}}$, $x\neq 0, y\neq 5$ [2]



locus of M.
(parabolic).

[12 Marks]

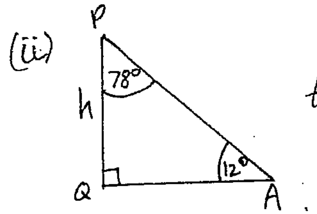
6 (a) (i) Plan view:



[1]

from diagram → $\angle AQB = 51° + 90° = 141°$ as required

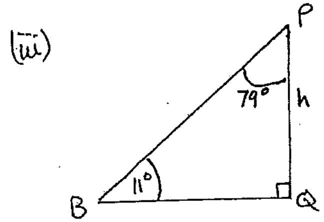
(ii)



$\tan 12° = \frac{h}{AQ}$ but $\tan 78° = \frac{AQ}{h}$

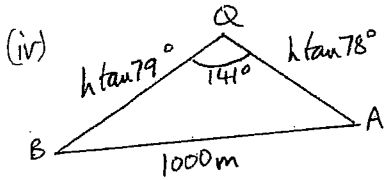
$\therefore AQ = h\tan 78°$ as required. [1]

(iii)



$\tan 79° = \frac{BQ}{h}$ $\therefore$ $BQ = h\tan 79°$ [1]

(iv)



$1000^2 = h^2\tan^2 79° + h^2\tan^2 78° - 2h^2\tan 79 \tan 78 \cos$

$h^2(\tan^2 79° + \tan^2 78° - 2\tan 79° \tan 78° \cos 141°) = 1000000$

Hence $h = \dfrac{1000}{\sqrt{\tan^2 79° + \tan^2 78° - 2\tan 79° \tan 78° \cos 141°}}$ [3]

$h \approx 107.695826$ ≈ 108m (nearest m)

(b) $\sec 15° = \frac{1}{\cos 15°} = \frac{1}{\cos(45°-30°)}$ ... ①

$\cos(45°-30°) = \cos 45° \cos 30° - \sin 45° \sin 30° = \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$ [2]

Therefore from ①, $\sec 15° = \frac{2\sqrt{2}}{\sqrt{3}-1} = \frac{2\sqrt{2}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{2\sqrt{6}+2\sqrt{2}}{2} = \sqrt{6}+\sqrt{2}$

(c) (i) $y = 2\cos^{-1}\left(\frac{x}{2}\right)$ by inspection of domain & range. [1]

$2\cos\frac{y}{2} = x$

(ii) $A = \int_0^2 2\cos^{-1}\left(\frac{x}{2}\right)dx = \int_0^{\pi} 2\cos\left(\frac{y}{2}\right)dy = \left[\sin\frac{y}{2}\right]_0^{\pi} = 1$ [2]

Notes on Q 5+6

5 a) Refer back to diagram and note gradient is positive

b i) ii) ii) Very well done

iv) Again reference to diagram makes these very easy marks. No need for distance formula.

v) Well done

vi) Disappointing that some did not know what to do with this simple elimination of the parameter

Those students need to do some serious work as more difficult parametric Q's are likely in the HSC.

6 a) When asked to show/explain/prove overdo the steps rather than underdo!

apart from this (a) was well done.

b) Easy marks for most but those who did not know $\sec\theta = \frac{1}{\cos\theta}$ and $\cos(a-b) = \cos a\cos b + \sin a \sin b$ it's time to LEARN these!!! ☹

c) Likewise for this section learn the basic rules and structures for the inverse trig functions and do remember that the differentiation of $\cos^{-1}x$ is $\frac{-1}{\sqrt{1-x^2}}$

NOT the integral ☺ ☺

7. (a) (i) $x = \sqrt{3}\sin 2t - \cos 2t = R\sin(2t - \alpha)$

$= R\sin 2t\cos\alpha - R\cos 2t\sin\alpha$

Comparing coefficients of $\sin 2t$ & $\cos 2t$, $\sqrt{3} = R\cos\alpha$ & $1 = R\sin\alpha$

Hence $R = 2$ by Pythagoras and $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ [1]

Thus $x = 2\sin\left(2t - \frac{\pi}{6}\right)$

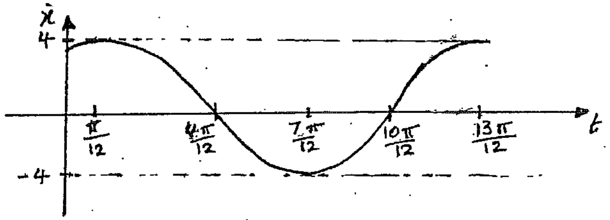
(ii) $\dot{x} = 4\cos\left(2t - \frac{\pi}{6}\right)$ & $\ddot{x} = -8\sin\left(2t - \frac{\pi}{6}\right)$

Hence $\ddot{x} = -4x$, S.H.M. cf: $\ddot{x} = -n^2 x$, $n = 2$

where amplitude $= 2$ and Period $= \frac{2\pi}{n} = \pi$ seconds [2]

(iii) $|\dot{x}|_{max} = 4$ m/s by inspection of $\dot{x} = 4\cos\left(2t - \frac{\pi}{6}\right)$ [1]

(iv)



[2]

(v) since period $= \pi$ seconds, the particle will have undergone two complete oscillations in 2π seconds.

In each oscillation, the particle moves through $4 \times$ amplitude $= 8$m

Hence total distance travelled in 2π seconds $= 16$M [2]

(b) Intersections are when $\frac{2}{\sqrt{1-x^2}} = 4 \quad \therefore 1 - x^2 = \frac{1}{4}$

ie. $x = \pm\frac{\sqrt{3}}{2}$

$$\text{Area} = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \left(4 - \frac{2}{\sqrt{1-x^2}}\right)dx = 2\int_0^{\frac{\sqrt{3}}{2}} 4 - \frac{2}{\sqrt{1-x^2}}\,dx \text{ by symmetry.}$$

$$\text{Area} = 2\Big[4x - 2\sin^{-1}x\Big]_0^{\frac{\sqrt{3}}{2}} = 2\left[2\sqrt{3} - 2\left(\frac{\pi}{3}\right) - (0 - 0)\right]$$

$$\text{Area} = 4\sqrt{3} - \frac{4\pi}{3} = \frac{12\sqrt{3} - 4\pi}{3} = \frac{4(3\sqrt{3} - \pi)}{3}$$ [4]

Question 7

18

(a) (i) Good

(ii) Many did not read the question (amplitude=?, period=?). When showing SHM, it is necessary to show that $\ddot{x} \propto x$, rather than simply stating that sinusoidal functions exhibit SHM. Some students wrote the amplitude as $-2 \le x \le 2$, which is wrong. Amplitude=2.

(iii) This was OK, but was only worth 1 mark and could be done by inspection, looking at the velocity function $\dot{x} = 4\cos(2t - \frac{\pi}{6})$, which has amplitude 4. Many used calculus and others found $t = \frac{\pi}{12}$ for $x = 0$ and substituted into the velocity function. Always look for quick and easy methods (especially when there is only 1 mark given!)

(iv) There were some reasonable graphs, but generally most students exhibited poor drawing skills, most not using pencil and ruler. Axes were unlabelled, values were absent on both axes. Many drew either a regular (unshifted) cosine graph and others drew a sine graph. Few students correctly identified the phase shift of $\frac{\pi}{12}$ in the positive t direction.

(v) This was very poor. Most simply substituted the value of $t = 2\pi$ into the equation for x and obtained -1. This part showed a lack of understanding of the concept of SHM (ie. a body oscillating with amplitude 'a' through a fixed point and travelling a total distance of $4a$ each period. This was a very easy 2 marks for those students who really understood this concept.

(b) This was well done by some, but many made the mistake of trying to use $\int_2^4 f(y)dy$ but couldn't successfully integrate the resulting integral. Others integrated the function between $x = -1$ and $x - 1$.