Name: ____________

Teacher: ____________

Class: ____________

FORT STREET HIGH SCHOOL

2010

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1

TIME ALLOWED: 2 HOURS
(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1, 2	
Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry and polynomials	3, 4, 5	
Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay	6	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	7	

Question	1	2	3	4	5	6	7	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/84	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started on a new page

Question 1 **(12 marks)**

a) Differentiate $\cos^{-1}\left(\frac{3x}{2}\right)$ with respect to x. **2**

b) Find the acute angle between the lines $2x+y-3=0$ and $x=1$. **2**

Correct your answer to the nearest degree.

c) Find the coordinates of the point P which divides the interval AB with endpoints $A(2,3)$ and $B(7,-7)$ externally in the ratio 4:9. **3**

d) Evaluate $\int_0^1 \frac{x+1}{x^2+1}\,dx$ **3**

e) Find all the real values of a for which ax^3-8x^2-9 is divisible by $(x-a)$ **2**

Question 2 **(12 marks)** *Start a separate booklet*

a) Solve the inequality $\dfrac{2x-3}{x+2} \geq 3$ **3**

b) i) Show that a solution for $3\sin x - x = 0$ lies between $x = 2{\cdot}2$ and $x = 2{\cdot}4$. **1**

ii) Taking $x = 2{\cdot}3$ as an initial approximation for a solution to $3\sin x - x = 0$, apply Newton's method once to find a better approximation correct to three decimal places. **3**

c) Find the values for a for which $f(x) = e^{-ax}(x-a)$ has a stationary point at $x = \dfrac{5}{2}$. **2**

d) Use the substitution $x = \log_e u$ to find $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$ **3**

Question 3 **(12 marks)** *Start a separate booklet*

a) Let $f(x) = 2x - x^2$ for $x \leq 1$.

This function has an inverse $f^{-1}(x)$.

i) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. **2**

ii) Find an expression for $f^{-1}(x)$ **2**

iii) Evaluate $f^{-1}\left(\dfrac{3}{4}\right)$. **1**

b) i) Express $\sin x - \cos x$ in the form $A\sin(x-\alpha)$ where $0 < \alpha < \dfrac{\pi}{2}$ **1**

ii) Determine $\displaystyle\lim_{x \to \frac{\pi}{4}} \left[\frac{\sin x - \cos x}{x - \frac{\pi}{4}}\right]$ **2**

c) Use the method of mathematical induction to prove that if $y = xe^x$ then **4**

$$\frac{d^n y}{dx^n} = e^x(x+n), \text{ for all positive integers } n.$$

Question 4 **(12 marks)** *Start a separate booklet*

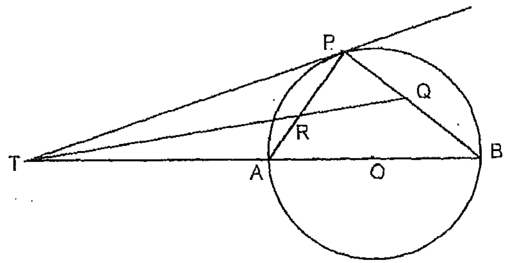
a) i) A particle moves from the origin with initial velocity $u\ ms^{-1}$ and experiences a retardation of magnitude $v+2v^2$, where v is the velocity of the particle at time t. **1**

Show that when it is at position x, $\frac{dv}{dx}=-(1+2v)$

ii) Find its distance from the origin when it comes to rest. **3**

b) O is the centre of the circle and TQ bisects $\angle OTP$. **4**

TB, AP and BP are straight lines and TP is a tangent to the circle at P.



Let $\angle PTQ=\alpha$ and $\angle TBQ=\beta$

Show $\angle TQP=45^\circ$

c) $P(2at,at^2)$ is a variable point on the parabola $x^2=4ay$ whose focus is S.

$Q(x,y)$ divides the interval from P to S in the ratio $t^2:1$.

i) Find the coordinates of Q in terms of a and t. **1**

ii) Verify that $\frac{y}{x}=t$ **1**

iii) Prove that as P moves on the parabola, Q moves on a circle and state its centre and radius. **2**

Question 5 **(12 marks)** *Start a separate booklet*

a) Solve the equation $x^3-3x+2=0$, given it has a double root. **2**

b) i) Show that $\cos 3x=4\cos^3 x-3\cos x$ **2**

ii) Show that the solution of $\cos 3x-\sin 2x=0,\ for\ 0<x<\frac{\pi}{2}$ is given by **3**

$$\sin x=\frac{\sqrt{5}-1}{4}$$

iii) Verify that $x=\frac{\pi}{10}$ is a solution to $\cos 3x=\sin 2x$. **2**

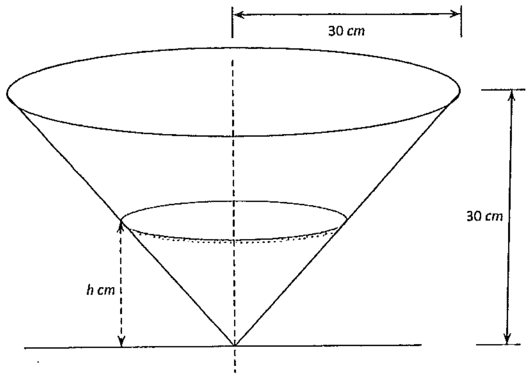
iv) Using the results obtained in parts (ii) and (iii) prove **3**

$$\sin\frac{\pi}{5}\cos\frac{\pi}{10}=\frac{\sqrt{5}}{4}$$

Question 6 **(12 marks)** *Start a separate booklet*

a) On a certain day the depth of water in a bay at high tide is 11 metres. At low tide, 6·25 hours later, the depth of the water is 7 metres. If high tide is due at 3:20 pm, what is the earliest time at which a ship needing a depth of 10 metres of water can enter the bay? (It may be assumed that the rise and fall of the water level is in simple harmonic motion). **4**

b) **4**



Water is poured into a conical vessel at a constant rate of $24\ cm^3$ *per second.*

The depth of the water is h cm at time t seconds.

What is the rate of increase of the area of the surface of the liquid when the depth is $16\ cm$?

c) A particle is projected in a straight line from an origin with velocity $2\ ms^{-1}$.

When x metres from the origin, its acceleration is $\left(2-e^{-\frac{x}{2}}\right)\ ms^{-2}$.

i) Show that, when x metres from the origin, its velocity, $v\ ms^{-1}$, is given by **2**

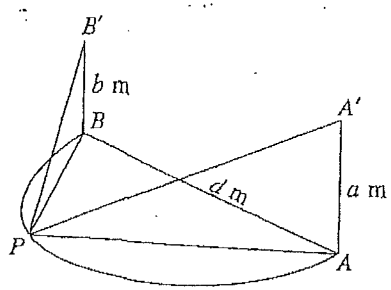
$$v^2 = 4x + 4e^{-\frac{x}{2}}$$

ii) Explain why, for large positive values of x, $v \approx 2\sqrt{x}$. **1**

iii) Prove that the particle will move from $x = 100$ to $x = 121$ in approximately 1 second. **1**

Question 7 **(12 marks)** *Start a separate booklet*

a) APB is a horizontal semicircle, diameter d metres. At A and B are vertical posts of height a m and b m. From P, the angle of elevation of the tops of both posts is θ



i) Prove that $d^2 = \dfrac{a^2}{\tan^2\theta} + \dfrac{b^2}{\tan^2\theta}$. **2**

ii) From B, the angle of elevation of A' is α and from A, the angle of elevation of B' is β. **2**

Prove that $\tan^2\alpha + \tan^2\beta = \tan^2\theta$.

b) Two particles are projected at different times from the same point with speed V. The angles of projection of the two particles are α° and $(90-\alpha)^\circ$ respectively. The greatest heights they reach above the horizontal plane through the point of projection are h_1 and h_2 respectively.

i) Show that for any angle α, $h_1 + h_2 = \dfrac{R}{2}$, where R is the maximum range. **4**

ii) If $\tan\alpha = \dfrac{3}{4}$ and $v = 196\ m/s$, what time must elapse between the instants of projection if the particles collide as they hit the horizontal plane? (Take $g = 9{\cdot}8\ ms^{-2}$). **4**

...L HSC 2010 : EXTENSION I SOLUTIONS

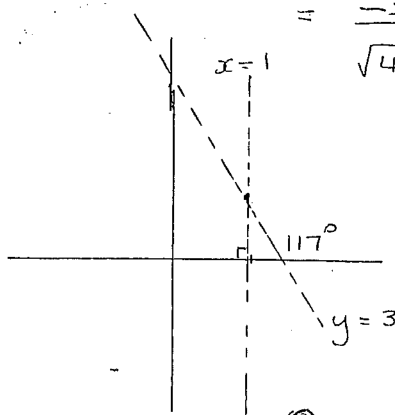
Question 1

COMMENTS

a) $\frac{d}{dx}\cos^{-1}\frac{3x}{2} = \frac{-1}{\sqrt{1-\frac{9x^2}{4}}} \times \frac{3}{2}$ ①

$= \frac{-3}{\sqrt{4-9x^2}} \qquad \frac{-1}{\sqrt{\frac{4}{9}-x^2}}$ ①

b)



$\tan^{-1} -2 = 117°$ ①

Angle between 2 lines is $27°$ (angle sum Δ) ①

c) $P = \left(\frac{-9\times 2 + 4\times 7}{-5}, \frac{-9\times 3 + 4\times -7}{-5}\right)$ ① ①

$= (-2, 11)$ ①

d) $\int_0^1 \frac{x+1}{x^2+1}\,dx = \int_0^1 \frac{x\,dx}{x^2+1} + \int_0^1 \frac{1\,dx}{x^2+1}$

$= \frac{1}{2}\ln(x^2+1)\Big]_0^1 + \tan^{-1}x\Big]_0^1$ ① ①

$= \frac{1}{2}\ln 2 + \frac{\pi}{4}$ ①

e) $P(a) = a^4 - 8a^2 - 9$

$= (a^2-9)(a^2+1)$ ①

If $(x-a)$ is a factor $P(a) = 0$

$\therefore a = \pm 3$ ①

TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 2.

COMMENTS

a) $\frac{(2x-3)}{(x+2)} \times (x+2)^2 \geq 3(x+2)^2$

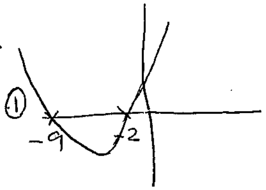
$(2x-3)(x+2) \geq 3(x+2)^2$ ①

$(x+2)[(2x-3) - 3(x+2)] \geq 0.$

$-(x+2)(x+9) \geq 0.$

$(x+2)(x+9) \leq 0$ ①

$\therefore -9 \leq x < -2$ ①



Mostly well done. Students should realise that $x \neq -2$

b)(i) When $x = 2.2$ $\quad 3\sin x - x = 0.2254\ldots$

$x = 2.4$ $\quad 3\sin x - x = -0.373$

Since the sign changes, a solution must lie between these two values. ①

(ii) $x_2 = 2.3 - \frac{f(2.3)}{f'(2.3)}$ ①

$= 2.3 - \frac{3\sin(2.3) - 2.3}{3\cos(2.3) - 1}$ ①

$= 2.279$ (to 3 d. pl). ①

* Everything (values + conclusion) had to be right for 1 mark.

Parts (i) & (ii) were poorly done because

1. Students did not know Newton's Formula (−2 marks)
2. Students do not seem to know how to work with radians ~ many calculator errors +
3. $\frac{d(\sin x)}{dx} = \cos x$ & not $-\cos x$

c) $f'(x) = e^{-ax} \times 1 + (x-a) \times -a e^{-ax}$

$= e^{-ax}(1 - ax + a^2)$

If stationary point exists at $\frac{5}{2}$

then

$f'\left(\frac{5}{2}\right) = 0$

TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 2 (cont'd)

COMMENTS

$$f'\left(\frac{5}{2}\right) = e^{-\frac{5a}{2}}\left(1 - \frac{5a}{2} + a^2\right)$$

$$= \frac{e^{-\frac{5a}{2}}}{2}\left(2a^2 - 5a + 2\right) \quad ①$$

$= 0$ if

$$= \frac{e^{-\frac{5a}{2}}}{2}(2a-1)(a-2)$$

$= 0$ if $a = \frac{1}{2}$ or 2. ①

* Should use product rule to differentiate.

* realise that $e^{-\frac{5a}{2}} \neq 0$, $e^{-\frac{5}{2}a} > 0$ } marks were not deducted if this was not written

d) $x = \log_e u$

$$\frac{dx}{du} = \frac{1}{u}$$

$$\int \frac{e^x \, dx}{\sqrt{1-e^{2x}}} = \int \frac{u \times \frac{du}{u}}{\sqrt{1-u^2}} \quad ①$$

$$= \int \frac{du}{\sqrt{1-u^2}} \quad ①$$

$$= \sin^{-1} u + c.$$

$$= \sin^{-1}(e^x) + c \quad ①$$

← Answer should be given in terms of 'x'. Lost 1 mark if integration constant was forgotten.

OR

$x = \ln u \rightarrow u = e^x \therefore du = e^x dx$

$$\therefore \int \frac{e^x \, dx}{\sqrt{1-e^{2x}}} = \int \frac{du}{\sqrt{1-u^2}}$$

$$= \sin^{-1} u + C$$

$$= \sin^{-1}(e^x) + c$$

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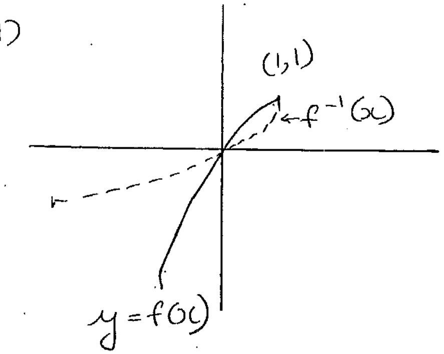
Question 3.

COMMENTS

a) (i)



②

(ii) For $y = 2x - x^2$

inverse is $x = 2y - y^2$

$$x + 1 = -(y^2 - 2y + 1) + 1$$

$$x + 1 = 1 - (y-1)^2.$$

$$(y-1)^2 = 1 - x. \quad ①$$

$$y = 1 \pm \sqrt{1-x}$$

Try to make y subject ①.

From graph we see

$$y = 1 - \sqrt{1-x}. \quad ①$$

(iii) $f^{-1}\left(\frac{3}{4}\right) = 1 - \sqrt{1 - 3/4}$ ①

$$= \frac{1}{2}.$$

b) (i) $\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$ ①

$A = \sqrt{1^2 + 1^2} = \sqrt{2}$
$\alpha = \tan^{-1}(-1)$
$\alpha = -\frac{\pi}{4}$

(ii) $\lim_{x \to \frac{\pi}{4}} \left(\frac{\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}}\right) = \sqrt{2}$ ②

Question 3 (contd) | COMMENTS

c) $y = xe^{x}$

Test $n = 1$

$$\frac{dy}{dx} = xe^{x} + e^{x} \quad (1)$$
$$= e^{x}(x+1) \quad \therefore \text{ true for } n = 1$$

Assume true for $n = k$

$$\frac{d^{k}y}{dx^{k}} = e^{x}(x+k) \quad (1)$$

Consider $n = k+1$

$$\frac{d^{k+1}y}{dx^{k+1}} = e^{x} + (x+k)e^{x}$$
$$= e^{x}(1+x+k)$$
$$= e^{x}(x+(k+1)) \quad (1)$$

This is of the same form as for $n=k$, therefore if true for $n=k$ it is also true for $n=k+1$. Since it is true for $n=1$, it is true for $n=2$ and hence all following (1) positive integers.

TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 4 | COMMENTS

a) (I) $v\frac{dv}{dx} = -(v+2v^2)$ $\quad a = -(v+2v^2)$

$$\therefore \frac{dv}{dx} = -(1+2v) \quad (1)$$

This needed to be demonstrated not just written down.

(II) $\frac{dx}{dv} = \frac{-1}{1+2v}$

$$x = -\int \frac{1}{1+2v}\,dv$$
$$x = -\frac{1}{2}\ln(1+2v) + c.$$

– many students didn't '+c' & did not use conditions to evaluate 'c'

when $x = 0$, $v = u$

$$0 = -\frac{1}{2}\ln(1+2u) + c.$$
$$\therefore c = \frac{1}{2}\ln(1+2u)$$
$$x = \frac{1}{2}\ln(1+2u) - \frac{1}{2}\ln(1+2v)$$
$$= \frac{1}{2}\ln\left(\frac{1+2u}{1+2v}\right)$$

* some students evaluated a definite integral which was fine

When comes to rest $v = 0$

– some students ignored this part

$$\therefore x = \frac{1}{2}\ln(1+2u)$$

b) $\angle PTQ = \angle QTB = \alpha$ (given)
$\angle APB = 90^\circ$ (angle in a semi-circle) (1)
$\angle PBT = \beta$ given.
$\angle TPA = \beta$ (angle between chord and tangent equals angle in alternate segment). (1)

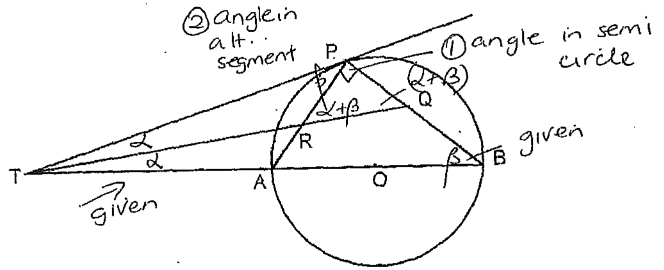
$\therefore \angle PRQ = \alpha + \beta$ (exterior angle to ΔTPR) (1)
$\angle PQR = \alpha + \beta$ (" " " ΔTBQ)

In ΔPRQ both base angles are $(\alpha+\beta)$
$\therefore \Delta PRQ$ is isosceles
so each must be 45°. (1)
$\therefore \angle QPR = 45^\circ$ as req'd.

O'all poorly done. Many made proof unnecessarily lengthy & convoluted

Note student MUST LEARN proper geometric reasoning & state them succinctly.

Question 4 (contd) | COMMENTS



I was extremely liberal, but you will not be so lucky in the HSC

c) (i) $P = (2at, at^2)$ $\quad S = (0, a)$ $\quad k : l$, $t^2 : 1$

$x = \frac{kx_2 + lx_1}{k+l}$ $\quad y = \frac{ky_2 + ly_1}{k+l}$

$\therefore Q = \left(\frac{2at}{t^2+1}, \frac{2at^2}{t^2+1}\right)$ ①

← learn formula

(ii) $y = \frac{2at^2}{t^2+1}$

$= \left(\frac{2at}{t^2+1}\right)t$

$= xt$. ①

$\therefore \frac{y}{x} = t$

← many used x and y showed it was t. (must use L.H.S. / RHS setting out)

(iii) At Q

$x = \frac{2at}{t^2+1}$

$= \frac{2a\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)^2 + 1}$ ①

$= \frac{\frac{2ay}{x}}{\frac{y^2+x^2}{x^2}}$

$= \frac{2ay}{x} \cdot \frac{x^2}{y^2+x^2}$

$= \frac{2ayx}{y^2+x^2}$

poorly done.

KIAL HSC 2010 : EXTENSION 1 SOLUTIONS

Question 4 (contd) | COMMENTS

$1 = \frac{2ay}{y^2+x^2}$.

$x^2 + y^2 - 2ay = 0$.

$x^2 + (y^2 - 2ay + a^2) = a^2$

$x^2 + (y-a)^2 = a^2$.

← some students got to here but did not complete the square to find the true centre & radius

$\therefore$ Q lies on circle whose centre is $(0, a)$ and radius is a units. ①

TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 5.

COMMENTS

a) Roots of $x^3 - 3x + 2 = 0$ are α, α and β.

$\therefore 2\alpha + \beta = 0. \Rightarrow \beta = -2\alpha$

$\alpha^2 + 2\alpha\beta = -3$ ①

$\alpha^2\beta = -2$

$\alpha^2 \times -2\alpha = -2$

$\alpha^3 = 1$

$\therefore \alpha = 1$ and $\beta = -2$. ①.

$\therefore$ Roots are $1, 1$ and -2.

Many Ext 1 students used factor theorem & polynomial div. Many Ext 1 students used $f(x) = 0$ $f'(x) = 0$ for double root.

b) (i) $\cos 3x = \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$= (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x$ ①

$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$

$= 4\cos^3 x - 3\cos x.$ ①

well done

(ii) $\cos 3x - \sin 2x = 0 \qquad 0 < x < \frac{\pi}{2}$

$4\cos^3 x - 3\cos x - 2\sin x \cos x = 0.$

$\cos x(4\cos^2 x - 3 - 2\sin x) = 0.$

$\cos x[4(1 - \sin^2 x) - 3 - 2\sin x] = 0$

$\cos x(4\sin^2 x + 2\sin x - 1) = 0.$ ①

$\cos x = 0$ when $x = \frac{\pi}{2}$ which is not in domain

$\therefore 4\sin^2 x + 2\sin x - 1 = 0$ ie

$\sin x = \dfrac{-2 \pm \sqrt{4+16}}{8}$ ①

$= \dfrac{\pm\sqrt{5} - 1}{4}$

A good number of students ignored the soln to $\cos x = 0$ and lost a mark.

TRIAL HSC 2010 : EXTENSION I SOLUTIONS

Question 5 (contd)

COMMENTS

$\sin x = \dfrac{-\sqrt{5} - 1}{4}$ is also outside required range ①.

$\therefore \sin x = \dfrac{\sqrt{5} - 1}{4}.$

(iii) If $x = \frac{\pi}{10}$, $\cos 3x = \cos\frac{3\pi}{10}$

$= \sin\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$ ①

$= \sin\frac{2\pi}{10}.$ ①

$\therefore x = \frac{\pi}{10}$ is a solution.

Many students used calculator approximations rather than properly showing this simple trig result, and lost a mark

(iv) $\sin\frac{\pi}{5}\cos\frac{\pi}{10} = \sin\frac{2\pi}{10} \cdot \cos\frac{\pi}{10}.$

$= 2\sin\frac{\pi}{10}\cos\frac{\pi}{10}\cos\frac{\pi}{10}$

$= 2\sin\frac{\pi}{10}\cos^2\frac{\pi}{10}.$ ①

$= 2\sin\frac{\pi}{10}\left(1 - \sin^2\frac{\pi}{10}\right)$

$= 2 \times \left(\frac{\sqrt{5}-1}{4}\right)\left(1 - \left(\frac{\sqrt{5}-1}{4}\right)^2\right)$

$= \dfrac{\sqrt{5}-1}{2}\left(\dfrac{16 - (5 + 1 - 2\sqrt{5})}{16}\right)$

$= \dfrac{\sqrt{5}-1}{2} \times \dfrac{10 + 2\sqrt{5}}{16}$

$= \dfrac{\sqrt{5}-1}{2} \times \dfrac{5 + \sqrt{5}}{8}$

$= \dfrac{5\sqrt{5} + 5 - 5 - \sqrt{5}}{16}$

$= \dfrac{4\sqrt{5}}{16}$ ①.

$= \dfrac{\sqrt{5}}{4}.$

Part (iv) not attempted by many students, only completed by ① most able.

Question 6.

SOLUTIONS

b) Using similar triangles
radius of water surface $= h$.

$$\left[\begin{array}{l}\text{vol of cone} \\ = \frac{1}{3}\pi r^2 h \\ = \frac{1}{3}\pi h^3\end{array}\right]$$

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$$\boxed{24 = \pi h^2 \frac{dh}{dt}.} \quad ①$$

$$\therefore \frac{dh}{dt} = \frac{24}{\pi h^2}$$

At $h = 16$ $\quad \frac{dh}{dt} = \frac{24}{\pi \times 16^2}$

$$\boxed{\frac{dh}{dt} = \frac{3}{32\pi} \text{ cm/s.}} \quad ①$$

$$S = \pi h^2$$

$$\frac{dS}{dh} = 2\pi h.$$

$$\frac{dS}{dt} = \frac{dS}{dh} \cdot \frac{dh}{dt}$$

$$\boxed{\frac{dS}{dt} = 2\pi h \times \frac{dh}{dt}} \quad ①$$

At $h = 16$

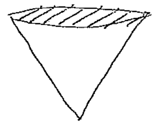
$$\frac{dS}{dt} = 2\pi \times 16 \times \frac{3}{32\pi}$$

$$\boxed{\frac{dS}{dt} = 3 \text{ cm}^2/\text{s}.} \quad ①$$

COMMENTS.

① students did not change $\frac{1}{3}\pi r^2 h$ to $\frac{1}{3}\pi h^3$ instead they used $\frac{dV}{dr} = \frac{d\left(\frac{1}{3}\pi r^2 h\right)}{dr}$ eventually this gave an answer $\frac{ds}{dt} = 4.5$.

② Students found the surface area of a cone. They should have concentrated on the surface of the area



Question 6

SOLUTIONS

a) If high tide is at 3.20pm.
Low tide would occur at 9.05 am
(6¼ hours earlier)

$$\frac{2\pi}{n} = \frac{25}{2}.$$

$$25n = 4\pi$$

$$n = \frac{4\pi}{25} \quad ①$$

$$a = 2 \quad ①$$

$$\therefore x = -2\cos \frac{4\pi}{25} t.$$

When $x = 1$ $\quad \boxed{10 = 9 - 2\cos \frac{4\pi}{25} t}$

$$1 = -2\cos \frac{4\pi}{25} t$$

$$\cos \frac{4\pi t}{25} = \frac{-1}{2}$$

$$\frac{4\pi t}{25} = \frac{2\pi}{3} \quad ①$$

$$t = \frac{25}{6}$$

$= 4$ hours 10 mins

$\therefore$ Ship can safely enter at
1.15 pm. ①

COMMENTS.

①
$T = 13$ hrs
$6.25 + 6.25 = 12.5$

② students did not calculate the first time that $x=10$ but the second time

② students made $T = 6.25$

Question 6 (contd)

c) (i) $\frac{d}{dx}\frac{1}{2}v^2 = 2 - e^{-x/2}$

$\boxed{\frac{1}{2}v^2 = 2x + 2e^{-x/2} + c}$ (1)

$x = 0 \quad v = 2$

$2 = 2 + c \quad \therefore \quad c = 0$

$\boxed{v^2 = 4x + 4e^{-x/2}}$ (1)

Comment: (i) students forgot to write the constant and lost 1 mark.

(ii) As $x \to \infty \quad e^{-x/2} \to 0$

$\therefore \quad v^2 \to 4x$

$\boxed{v \doteqdot 2\sqrt{x}}$ (1).

Comment: Well done

(iii) when $x = 100 \quad v = 20$

$x = 121 \quad v = 22$

$\therefore \quad t = \frac{d}{s}$

$t = \frac{21}{21.5}$

$\boxed{t \doteqdot 1}$

Comment: some students seemed completeley stumped, and did not even attempt this question.

(1) For the ones that did, it was mostly well done.

Question 7.

a) (i) Since $\angle BPA = 90^\circ$ (angle in a semi-circle)

$d^2 = BP^2 + AP^2$ (1)

$\tan\theta = \frac{b}{PB} \qquad \tan\theta = \frac{a}{PA}$

$\therefore \quad d^2 = \frac{b^2}{\tan^2\theta} + \frac{a^2}{\tan^2\theta}$. (1).

Comment: Need to state this to show why you can use Pythagoras' theorem.

(ii) $\tan\alpha = \frac{a}{d}, \quad \tan\beta = \frac{b}{d}$. (1)

$a^2 = d^2\tan^2\alpha, \qquad b^2 = d^2\tan^2\beta$.

$\therefore \quad d^2 = \frac{d^2\tan^2\beta}{\tan^2\theta} + \frac{d^2\tan^2\alpha}{\tan^2\theta}$.

$\tan^2\theta = \tan^2\beta + \tan^2\alpha$. (1)

b) (i) For the first particle

$\ddot{x} = 0 \qquad \ddot{y} = -g$

$\dot{x} = V\cos\theta \qquad \dot{y} = -gt + V\sin\alpha$

$x = Vt\cos\theta \qquad y = Vt\sin\alpha - \frac{gt^2}{2}$.

Max. height occurs when $\dot{y} = 0$.

$V\sin\alpha = gt$

$t = \frac{V\sin\alpha}{g}$. (1)

COMMENTS:

Question 7 (cont'd)

$\therefore y = V.\frac{V\sin\alpha}{g} - \frac{1}{2}g.\frac{V^2\sin^2\alpha}{g^2}$

$= \frac{V^2\sin^2\alpha}{2g}.$

$\therefore h_1 = \frac{V^2\sin^2\alpha}{2g}$ (1)

Similarly

$h_2 = \frac{V^2\sin^2(90-\alpha)}{2g}.$

Many failing to recognise $\sin(90-\alpha) = \cos\alpha$.

$h_1 + h_2 = \frac{V^2\sin^2\alpha}{2g} + \frac{V^2\cos^2\alpha}{2g}$ (1)

$= \frac{V^2}{2g}.$

Maximum range is when $y=0$.

i.e. $0 = Vt\sin\alpha - \frac{gt^2}{2}$

$= t\left(V\sin\alpha - \frac{gt}{2}\right)$

Need to derive these outcomes

$\therefore t = \frac{2V\sin\alpha}{g}$

$x = V.\frac{2V\sin\alpha}{g}.\cos\theta$

$= \frac{V^2\sin 2\alpha}{g}$ (1)

COMMENTS:

Question 7 (cont'd)

Max value of $\sin 2\alpha = 1$.

$\therefore$ max $R = \frac{V^2}{g}$

$\frac{R}{2} = \frac{V^2}{2g}$ (1)

$= h_1 + h_2$

(ii) Particle 1 – hits horizontal plane when $y=0$. Since $\tan\alpha = 3/4$, $\sin\alpha = 3/5$, $\cos\alpha = \frac{4}{5}$.

Many making this more complicated than really is

i.e. $0 = 196t \times \frac{3}{5} - 9.8 \times \frac{t^2}{2}$ (1)

$= t\left(196 \times \frac{3}{5} - 4.9t\right)$

$\therefore t = 0$ or 24 seconds. (1)

Particle 2 – hits horizontal plane when $y=0$

i.e. $y = 196t \times \frac{4}{5} - \frac{9.8t^2}{2}$ (1)

$= t\left(196 \times \frac{4}{5} - 4.9t\right)$

$\therefore t = 0$ or 32 s.

Time lapse $= 32 - 24$ (1)

$= 8$ s.