Neap:

HSC Trial Examination 2010

Mathematics

This paper must be kept under strict security and may only be used on or after the morning of 9 August, 2010 as specified in the Neap Examination Timetable.

General Instructions

Reading time – 5 minutes

Working time – 3 hours

Write using black or blue pen

Board-approved calculators may be used

A table of standard integrals is provided at the back of this paper

All necessary working should be shown in every question

Total marks – 120

Attempt questions 1–10

All questions are of equal value

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2010 HSC Mathematics Examination.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note: $\ln x = \log_e x, \quad x > 0$

Total marks 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\frac{\sqrt{5^2+144}}{13-6}$ to two decimal places. **1**

(b) Simplify the following expression: $\left(x-\frac{1}{x}\right)^2$. **1**

(c) Factorise $2a^2+9a-35$. **1**

(d) Rationalise the denominator and simplify: $\frac{1}{2-\sqrt{3}}-\frac{1}{2+\sqrt{3}}$. **2**

(e) Evaluate $\lim_{x\to 3}\frac{x^3-27}{x-3}$. **2**

(f) In a particular country town, the probability of an adult contracting swine flu is 0.2 during 2010.

Two adults in this town are selected at random.

(i) Draw a tree diagram to show the different outcomes. **1**

(ii) What is the probability that they will both contract swine flu in 2010? **1**

(iii) What is the probability that only one of the adults will contract swine flu in 2010? **1**

(g) On a diagram, indicate the region where the following inequalities hold simultaneously: $y+1\geq 0,\ x+y-2\leq 0$ and $x\geq 2$. **2**

Marks

Question 2 (12 Marks) Use a SEPARATE writing booklet.

(a) Graph on a number line the solution set of: $|2x-1|\geq 7$. **2**

(b) If $f(x)=2cx+d$, find the values of c and d given that $f(-1)=2$ and $f(2)=-1$. **2**

(c) The coordinates of the points A, B and C are $(0,-2)$, $(4,0)$ and $(6,-4)$ respectively.

(i) Find the length AB, and the gradient of AB. **2**

(ii) Show that the equation of the line L, drawn through C parallel to AB is $x-2y-14=0$. **1**

(iii) Find the coordinates of D, the point where L intersects the x-axis. **1**

(iv) Find the perpendicular distance of the point A from the line L. **2**

(v) Find the area of the quadrilateral $ABDC$. **2**

Question 3 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the equation of the locus of $P(x, y)$, if P is always equidistant from $A(3, 1)$ and $B(1, 3)$. **3**

Give a geometric description of this locus.

(b) Determine the minimum value of $5 + 7\cos x$. **2**

(c) Find all real numbers, x, which satisfy the equation $9^x - 10(3^x) + 9 = 0$. **2**

(d) For the parabola $x^2 = 16y$ find the:

(i) focal length **1**

(ii) coordinates of the focus **1**

(iii) equation of the directrix **1**

(e) Determine if the line $x + y + 3 = 0$ is a tangent to the parabola $y = 2x^2 + 3x - 1$. **2**

Question 4 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) In ΔXYZ, $YZ = 3$ cm, $XZ = 5$ cm and $\angle XZY = 150°$.

Find the length of XY to one decimal place. **2**

(b) Differentiate with to respect to x:

(i) $3\tan x$ **1**

(ii) $\dfrac{x-1}{x+1}$ **2**

(iii) $xe^{\cos x}$ **2**

(c) If $f(x) = \log_e x$, find $f'(x)$ and hence $f'(5)$. **2**

(d) The sum to n terms of an arithmetic series is $n^2 + 8n$ for positive values of n.

(i) Show that $T_n = 2n + 7$. **2**

(ii) Which term is the first term greater than 1000? **1**

Marks

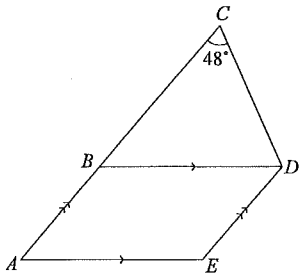
Question 5 (12 Marks) Use a SEPARATE writing booklet.

(a) Simplify $\operatorname{cosec}(90-A)\cos(90-A)$. **2**

(b) For $\frac{\pi}{2}<\beta<\frac{3\pi}{2}$ find β if $2\sin^2\beta=\frac{1}{2}$. **3**

(c) A, B and C are collinear points. $BD \parallel AE$, $BA \parallel DE$, $BC=BD$ and $\angle BCD=48°$.

Copy the diagram on your answer sheet and find the magnitude of $\angle DEA$, giving reasons. **2**



NOT TO SCALE

(d) Using the limiting sum of a geometric series, find a fraction for $0.\dot{1}\dot{2}$ in simplest form. **2**

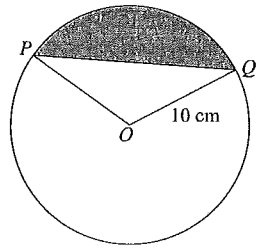
(e) Sam wants to save for a holiday in 10 years' time. The interest rate is fixed at 9.08% p.a. compounding yearly for the first 5 years and will change to 10% p.a. compounding yearly for the next 5 years.

How much should Sam invest now so that he has \$5000 in 10 years from now? Give your answer to the nearest dollar. **3**

Marks

Question 6 (12 Marks) Use a SEPARATE writing booklet.

(a)



(i) The area of sector OPQ is $\frac{100\pi}{3}$ cm^2. Given that the radius of the circle is 10 cm, find the angle α, leaving your answer in exact form. **2**

(ii) Hence, or otherwise, find the area of the shaded segment. **2**

(b) Find the following indefinite integrals:

(i) $\int 3\cos 2x\ dx$ **1**

(ii) $\int \frac{8x+10}{2x^2+5x}\ dx$ **2**

(iii) $\int \frac{x+1}{\sqrt{x}}\ dx$ **2**

(c) Find the exact volume of the solid formed when the curve $y=3\sec 2x$ is rotated about the x-axis between the lines $x=0$ and $x=\frac{\pi}{8}$. **3**

Marks

Question 7 (12 Marks) Use a SEPARATE writing booklet.

(a) A party of hikers leaves its camp, S, at 5:00 am and walks in the direction 150° T until they reach point B at the base of a cliff. Unable to ascend the cliff, they follow it on a bearing of 015° T and set up camp for the night when they reach point C at 2:00 pm. The bearing and distance of C from S are 120° T and 7.5 km respectively.

(i) Draw a diagram showing the above information. **1**

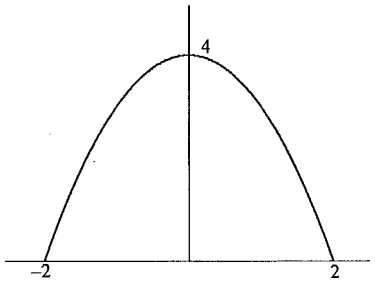
(ii) Show that $\angle SBC = 45°$. **1**

(iii) Calculate the average speed of the hike from S to C, correct to one decimal place. **2**

(b) (i) Draw the graph of $f(x) = \tan x$ for $-\pi \le x \le \pi$. **1**

(ii) Use Simpson's Rule with three function values to approximate the area enclosed between the curve $y = f(x)$ and the lines $x = 0$ and $x = \frac{\pi}{3}$. Leave your answer as an exact value. **2**

(c) A builder wishes to install an arch window which is in the shape of a parabola, as shown in the diagram. The parabola has a window sill 4 m wide and is 4 m tall.



Calculate the area of the window. **2**

(d) Evaluate:

(i) $\int_0^1 \frac{dx}{2x+1}$ **1**

(ii) $\int_0^1 \frac{1}{(2x+1)^2}\,dx$ **2**

Marks

Question 8 (12 Marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = 1 - 3x + x^3$, in the domain $-2 \le x \le 3$.

(i) Find the coordinates of the turning points and determine their nature. **3**

(ii) Draw a sketch of the curve $y = f(x)$ in the domain $-2 \le x \le 3$, clearly showing all its essential features. **2**

(iii) What is the maximum value of the function $f(x)$ in the domain $-2 \le x \le 3$? **1**

(b) For what value of x is the tangent to the curve $y = e^{3x}$ parallel to the line $y = 6x$? **3**

(c) A particle moves in a straight line such that its displacement in metres, x, after t seconds is given by $x = t^3 + 3t^2 - 9t - 1$.

(i) When and where does the particle first change direction? **2**

(ii) What is the average speed of the particle in the first second? **1**

Question 9 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Sketch the curves $y = 2\sin x$ and $y = -2\cos x$ on the same diagram for $0 \le x \le \pi$. Label clearly any point(s) of intersection of the two curves. **2**

(ii) Calculate the area bounded by the curves $y = 2\sin x$ and $y = -2\cos x$ and the y-axis. **2**

(b) A ball is projected vertically from the top of a tower. The height of the ball above ground level is given by $h(t) = 24 + 8t - 2t^2$, where h is height in metres and t is time in seconds.

(i) Find the initial velocity. **1**

(ii) Calculate the time taken for the ball to hit the ground. **2**

(iii) Explain why the acceleration of the ball is constant. **1**

(c) The size of a colony of bees is given by the equation $P = 5000e^{kt}$ where P is the population after t weeks.

(i) If there are 6000 bees after one week, find the value of k correct to 2 decimal places. **1**

(ii) When will the colony (to the nearest day) triple in size? **1**

(iii) What is the growth rate of the population after two weeks? **2**

Question 10 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Given that $m^2 + n^2 = 14mn$, simplify $\left(\frac{(m+n)}{4}\right)^2$. **2**

Hence show that $\log\left(\frac{m+n}{4}\right) = \frac{1}{2}[\log m + \log n]$.

(b) (i) Sketch the curve $y = xe^{2x}$ showing all stationary points and asymptotes. **2**

(ii) Hence find value(s) of k for which $xe^{2x} = k$ has:

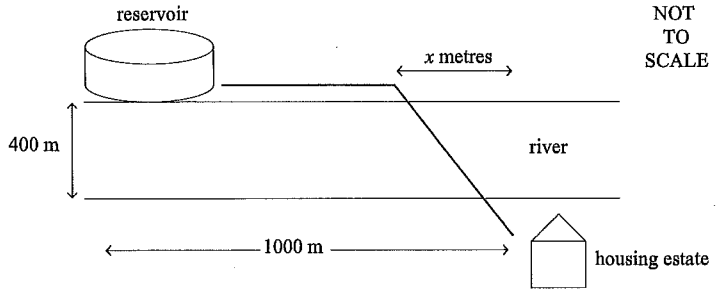
(α) no solutions **1**

(β) one solution **1**

(c) An underground water pipe is being built from a reservoir to a new housing estate on the other side of a river, 1000 metres downstream. The river is 400 metres wide and has straight banks.

The diagram shows the proposed route of the water pipe. It follows the river bank for some distance before crossing the river to the housing estate.

The cost of laying the pipe underground is \$800 per metre and the cost of laying the pipe underwater is \$1000 per metre.



(i) Show that the cost (C) of the pipeline, in terms of x, is given by: **1**

$C = 800(1000 + x) + 1000\sqrt{400^2 + x^2}$.

(ii) Find the value of x that gives minimum cost. **4**

(iii) What is the minimum cost of the pipeline? **1**

End of paper

Neap:

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Mathematics

Solutions and marking guidelines

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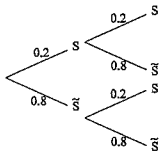
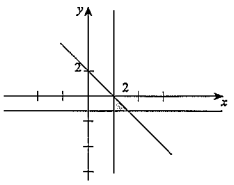
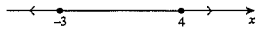
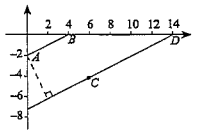
Question 1

Sample answer	Syllabus outcomes and marking guide
(a) 1.86	P3 • Gives the correct answer 1
(b) $\left(x-\frac{1}{x}\right)^2 = x^2 - 2\cdot x\cdot\frac{1}{x} + \frac{1}{x^2}$ $= x^2 - 2 + \frac{1}{x^2}$	P3 • Gives the correct answer 1
(c) $(2a-5)(a+7)$	P4 • Correctly factorises 1
(d) $\frac{1}{2-\sqrt{3}} - \frac{1}{2+\sqrt{3}}$ $= \frac{(2+\sqrt{3})-(2-\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$ $= \frac{2\sqrt{3}}{1}$	P4 • Gives the common denominator. • Gives the correct answer ..... 2 • Gives common denominator. OR • Gives correct answer 1
(e) $\lim_{x\to 3}\frac{(x^3-27)}{x-3}$ $= \lim_{x\to 3}\frac{(x-3)(x^2+3x+9)}{(x-3)}$ $= 3^2+3(3)+9$ $= 27$	P3 • Correctly factorises. • Gives correct answer ..... 2 • Correctly factorises. OR • Gives correct answer 1
(f) (i) 	H4, H5, H9 • Correctly draws tree diagram. 1
(ii) $P(SS) = 0.2 \times 0.2$ $= 0.04$	H4, H5 • Gives the correct probability 1
(iii) $P(S\tilde{S} \text{ or } \tilde{S}S) = 0.2 \times 0.8 + 0.8 \times 0.2$ $= 0.32$	H4, H5 • Gives the correct probability 1
(g) 	H9 • Correctly indicates two or three lines. • Shows correct shading ..... 2 • Correctly indicates two or three lines. OR • Shows correct shading 1

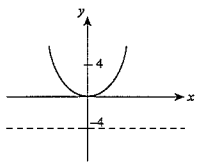
Question 2

Sample answer	Syllabus outcomes and marking guide
(a) $\lvert 2x-1\rvert \geq 7$ $2x-1 \geq 7$ $\quad -(2x-1) \geq 7$ $2x \geq 8$ $\quad -2x+1 \geq 7$ $x \geq 4$ $\quad -2x \geq 6$ $x \leq -3$ 	P4, H9 • Gives one correct inequality. • Draws correct number line. ..... 2 • Gives one correct inequality. OR • Draws correct number line. 1
(b) (1) $f(-1) = -2c+d \quad \therefore -2c+d \quad (1)$ (2) $f(2) = 4c+d \quad \therefore 4c+d=-1 \quad (2)$ Solving simultaneously (1) − (2) $-6c = 3$ $c = -\frac{1}{2}$ substitute into (1) $-2\left(-\frac{1}{2}\right)+d=2$ $\therefore d = 1$	P5 • Gives correct value of c. • Gives correct value of d ..... 2 • Gives correct value of c. OR • Gives correct value of d 1
(c)  (i) $AB = \sqrt{20}$, $m_{AB} = \frac{1}{2}$	P5, P9 • Gives correct length AND gradient ..... 2 • Gives one correct value 1
(ii) $y+4 = \frac{1}{2}(x-6)$ $2y+8 = x-6$ $x-2y-14 = 0$	P5, H9 • Correctly shows equation of L 1
(iii) $D(14, 0)$	• Gives correct co-ordinate 1
(iv) $d = \frac{\lvert ax+by+c\rvert}{\sqrt{a^2+b^2}}$ $d = \frac{\lvert 1(0)+(-2(-2))+(-14)\rvert}{\sqrt{1^2+(-2)^2}}$ $d = \frac{\lvert -10\rvert}{\sqrt{5}} = 2\sqrt{5}$	P5, H9 • Gives correct substitution. • Gives correct perpendicular distance ..... 2 • Gives correct substitution. OR • Gives correct perpendicular distance 1

Question 2

Sample answer	Syllabus outcomes and marking guide
(v) Area $= \frac{1}{2}(a+b)h$ $= \frac{1}{2}(\sqrt{20}+\sqrt{80}) \times 2\sqrt{5}$ $= \frac{10}{\sqrt{5}}$ $= 30\ u^2$	P5, H9 • Gives correct length of CD. • Gives correct area of $ABDC$. ..... 2 • Gives correct length of CD. OR • Gives correct area of $ABDC$. 1

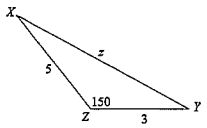
Question 3

Sample answer	Syllabus outcomes and marking guide
(a) $AP = PB$ $AP^2 = PB^2$ $\therefore (x-3)^2 + (y-1)^2 = (x-1)^2 + (y-3)^2$ $x^2 - 6x + 9 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 - 6y + 9$ $-6x - 2y = -2x - 6y$ $4y = 4x$ $y = x$ i.e. a straight line which is the perpendicular bisector of AB.	H4 • Correctly sets up $AP = BP$ or $AP^2 = BP^2$. • Correctly substitutes and simplifies. • Gives correct geometric description .... 3 • Any two of the above ..... 2 • Any one of the above 1
(b) $5 + 7\cos x$ has a minimum when $\cos x$ is a minimum, i.e. when $x = \pi$ $\therefore min[5 + 7\cos x] = 5 + 7(-1)$ $= -2$	H5 • Determines when $\cos x$ is a minimum. • Gives correct minimum value ..... 2 • Determines when $\cos x$ is a minimum. OR • Gives correct minimum value 1
(c) $9^x - 10(3^x) + 9 = 0$ $(3^x)^2 - 10(3^x) + 9 = 0$ $(3^x - 9)(3^x - 1) = 0$ $3^x = 9,\ 3^x = 1$ $x = 2,\ 0$	H3 • Correctly factorises. • Gives correct values for x. ..... 2 • Correctly factorises. OR • Gives correct values for x. 1
(d)  (i) $4a = 16$ $a = 4$	P5 • Gives correct focal length 1
(ii) $S(0, 4)$	P5 • Gives correct focus 1
(iii) $y = -4$	P5 • Gives correct directrix 1

Question 3 (Continued)

Sample answer	Syllabus outcomes and marking guide
(e) $x + y + 3 = 0$ (1) $y = 2x^2 + 3x - 1$ (2) $\therefore -x - 3 = 2x^2 + 3x - 1$ $2x^2 + 4x + 2 = 0$ $x^2 + 2x + 1 = 0$ $(x+1)^2 = 0$ $x = -1, y = -2$ OR $\Delta = 2^2 - 4.(1).(1)$ $\Delta = 0$ $\therefore x + y + 3 = 0$ is a tangent	P5 • Gives simultaneous solutions. • Gives correct conclusion ..... 2 • Gives simultaneous solutions. OR • Gives correct conclusion 1

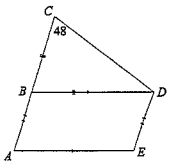
Question 4

Sample answer	Syllabus outcomes and marking guide
(a)  Using cosine rule $z^2 = 34 + 15\sqrt{3}$ $z^2 = 59.98\ldots$ $z = 7.74472\ldots$ $z = 7.7$ cm	P4 • Shows correct use of cosine rule. • Gives correct answer ..... 2 • Shows correct use of cosine rule. OR • Gives correct answer 1
(b) (i) $\frac{d}{dx}[3\tan x] = 3\sec^2 x$	P7 • Gives correct answer 1
(ii) $\frac{d}{dx}\left[\frac{x-1}{x+1}\right] = \frac{d}{dx}\left[\frac{x+1-2}{x+1}\right]$ $= \frac{d}{dx}\left[1 - \frac{2}{x+1}\right]$ $= \frac{2}{(x+1)^2}$ OR Use quotient rule: $\frac{vu' - uv'}{v^2} = \frac{(x+1)\cdot 1 - ((x-1)\cdot 1)}{(x+1)^2}$ $= \frac{2}{(x+1)^2}$	P7 • Uses correct technique. • Gives correct answer ..... 2 • Uses correct technique. OR • Gives correct answer 1
(iii) $\frac{d}{dx}xe^{\cos x} = 1 \cdot e^{\cos x} + x \cdot -\sin x e^{\cos x}$ $= e^{\cos x}[1 - x\sin x]$	H5 • Shows correct use of product rule. • Gives correct answer ..... 2 • Shows correct use of product rule. OR • Gives correct answer 1
(c) $f'(x) = \frac{1}{x}$ $\therefore f'(5) = \frac{1}{5}$ or 0.2	H3 • Gives correct derivative. • Gives correct answer ..... 2 • Gives correct derivative. OR • Gives correct answer 1

Question 4 (Continued)

Sample answer	Syllabus outcomes and marking guide
(d) (i) Method 1: $T_n = S_n - S_{n-1}$, where $S_n = n^2 + 8n$ $T_n = (n^2 + 8n) - ((n-1)^2 + 8(n-1))$ $T_n = (n^2 + 8n) - (n^2 - 2n + 1 + 8n - 8)$ $T_n = 2n + 7$ Method 2: $S_1 = 9,\ S_2 = 20,\ S_3 = 33$, etc... $\therefore T_1 = 9,\ T_2 = 11,\ T_3 = 13$ So, $a = 9$ and $d = 2$ $\therefore T_n = 9 + (n-1) \times 2$ So we get $T_n = 2n + 7$	H5 • Gives correct algebra to find terms. • Gives correct answer ..... 2 • Gives correct algebra to find terms. OR • Gives correct answer 1
(ii) $T_n > 1000$ $2n + 7 > 1000$ $2n > 993$ $n > 141\frac{6}{7}$ $\therefore T_{142}$ is the first term greater than 1000	H5 • Gives correct answer 1

Question 5

Sample answer	Syllabus outcomes and marking guide
(a) $\operatorname{cosec}(90-A)\cdot\cos(90-A)$ $=\frac{1}{\cos A}\cdot\sin A$ $=\tan A$	P4 • Uses correct simplification. • Gives correct answer ..... 2 • Uses correct simplification. OR • Gives correct answer 1
(b) $2\sin^2\beta=\frac{1}{2}$ $\sin^2\beta=\frac{1}{4}$ $\sin\beta=\frac{1}{2}$ or $\sin\beta=-\frac{1}{2}$ $\beta=\frac{5\pi}{6}$ or $\beta=\frac{7\pi}{6}$ Solutions: $\beta=\frac{5\pi}{6}$ or $\frac{7\pi}{6}$	H5 • Correctly determines $\sin\beta=\pm\frac{1}{2}$. • Correctly determines $\sin^2\beta=\frac{1}{4}$. • Gives correct values for β ..... 3 • Gives two of three above ..... 2 • Gives one of three above ..... 1 • Correctly determines $\sin\beta=\pm\frac{1}{2}$. OR • Correctly determines $\beta=30°$ 1
(c)  $\angle CDB=48°$ (base angles of isosceles triangle are equal) $\angle CBD=84°$ (angle sum of Δ is 180°) $\angle ABD=180°-84°$ $=96°$ (angles in straight line add to 180°) $\therefore\angle DEA=96°$ (opposite angles of parallelogram are equal)	P4 • Correctly determines $\angle CBD$. • Correctly finds $\angle DEA$ ..... 2 • Correctly determines $\angle CBD$. OR • Correctly finds $\angle DEA$ 1
(d) $0.\dot{1}\dot{2}=\frac{12}{100}+\frac{12}{10\,000}+\frac{12}{100\,000}+\ldots$ $a=\frac{12}{100}$, $r=\frac{1}{100}$ Using $S_\infty=\frac{a}{1-r}$, where $\lvert r\rvert<1$ $S_\infty=\frac{\frac{12}{100}}{\left(1-\frac{1}{100}\right)}$ $S_\infty=\frac{12}{100-1}=\frac{12}{99}=\frac{4}{33}$	H4, H5 • Gives correct answer for a and r. • Finds correct limiting sum ..... 2 • Gives correct answer for a and r. OR • Finds correct limiting sum 1

Question 5 (Continued)

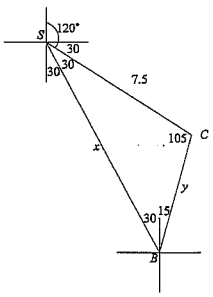
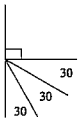
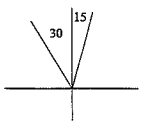
Sample answer	Syllabus outcomes and marking guide
(e) $5000=x\times(1.0908)^5\times(1.1)^5$ $x=\frac{5000}{(1.0908)^5\times(1.1)^5}$ $x=2010.392855\ldots$ $x=\$2010$ (nearest dollar)	H4, H5 • Uses correct rates. • Uses correct algorithm. • Gives correct answer ..... 3 • Gives two of three above ..... 2 • Gives one of three above 1

Question 6

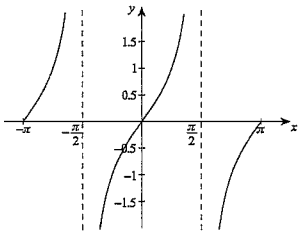
Sample answer	Syllabus outcomes and marking guide
(a) (i) $\frac{1}{2}r^2\alpha=\frac{100\pi}{3}$ $\frac{1}{2}\times100\times\alpha=\frac{100\pi}{3}$ $\alpha=\frac{2\pi}{3}$ radians	H5 • Uses area formula. • Gives correct exact answer ..... 2 • Uses area formula. OR • Gives correct exact answer 1
(ii) Area shaded $=\frac{1}{2}r^2(\alpha-\sin\alpha)$ $=\frac{1}{2}\times10^2\times\left(\frac{2\pi}{3}-\sin\left(\frac{2\pi}{3}\right)\right)$ $=50\times\left(\frac{4\pi-3\sqrt{3}}{6}\right)\text{ cm}^2$ $=\frac{25}{3}(4\pi-3\sqrt{3})$ $=61.418\ldots\text{ cm}^2$	H5 • Uses correct formula. • Gives correct answer ..... 2 • Uses correct formula. OR • Gives correct answer 1
(b) (i) $\int3\cos2x\cdot dx=3\cdot\frac{1}{2}\sin2x+c$ $=\frac{3}{2}\sin2x+c$	H5, H8 • Gives correct answer 1
(ii) $\int\frac{8x+10}{2x^2+5x}\cdot dx=2\int\frac{4x+10}{2x^2+5x}\cdot dx$ $=2\ln(2x^2+5x)+c$	H5, H8 • Shows correct simplification. • Gives correct answer ..... 2 • Shows correct simplification. OR • Gives correct answer 1
(iii) $\int\frac{x+1}{\sqrt{x}}\cdot dx=\int\sqrt{x}+\frac{1}{\sqrt{x}}\cdot dx$ $=\int x^{\frac{1}{2}}+x^{-\frac{1}{2}}\cdot dx$ $=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+c$ $=\frac{2}{3}x\sqrt{x}+2\sqrt{x}+c$	H5, H8 • Shows correct simplification. • Gives correct answer ..... 2 • Shows correct simplification. OR • Gives correct answer 1

(c) $V=\pi\int y^2dx$
$=\pi\int_0^{\frac{\pi}{8}}9\sec^2 2x\,dx$
$=9\pi\left[\frac{\tan2x}{2}\right]_0^{\frac{\pi}{8}}$
$=\frac{9\pi}{2}\left[\tan\frac{\pi}{4}-\tan0\right]$
$=\frac{9\pi}{2}u^3$
	H8
	• Correctly completes:
	  • substitution
	  • integral
	  • answer 3
	• Correctly completes two of the above ... 2
	• Correctly completes one of the above ... 1

Question 7

Sample answer	Syllabus outcomes and marking guide
(a) (i)  	H4, H9 • Correctly draws diagram 1
(ii) Diagram should indicate angles at S:  Diagram should indicate angles at B:  Clearly, alternate angles enable the students to show that $\angle SBC=45°$.	H4 • Correctly shows $\angle SBC=45°$ 1

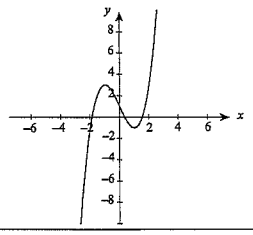
Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(iii) By angle sum of a triangle, $\angle SCB = 105°$ Using sine rule, $\frac{x}{\sin 105} = \frac{7.5}{\sin 45}$ $x = \frac{7.5\sin 75}{\sin 45}$ $x = 10.24519\ldots$ $\frac{y}{\sin 30} = \frac{7.5}{\sin 45}$ $y = \frac{7.5\sin 30}{\sin 45} = 5.303300\ldots$ $\therefore$ Average Speed $= \frac{x+y}{9}$ $= 1.7$ km/h	H4 • Shows correct use of sine rule. • Gives correct average speed. . . . . 2 • Shows correct use of sine rule. OR • Gives correct average speed. 1
(b) (i) 	H9 • Gives correct diagram 1
(ii) Simpson's Rule: Area $\approx \frac{b-a}{6}\{f_F + 4f_M + f_L\}$ Area $\approx \frac{\frac{\pi}{3}-0}{6}\left\{\tan(0) + 4\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{3}\right)\right\}$ $\approx \frac{\pi}{18}\{4+\sqrt{3}\}$ units^2	H8 • Shows correct substitution into Simpson's rule. • Gives correct answer . . . . . 2 • Shows correct substitution into Simpson's rule. OR • Gives correct answer 1

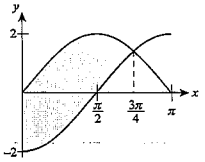
Question 7 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) Area $= \int_{-2}^{2} -x^2+4\ dx$ $= \left[-\frac{x^3}{3}+4x\right]_{-2}^{2}$ $= \left(-\frac{8}{3}+8\right) - \left(\frac{8}{3}-8\right)$ $= 16 - \frac{16}{3}$ $= \frac{32}{3}$ or $10\frac{2}{3}$ units^2 Alternatively, Area $= 2\int_0^2 4-x^2\cdot dx$ $= 2\left[4x - \frac{1}{3}x^3\right]_0^2$ $= 2\left[\left(8-\frac{8}{3}\right) - (0-0)\right]$ $= 10\frac{2}{3}u^2$	H8 • Correctly states an integral. • Gives correct answer . . . . . 2 • Correctly states an integral. OR • Gives correct answer 1
(d) (i) $\int_0^1 \frac{dx}{2x+1} = \frac{1}{2}[\ln(2x+1)]_0^1$ $= \frac{1}{2}[\ln 3 - \ln 1]$ $= \frac{1}{2}\ln 3$	H8 • Gives correct integral. • Gives correct answer 1
(ii) $\int_0^1 \frac{dx}{(2x+1)^2} = \int_0^1 (2x+1)^{-2}dx$ $= \left[\frac{(2x+1)^{-1}}{-2}\right]_0^1$ $= -\frac{1}{2}\left[\frac{1}{2x+1}\right]_0^1$ $= -\frac{1}{2}\left(\frac{1}{3}-1\right)$ $= \frac{1}{3}$	H8 • Gives correct integral. • Gives correct answer . . . . . 2 • Gives correct integral. OR • Gives correct answer 1

Question 8

Sample answer	Syllabus outcomes and marking guide
(a) (i) $f'(x) = -3+3x^2$, $f''(x) = 6x$ For turning points $f'(x) = 0$, hence $-3+3x^2 = 0$, $x^2 = 1$, $x = \pm 1$ Turning points are **(1, –1)** and **(–1, 3)** Nature of these points: $f''(1) = 6$, $f''(-1) = -6$ Hence, **(1, –1) is a minimum and (–1, 3) is a maximum.**	H6 • Correctly shows the derivatives. • Correctly gives the coordinates of each turning point. • Determines the nature of each turning point. . . . . 3 • Gives any two of the above . . . . . 2 • Gives one of the above. 1
(ii) 	H9 • Shows correct intercepts. • Gives correct graph of $y = f(x)$ . . . . . 2 • Shows correct intercepts. OR • Gives correct graph of $y = f(x)$ 1
(iii) $\max f(x) = 19$, when $x = 3$.	H9 • Gives correct maximum value 1
(b) $y = e^{3x}$ $\quad y = 6x$ $y' = 3e^{3x}$ $\quad m = 6$ $\therefore$ parallel $\Rightarrow 3e^{3x} = 6$ $e^{3x} = 2$ $3x = \ln 2$ $x = \frac{1}{3}\ln 2$	H6 • Correctly finds y'. • Equates $3e^{3x} = 6$. • Gives correct value for x . . . . . 3 • Gives two of the above. . . . . 2 • Correctly finds y' 1
(c) (i) $x(t) = t^3+3t^2-9t-1$ $\dot{x} = 3t^2+6t-9$ $= 3(t^2+2t-3)$ $= 3(t+3)(t-1)$ $\therefore \dot{x} = 0$ when $t = 1$ second (since $t \geq 0$) $x(1) = 1+3-9-1$ $= -6$ metres, i.e. 6 m on LHS of origin	H3, H4 • Gives the correct time. • Gives the correct place. . . . . 2 • Gives the correct time. OR • Gives the correct place. 1
(ii) Average speed $= \frac{\lvert(-6)-(-1)\rvert}{1}$ $= 5$ m s^{-1}	H3, H4 • Gives correct answer 1

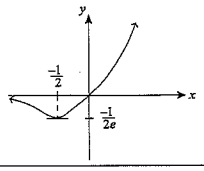
Question 9

Sample answer	Syllabus outcomes and marking guide
(a) (i)  $2\sin x = -2\cos x$ $\tan x = -1$ $x = \frac{3\pi}{4}$ $(0 \leq x \leq \pi$ only) $\therefore$ Point of intersection is $\left(\frac{3\pi}{4}, \sqrt{2}\right)$	H9 • Correctly draws graph. • Correctly identifies points of intersection. . . . . 2 • Correctly draws graph. OR • Correctly identifies points of intersection. 1
(ii) Area shaded $= \int_0^{\frac{3\pi}{4}} 2\sin x - (-2\cos x)\cdot dx$ $= 2\int_0^{\frac{3\pi}{4}} \sin x + \cos x \cdot dx$ $= 2[-\cos x + \sin x]_0^{\frac{3\pi}{4}}$ $= 2\left[\left(-\cos\frac{3\pi}{4} + \sin\frac{3\pi}{4}\right) - (-\cos 0 + \sin 0)\right]$ $= 2\left[\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) - (-1+0)\right]$ $= 2\left(\frac{2}{\sqrt{2}}+1\right)$ Area $= 2\sqrt{2}+2u^2$	H8 • Correctly gives integral. • Gives correct area . . . . . 2 • Correctly gives integral. OR • Gives correct area 1
(b) (i) $\frac{dh}{dt} = 8-4t$ $t = 0$ $\quad \frac{dh}{dt} = 8$ ms^{-1}	H4 • Gives correct answer 1
(ii) $h = 0$ $\therefore 24+8t-2t^2 = 0$ $2t^2-8t-24 = 0$ $2(t+2)(t-6) = 0$ $\therefore t = 6$ seconds $\quad (t > 0)$	H4 • Correctly factorises. • Gives correct time . . . . . 2 • Correctly factorises. OR • Gives correct time 1
(iii) $\frac{dh}{dt} = 8-4t$, therefore $\frac{d}{dt}\left(\frac{dh}{dt}\right) = -4$ i.e. acceleration is independent of t.	H4 • Shows $\frac{d^2h}{dt^2} = -4$ or states $h = -4$ 1

Question 9 (Continued)

Sample answer	Syllabus outcomes and marking guide
(c) (i) $6000 = 5000e^{kt}$ $k = \ln\left(\frac{6}{5}\right)$ $k = 0.18$ (2 decimal points)	H3, H4 • Gives correct value for k 1
(ii) $P = 15000$ $\therefore 15000 = 5000e^{kt}$ $3 = e^{kt}$ $kt = \ln 3$ $t = \frac{1}{k}\ln 3$ $t = \frac{\ln 3}{\ln\left(\frac{6}{5}\right)} = 6.025\ldots$ $t = 42$ days or 6 weeks	H3, H4 • Gives correct number of days 1
(iii) $P(2 \text{ weeks}) = 5000e^{2k}$ $= 5000e^{2\ln\left(\frac{6}{5}\right)}$ $= 7200$ $\therefore \frac{dP}{dt} = kP$ $= \ln\left(\frac{6}{5}\right) \cdot 7200$ $= 1312.7$ $= 1313$ bees/week	H3, H4 • Gives correct population after two weeks. • Gives correct rate of growth..... 2 • Gives correct population after two weeks. OR • Gives correct rate of growth..... 1

Question 10

Sample answer	Syllabus outcomes and marking guide
(a) $m^2 + n^2 = 14mn$ $\left(\frac{m+n}{4}\right)^2 = \frac{m^2+2mn+n^2}{16}$ $= \frac{(m^2+n^2)+2mn}{16}$ $= \frac{16mn}{16}$ $= mn$ Now, $2\log\left(\frac{m+n}{4}\right) = \log\left(\frac{m+n}{4}\right)^2$ $= \log(mn)$ $\therefore \log\left(\frac{m+n}{4}\right) = \frac{1}{2}(\log m + \log n)$, as required	P3, H3 • Gives correct expression. • Shows result as required ..... 2 • Gives correct expression. OR • Shows result as required 1
(b) (i) $y = xe^{2x}$ $y' = 1 \cdot e^{2x} + x \cdot 2e^{2x}$ $= e^{2x}[1 + 2x]$  	H3, H9 • Correctly sketches the graph 2
(ii) (α) $xe^{2x} = k$ will have no solutions when $k < -\frac{1}{2e}$.	H3, H9 • Gives correct answer 1
(β) $xe^{2x} = k$ will have one solution when $k = -\frac{1}{2e}$ and $k \geq 0$.	H3, H9 • Gives correct answer 1
(c) (i) Total cost, C = cost of under ground + cost of under water $= \$800 \times (1000 - x) + \$1000 \times \sqrt{400^2 + x^2}$ $= 800(100 - x) + 100\sqrt{400^2 + x^2}$, as required	H4, H5 • Shows total cost of pipeline..... 1

Question 10 (Continued)

Sample answer	Syllabus outcomes and marking guide
(ii) $\frac{dC}{dx} = -800 + 1000 \times \frac{1}{2}(400^2 + x^2)^{-\frac{1}{2}} \times 2x$ $\frac{dC}{dx} = -800 + \frac{1000x}{\sqrt{400^2 + x^2}}$ For minimum/maximum, $\frac{dC}{dx} = 0$ $\therefore -800 + \frac{1000x}{\sqrt{400^2 + x^2}} = 0$ $1000x = 800\sqrt{400^2 + x^2}$ squaring both sides, $1\,000\,000x^2 = 640\,000(400^2 + x^2)$ $360\,000x^2 = 102\,400\,000\,000$ $x^2 = 284\,444\frac{4}{9}$ $x = 533\frac{1}{3}$ metres Alternatively, $1000x = 800\sqrt{400^2 + x^2}$ $\sqrt{400^2 + x^2} = \frac{1000x}{800}$ $= \frac{5}{4}x$ Squaring both sides, $400^2 + x^2 = \frac{25}{16}x^2$ $\frac{9}{16}x^2 = 400^2$ $x = \frac{400 \times 4}{3}, x > 0$ $x = 533\frac{1}{3}$ metres Sign change in first derivative indicates that $x = 533\frac{1}{3}$ metres gives minimum cost. x: 500 | 533.333 | 600 $\frac{dC}{dx}$: ~ -800 | 0 | ~ 32	H4, H5 • Correctly differentiates total cost expression. • States $\frac{dC}{dx} = 0$ to minimise cost. • Finds $x = 533\frac{1}{3}$. • Correctly identifies minimum ..... 4 • Correctly gives three of four above ..... 3 • Correctly gives two of four above ..... 2 • Correctly gives one of four above 1
(iii) Minimum cost $= \left(1000 - 533\frac{1}{3}\right) \times 800 + \sqrt{400^2 + 533\left(\frac{1}{3}\right)^2} \times 1000$ $= \$1\,040\,000$ (or \$1.04 million)	H4, H5 • Gives correct answer 1