NEWINGTON COLLEGE

Mid Year Examination 2009

YEAR 8 MATHEMATICS

Time allowed: 90 minutes

NAME: _____ CLASS: _____

Outcomes being assessed:

1. Operates with percentages.
2. Factorises simple algebraic expressions and uses this to simplify simple algebraic fractions.
3. Uses and applies Pythagoras' Theorem.
4. Constructs, reads and interprets graphs, tables, chart and statistical information.
5. Uses algebraic techniques to solve linear equations and simple inequalities.

Directions to candidates

All questions may be attempted.
In each question, show all necessary working.
The use of hand-held non-programmable calculators is permitted.
Marks will be deducted for careless or badly arranged work.

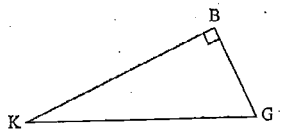
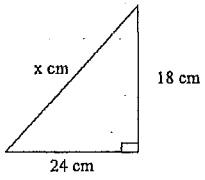
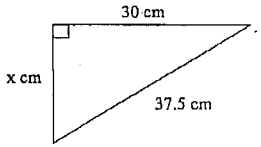
Outcome	Mark
Percentages	/10
Algebra	/15
Pythagoras' Theorem	/25
Data	/25
Equations	/25
Total	/100

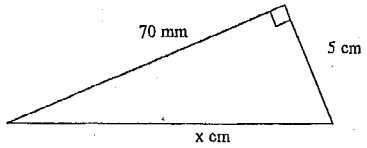
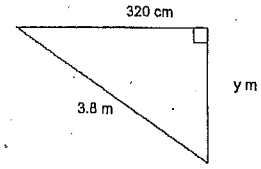
Teacher's Comment:

SECTION 1 PERCENTAGES	**10 Marks**
1. Complete the table below Percentage | Fraction | Decimal 29% | | | | 0.07	2
2. Find 16% of 2.4 tonnes.	1
3. Increase \$78.50 by 8%	2
4. What percentage is 209 mm of 38 cm?	1
5. A cake is made up of 1 part flour, 2 parts chocolate and 2 parts sugar. What percentage of the cake is chocolate?	2
6. If after a $6\frac{1}{4}$ % pay rise I received \$680, how much did I earn before the pay rise?	2

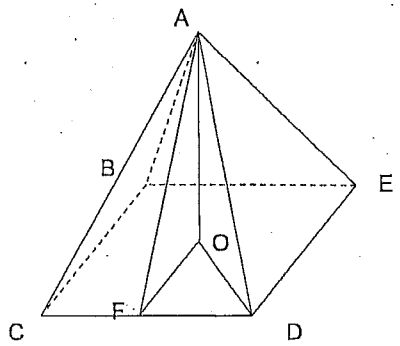
SECTION 2 ALGEBRA	15 Marks
1. Write an algebraic expression for the following: "The sum of $2x$ and $3y$ minus $4z$ "	1
2. Circle the unlike term $4xy^2,\ 6y^2x,\ -x^2y,\ \frac{7xy^2}{3}$	1
3. Simplify by collecting like terms: (a) $4x-3x$ (b) $2m-9k-8m+3k$	2
4. Simplify these expressions: (a) $4k \times -2k$ (b) $p^6 \times p^2$ (c) $8m \div 12m^2$ (d) $(3n^2)^3$	4
5. Expand and simplify $3p+4(p+5)-7$	2
6. Factorise fully $9my^2-12my$	1

7. Simplify completely (a) $\frac{a}{15} \times \frac{25}{2a}$ (b) $\frac{3x}{5}+\frac{2x-3}{4}$	4

SECTION 3 PYTHAGORAS' THEOREM	25 Marks
1. Name the hypotenuse. 	1
2. Write Pythagoras' Theorem in words.	1
3. Find the exact length of the unknown side. a)  b) 	4

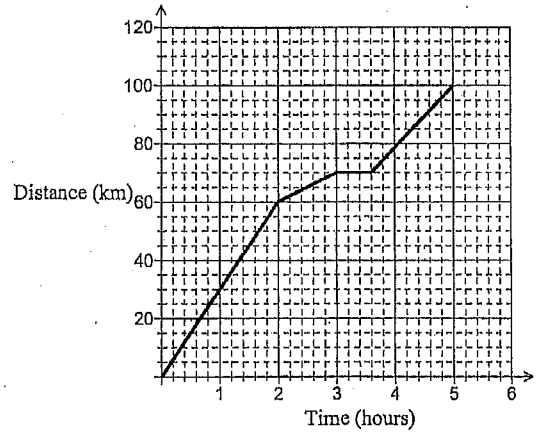
4. Find the length of the unknown side, correct to two decimal places. a)  b) 	4
5. An equilateral triangle has a perpendicular height of 12 metres. Calculate the length of the sides of the equilateral triangle. Answer correct to 1 decimal place.	3

6. Ben rode his horse 3 km north and then turned and rode 2 km west. He then returned to his starting point. a) Draw a diagram to represent this information, clearly showing the distances. b) What was the total distance that Ben travelled? Give your answer correct to one decimal place.	3
7. The diagonals of a rhombus bisect each other at right angles. Find the perimeter of the rhombus with diagonals of lengths 18cm and 24cm. Draw a diagram showing all this information.	4

8. Given Information for diagram: O is the centre of the square base. Angle AOD and Angle AOF are perpendicular (90 degrees) Line AO is 80 cm long. Lines CD and ED are sides of the square base of 50 cm each. Point F is in the middle of CD (i) Show this information on the diagram below. (ii) Using Pythagoras' theorem, calculate the difference in length between AD and AF accurate to three decimal places.  	5

SECTION 4 DATA REPRESENTATION — 25 marks

1.



The graph above represents Mr Brown's charity bike ride.
(Hint: Average Speed = distanced travelled÷ time taken)

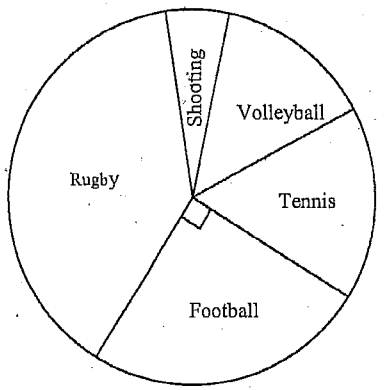
a) What is his speed in the first two hours of his bike ride? — 1

b) After those two hours, does he speed up or slow down? — 1

c) How long does he rest for? — 1

d) Find Mr Brown's average speed for the whole 5 hour ride. — 1

2. This sector graph shows the first choice of winter sport for the Year 8 students at Newington College.



a) What is the most popular sport? — 1

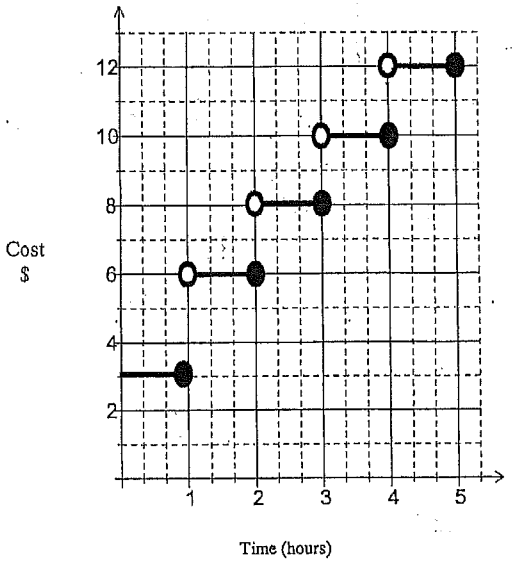
b) By measuring, what is the angle at the centre of the volleyball sector? — 1

c) If there are 180 students in year 8, how many play tennis? — 1

d) What fraction of the students chose shooting? — 1

e) What percentage of the students chose rugby and football? — 1

3. The graph below shows the cost of parking a car at a city parking station.



a) How much does it cost to park your car for 1 hour? 1

b) How much does it cost to park your car for $2\frac{1}{2}$ hours? 1

c) Eric paid $12 to park his car. If he arrived at the parking station at 11am, what was the latest time he could have left the parking station? 1

4. Thirty test scores in a Mathematics examination are listed below:

30	15	37	21	45	52	14	23	48	36
35	51	35	46	25	44	17	41	33	16
53	57	41	18	18	35	28	16	21	13

i) Complete the stem and leaf plot 3

(UNORDERED)

STEM	LEAF
1	
2	
3	
4	
5	

(ORDERED)

STEM	LEAF

ii) What is the difference between the highest and lowest score? 1

iii) Which score occurs the most? 1

iv) What fraction of scores are greater than 30? 1

5. A survey of the number of CDs bought in the past month by a group of Year 8 students gave the following results shown below.

2 3 1 0 2 2 5 0 2 1 5

Display the results on a dot plot. 2

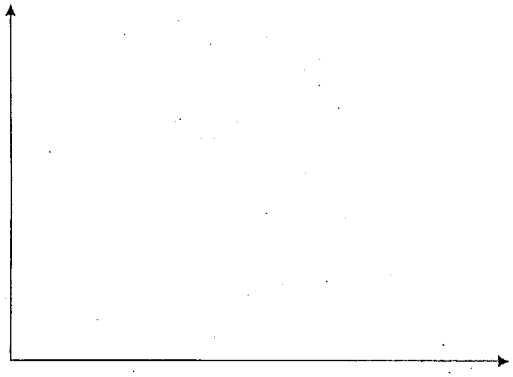
6. The data below shows the number of goals scored by a netball goal shooter in 20 games of netball. **5**

12 13 14 15 10 16 13 15 14 13
11 13 12 16 13 12 13 15 12 10

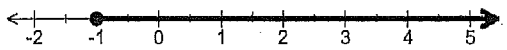
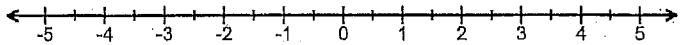
(a) Put the data into a frequency table.

Score	Frequency

(b) Display the results in a combined frequency histogram and polygon. Clearly label the axes.



SECTION 5 EQUATIONS, INEQUATIONS AND FORMULAE	25 marks
1. Show, by working out, that $x=-2$ is a solution to the equation $x-1=-3$.	1
2. Complete the steps needed to solve this equation: $4m-3=2m+1$ $4m = 2m$ _____ _____ $= 4$ $m =$ _____	3
3. Solve the following equations: (a) $6x=15$ (b) $\frac{k}{5}-3=8$ (c) $7(p-4)=7$ (d) $8+8n=5n-4$ (e) $2-5(x+3)=2x$	10

4. Write down the inequality that has been graphed on this number line:  	1
5. Is $x = 5$ a solution to the inequality $x < 9$?	1
6. If $T = a + (n-1)d$, $a = -3$, $n = 5$ and $d = -2$, find the value of T.	2
7. Given $y = mx + b$ when $y = -12$, $m = -2$ and $b = 8$, find the value of x.	2
8. Solve the inequality below and graph your solution on the number line $6 + 5m > -9$  	3
9. Gale sails one third of her trip at 4 km/h, the next third at 8 km/h and the last third at 6 km/h. The trip takes 9 hours and 45 minutes. By using an equation, how far was the complete trip?	2

MASTER COPY (SOLUTIONS)

SECTION 1 PERCENTAGES — 10 Marks

1. Complete the table below (1 for each correct line) — 2

Percentage	Fraction	Decimal
29%	$\frac{29}{100}$	0.29
7%	$\frac{7}{100}$	0.07

(1) (1)

2. Find 16% of 2.4 tonnes. — 1

$\frac{16}{100} \times 2.4\text{ t} = 0.384\text{ t}$ (or 384 kg) (1)

3. Increase \$78.50 by 8% — 2

108% × \$78.50 = \$84.78 (full 2 marks)

OR 8% × 78.50 = \$6.28 (1)

∴ 78.50 + 6.28 = \$84.78 (1)

4. What percentage is 209 mm of 38 cm? — 1

$\frac{209}{380} \times 100 = 55\%$ (1)

5. A cake is made up of 1 part flour, 2 parts chocolate and 2 parts sugar. What percentage of the cake is chocolate? — 2

* 5 PARTS (1)
* CHOCOLATE: $\frac{2}{5} \times 100 = 40\%$ (1)

6. If after a 6 ¼% pay rise I received \$680, how much did I earn before the pay rise? — 2

$106\frac{1}{4}\% \rightarrow \$680$

$1\% \rightarrow \$6.40$ (1)

$100\% \rightarrow \$640$ (1)

SECTION 2 ALGEBRA — 15 Marks

1. Write an algebraic expression for the following: — 1

"The sum of $2x$ and $3y$ minus $4z$"

$2x + 3y - 4z$ (1)

2. Circle the unlike term — 1

$4xy^2$, $6y^2x$, $(-x^2y)$, $\frac{7xy^2}{3}$ (1)

3. Simplify by collecting like terms: — 2

(a) $4x - 3x = 1x$ (1)

(b) $2m - 9k - 8m + 3k = -6m - 6k$ (1)

4. Simplify these expressions: — 4

(a) $4k \times -2k = -8k^2$ (1)

(b) $p^6 \times p^2 = p^8$ (1)

(c) $8m \div 12m^2 = \frac{8m}{12m^2} = \frac{2}{3m}$ (1)

(d) $(3n^2)^3 = 27n^6$ (1)

5. Expand and simplify (1 expansion, 1 collecting 2 like terms) — 2

$3p + 4(p + 5) - 7 = 3p + 4p + 20 - 7 = 7p + 13$

6. Factorise fully — 1

$9my^2 - 12my = 3my(3y - 4)$ (1)

7. Simplify completely 4

(1 for unsimplified answer)

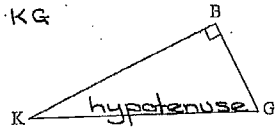
(a) $\frac{\cancel{a}}{\cancel{15}\,3} \times \frac{\cancel{25}\,5}{\cancel{2a}} = \frac{5}{6}$ or $\frac{25a}{30a} = \frac{5}{6}$ 2

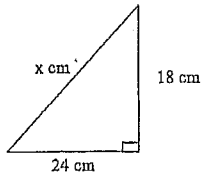
(b) $\frac{3x}{5} + \frac{2x-3}{4} = \frac{12x}{20} + \frac{5(2x-3)}{20}$ 1

$= \frac{12x + 10x - 15}{20}$

$= \frac{22x - 15}{20}$ 1

Penalise for rounding only once in this section

SECTION 3 PYTHAGORAS' THEOREM	25 Marks
1. Name the hypotenuse. KG 	1
2. Write Pythagoras' Theorem in words. In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.	1
3. Find the exact length of the unknown side.	4

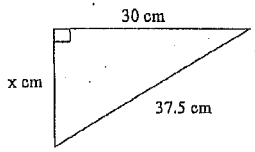
a) 

$x^2 = 24^2 + 18^2$ (1)

$= 900$

$x = \sqrt{900}$

$x = 30$ (1)

b) 

$x^2 = 37.5^2 - 30^2$ (1)

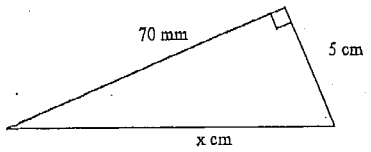
$= 506.25$

$x = \sqrt{506.25}$

$= 22.5$ (1)

4. Find the length of the unknown side, correct to two decimal places. 4

a)



$x^2 = 7^2 + 5^2$ ①

$= 74$

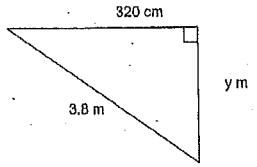
$x = \sqrt{74}$

$\doteq 8.602325267$

$x = 8.60$ to 2dp ①

∴ The length is 8.60cm to 2dp.

b)



$y^2 = 3.8^2 - 3.2^2$ ①

$= 4.2$

$y = \sqrt{4.2}$

$\doteq 2.049390153$

$= 2.05$ to 2dp ①

∴ The length is 2.05m to 2dp.

5. An equilateral triangle has a perpendicular height of 12 metres. Calculate the length of the sides of the equilateral triangle. Answer correct to 1 decimal place. 3

Let the length of the sides be $2x$

$(2x)^2 = x^2 + 12^2$ ①

$4x^2 = x^2 + 12^2$

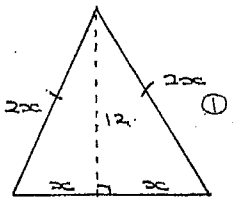
$3x^2 = 144$

$x^2 = 48$

$x = \sqrt{48}$

$\therefore 2x = 2\sqrt{48}$

$\doteq 13.85640646$ ①



∴ The length of each side is 13.9m to 1dp

* penalise only once for missing right angle

6. Ben rode his horse 3 km north and then turned and rode 2 km west. He then returned to his starting point. 3

a) Draw a diagram to represent this information, clearly showing the distances.

b) What was the total distance that Ben travelled? Give your answer correct to one decimal place.

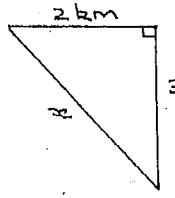
Let x be the distance

$x^2 = 3^2 + 2^2$ ①

$= 13$

$x \doteq 3.605551275$

$= 3.6$ to 1 dp



① must have the right angle

∴ Total distance $= 2 + 3 + 3.6$

$= 8.6$km to 1 dp ①

7. The diagonals of a rhombus bisect each other at right angles. Find the perimeter of the rhombus with diagonals of lengths 18cm and 24cm. Draw a diagram showing all this information. 4

Let x be the length of the side

$x^2 = 9^2 + 12^2$ ①

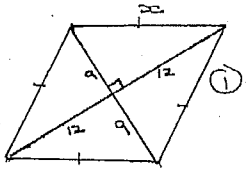
$= 225$

$x = \sqrt{225}$

$= 15$ ①

Perimeter $= 15 \times 4$

$= 60$cm ①

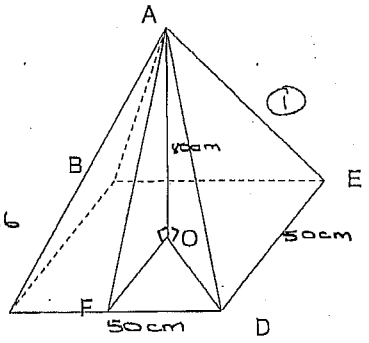


①

8. Given Information for diagram:
O is the centre of the square base.
Angle AOD and Angle AOF are perpendicular (90 degrees)
Line AO is 80 cm long.
Lines CD and ED are sides of the square base of 50 cm each.
Point F is in the middle of CD

(i) Show this information on the diagram below.
(ii) Using Pythagoras' theorem, calculate the difference in length between AD and AF accurate to three decimal places.

5



$BD^2 = BE^2 + ED^2$
$= 50^2 + 50^2$ (1)
$= 5000$
$BD = \sqrt{5000}$
$\doteq 70.71067812$
$OD = \frac{1}{2} BD$
$\doteq 35.35533906$

In $\triangle AOD$,
$AD^2 = AO^2 + OD^2$
$= 80^2 + OD^2$
$= 7650$
$AD = \sqrt{7650}$
$\doteq 87.46427842$ (1)

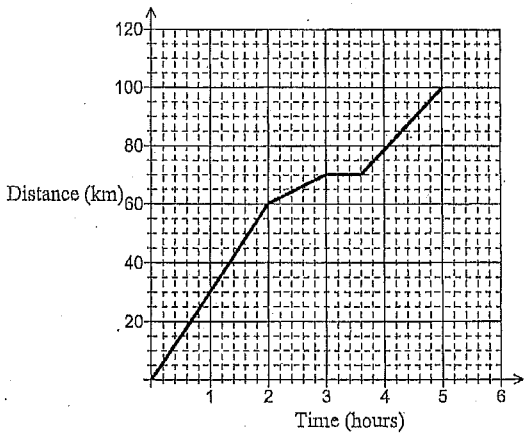
In $\triangle AOF$
$AF^2 = FO^2 + OA^2$
$= 25^2 + 80^2$
$= 7025$
$AF = \sqrt{7025}$
$\doteq 83.81527307$ (1)

Difference $= AD - AF$
$\doteq 3.649005351$ (1)

$\therefore$ Difference in length $= 3.649$cm to 3dp

SECTION 4 DATA REPRESENTATION — 25 marks

1.



The graph above represents Mr Brown's charity bike ride.
(Hint: Average Speed = distanced travelled÷ time taken)

a) What is his speed in the first two hours of his bike ride? — 1

Speed $= \frac{D}{T}$
$= \frac{60}{2}$ (1)
$= 30$ km/h

b) After those two hours, does he speed up or slow down? — 1

slow down (1)

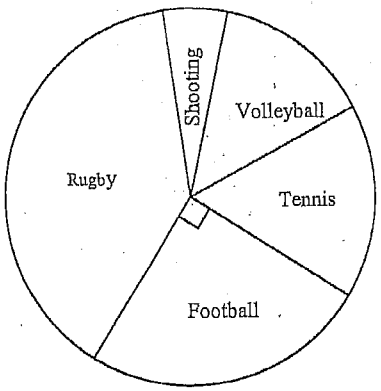
c) How long does he rest for? — 1

36 minutes (1)

d) Find Mr Brown's average speed for the whole 5 hour ride. — 1

speed $= \frac{100}{5}$
$= 20$ km/h (1)

2. This sector graph shows the first choice of winter sport for the Year 8 students at Newington College.



a) What is the most popular sport? rugby (1) — 1

b) By measuring, what is the angle at the centre of the volleyball sector? 50° (1) — 1

c) If there are 180 students in year 8, how many play tennis?

students playing tennis = $\frac{60}{360} \times 180$ (1)

$= 30$ — 1

d) What fraction of the students chose shooting?

$\frac{20}{360} = \frac{1}{18}$ (1) — 1

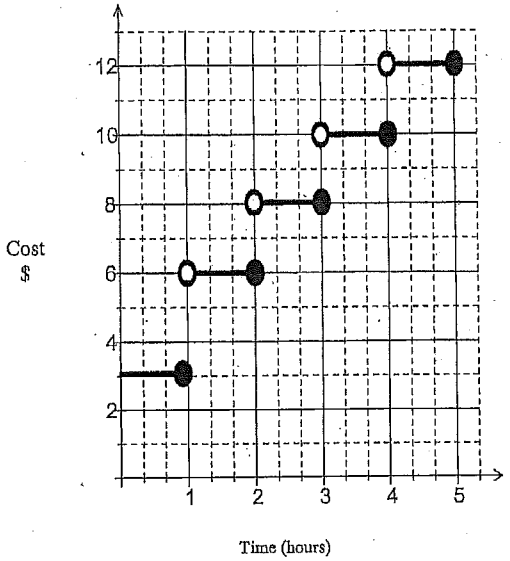
e) What percentage of the students chose rugby and football?

percentage = $\frac{140+90}{360} \times 100$

$= \frac{230}{360} \times 100$

$= 63\frac{8}{9}\%$ (1) — 1

3. The graph below shows the cost of parking a car at a city parking station.



a) How much does it cost to park your car for 1 hour? $3 (1) — 1

b) How much does it cost to park your car for $2\frac{1}{2}$ hours? $8. (1) — 1

c) Eric paid $12 to park his car. If he arrived at the parking station at 11am, what was the latest time he could have left the parking station? 4pm (1) — 1

4. Thirty test scores in a Mathematics examination are listed below:

30	15	37	21	45	52	14	23	48	36
35	51	35	46	25	44	17	41	33	16
53	57	41	18	18	35	28	16	21	13

i) Complete the stem and leaf plot

(UNORDERED) ①

STEM	LEAF
1	5 8 8 4 7 6 6 3
2	1 5 8 3 1
3	0 5 7 5 5 3 6
4	1 6 5 4 1 8
5	3 1 7 2

(ORDERED) ②

STEM	LEAF
1	3 4 5 6 6 7 8 8
2	1 1 3 5 8
3	0 3 5 5 5 6 7
4	1 1 4 5 6 8
5	1 2 3 7

3

ii) What is the difference between the highest and lowest score? 57-13=44 ① 1

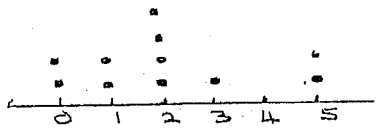
iii) Which score occurs the most? 35 ① 1

iv) What fraction of scores are greater than 30? $\frac{16}{30} = \frac{8}{15}$ ① 1

5. A survey of the number of CDs bought in the past month by a group of Year 8 students gave the following results shown below.

2 3 1 0 2 2 5 0 2 1 5

Display the results on a dot plot. 2



6. The data below shows the number of goals scored by a netball goal shooter in 20 games of netball. 5

12	13	14	15	10	16	13	15	14	13
11	13	12	16	13	12	13	15	12	10

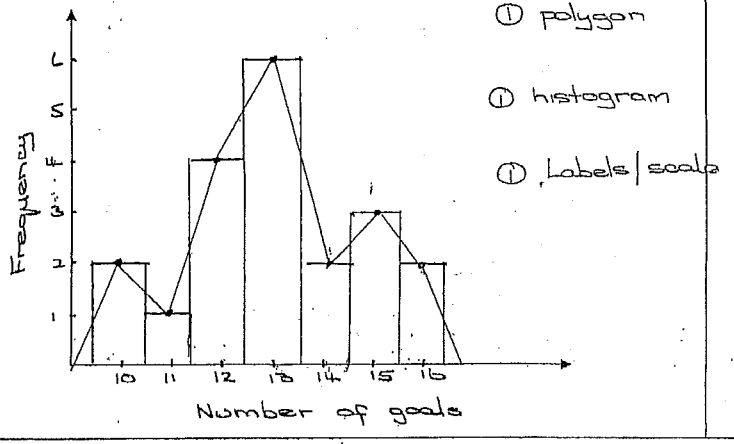
(a) Put the data into a frequency table.

Score	Frequency
10	2
11	1
12	4
13	6
14	2
15	3
16	2

②

(b) Display the results in a combined frequency histogram and polygon. Clearly label the axes.

Number of goals scored by a goal shooter



① polygon

① histogram

① Labels | scale

SECTION 5 EQUATIONS, INEQUATIONS AND FORMULAE — 25 marks

1. Show, by working out, that $x=-2$ is a solution to the equation $x-1=-3$. — 1

By substitution LHS $= -2-1$ ✓1
$= -3 =$ RHS ✓

2. Complete the steps needed to solve this equation: — 3

$4m-3=2m+1$

$4m = 2m + 4$ ✓1
$2m = 4$ ✓1
$m = 2$ ✓1

3. Solve the following equations: — 10

(a) $6x=15$ $x = \frac{15}{6} = 2\frac{1}{2}$ ✓1

(b) $\frac{k}{5}-3=8$

$\frac{k}{5} = 11$ ✓1
$k = 55$ ✓1

(c) $7(p-4)=7$

$7p - 28 = 7$ ✓1 expansion
$7p = 35$
$p = 5$ ✓1

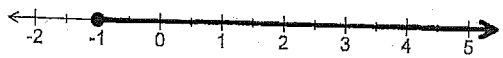
(d) $8+8n=5n-4$

$8 + 3n = -4$ ✓1
$3n = -12$
$n = -4$ ✓1

(e) $2-5(x+3)=2x$

$2 - 5x - 15 = 2x$ ✓1 expansion
$-7x - 13 = 0$
$-7x = 13$ ✓1
$x = -\frac{13}{7} = -1\frac{6}{7}$ ✓1

4. Write down the inequality that has been graphed on this number line: — 1



$x \geq -1$

5. Is $x=5$ a solution to the inequality $x<9$? — 1

Yes.

6. If $T=a+(n-1)d$, $a=-3$, $n=5$ and $d=-2$, find the value of T. — 2

$T = -3 + (5-1) \times -2$ ✓1 for substitution
$= -3 + 4 \times -2 = -11$ ✓1 for answer.

7. Given $y=mx+b$ when $y=-12$, $m=-2$ and $b=8$, find the value of x. — 2

$-2x + 8 = -12$ ✓1 for substitution
$-2x = -20$ ✓1 for answer.
$x = 10$.

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8. Gale sails one third of her trip at 4 km/h, the next third at 8 km/h and the last third at 6 km/h. The trip takes 9 hours and 45 minutes. — 2

By using an equation, how far was the complete trip?

TIME = DISTANCE / SPEED

Let x = total distance

$\frac{\frac{1}{3}x}{4} + \frac{\frac{1}{3}x}{8} + \frac{\frac{1}{3}x}{6} = 9\frac{3}{4}$ hours ✓1 for equation

$\frac{x}{12} + \frac{x}{24} + \frac{x}{18} = 9\frac{3}{4}$ ✓1 for solution

$\frac{13x}{72} = 9\frac{3}{4}$ $\therefore x =$ 54 km

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9. Solve the inequality below and graph your solution on the number line — 3

$6 + 5m > -9$ ✓2 for solving inequation
$5m > -15$ ✓1 for number line
$m > -3$