NORTH SYDNEY GIRLS HIGH SCHOOL

2010
TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84
Attempt Questions 1–7
All questions are of equal value

At the end of the examination, place your solution booklets in order and put this question paper on top. Submit one bundle. The bundle will be separated before marking commences so that anonymity will be maintained.

Student Number: ____________________ **Teacher:** ____________________

Student Name: ____________________

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

Total Marks – 84
Attempt Questions 1–7
All questions are of equal value

Begin each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The polynomial $P(x) = x^4 - 2x^3 + ax + b$ is exactly divisible by $x^2 - 1$. Find the values of a and b. **2**

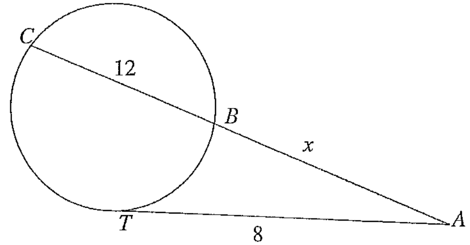
(b) Solve $\dfrac{x+1}{x-1} \geq 2$ **3**

(c) (i) In how many ways can the letters of the word BIOLOGIST be arranged? **1**

(ii) What is the probability the letters "I" will be next to each other? **2**

(d) The acute angle between the lines $y = 2x - 7$ and $y = mx + 1$ is 45°. Find the two possible values of m. **2**

(e)



The line AT is a tangent to the circle at T.
Given $AT = 8$, $BC = 12$ and $AB = x$, find the value of x. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int \frac{dx}{1+4x^2}$ **2**

(b) Find the roots of the following equation $4x^3 - 4x^2 - 29x + 15 = 0$ given one root is the difference between the other two roots. **3**

(c) $P(2p, p^2)$ is a point on the parabola $x^2 = 4y$ with focus S.
R is the point which divides the interval SP internally in the ratio 1:2.

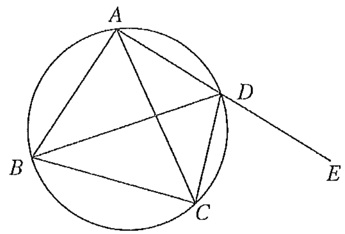
(i) Write down the coordinates of R in terms of p. **2**

(ii) Hence show that as P moves on the parabola $x^2 = 4y$, the locus of R is the parabola $9x^2 = 12y - 8$. **2**

(d) Express $\sin\theta$ and $\cos\theta$ in terms of t, where $t = \tan\frac{\theta}{2}$. **3**
Hence solve $2\sin\theta + 4\cos\theta = 3$, $0 \le \theta \le 2\pi$ correct to 2 decimal places.

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



In the diagram above, ABC is a triangle in which $BC = AC$.
D is a point on the minor arc AC of the circle passing through A, B and C.
AD is produced to E.

(i) Copy the diagram into your writing booklet.

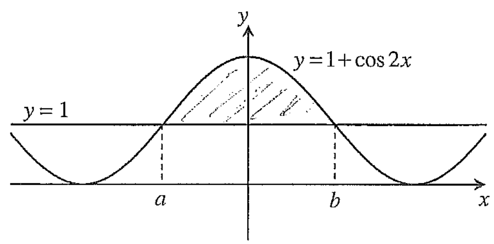
(ii) Give a reason why $\angle CDE = \angle ABC$. **1**

(iii) Prove that DC bisects $\angle BDE$. **2**

(b) Evaluate $\lim_{x \to 0}\left(\frac{\sin 2x}{3x}\right)$. **1**

(c) Using the substitution $u = e^x - 1$, find the value of $\int_0^{\ln 2} e^{2x}\sqrt{e^x - 1}\,dx$. **4**

(d)



Part of the curve $y = 1 + \cos 2x$ is shown above.
The curve cuts the line $y = 1$ at $x = a$ and $x = b$.

(i) Write down the values of a and b. **1**

(ii) The area between the curve $y = 1 + \cos 2x$ and $y = 1$ from $x = a$ to $x = b$ is rotated about the x-axis.
Find the exact volume of the solid generated. **3**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the coefficient of x in the expansion of $\left(x-\frac{3}{x^2}\right)^{16}$. **3**

(You may leave your answer in unsimplified form.)

(b) After t years, $t \geq 0$ the number N of individuals in a population is given by $N = A + Be^{-0.4t}$ for some constants $A > 0$ and $B > 0$. The initial population size is 500 individuals and the limiting population size is 100 individuals.

(i) Find values for A and B. **2**

(ii) Find the time taken for the population to fall within 10 of its limiting value, giving your answer correct to the nearest month. **2**

(c) Consider the function $y = 2\cos^{-1}(x-1)$.

(i) State the domain of $y = 2\cos^{-1}(x-1)$. **1**

(ii) Draw a neat sketch of $y = 2\cos^{-1}(x-1)$. **2**

(iii) Find the exact area enclosed by $y = 2\cos^{-1}(x-1)$ and the coordinate axes. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Prove by mathematical induction that **3**

$$2+5+9+\dots\dots+\left(2^{n-1}+2n-1\right)=2^n+n^2-1 \text{ for all integers } n \geq 1.$$

(b) Consider the function $f(x)=\frac{e^x}{e^x+4}$.

(i) Show that $y = f(x)$ has no stationary points. **2**

(ii) Explain why $y = f(x)$ has an inverse function for all values of x. **1**

(iii) Show that $0 < f(x) < 1$ for all x. **2**

(iv) Sketch the graph of $y = f^{-1}(x)$, the inverse function, given $y = f(x)$ has a point of inflexion at $\left(\ln 4, \frac{1}{2}\right)$. **2**

(v) Find the equation of the inverse function $y = f^{-1}(x)$. **2**

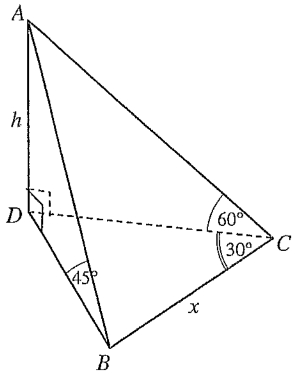
Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Write down the general solutions to the equation $2\cos 5\theta = 1$. **2**

(ii) Hence or otherwise find all solutions of the equation **1**

$$2\cos 5\theta = 1 \text{ for } 0 \le \theta \le \tfrac{\pi}{2}$$

(b) $ABCD$ is a triangular pyramid with base BCD and perpendicular height $AD = h$. $\angle BCD = 30^\circ$, $\angle ABD = 45^\circ$ and $\angle ACD = 60^\circ$.



(i) Use the cosine rule to show that $2h^2 + 3xh - 3x^2 = 0$. **2**

(ii) Hence show that $\dfrac{h}{x} = \dfrac{\sqrt{33} - 3}{4}$. **2**

(c) Consider the expansion $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots\dots\dots + \binom{n}{n}x^n$.

(i) By integrating both sides of the above expression show that **3**

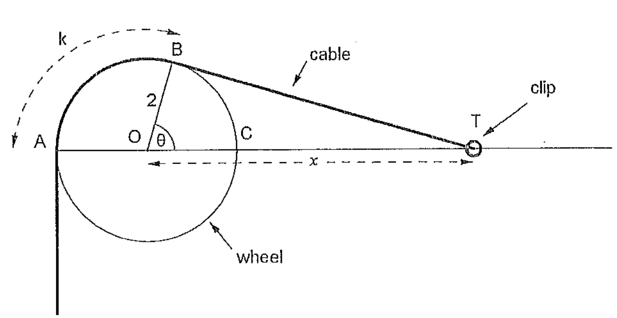
$$\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \dots\dots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1} - 1}{n+1}.$$

(ii) Hence show that **2**

$$\frac{1}{2}\binom{n}{1} + \frac{2}{3}\binom{n}{2} + \frac{3}{4}\binom{n}{3} + \dots\dots + \frac{n}{n+1}\binom{n}{n} = \frac{2^n(n-1)+1}{n+1}.$$

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



A long cable is wrapped over a wheel of radius 2 metres and one end is attached to a clip at T.
The centre of the circle is at O and the distance OT is x metres.
TB is a tangent to the circle and $\angle COB = \theta$ (in radians).
Let k = length of the cable from A to B.
Let s = length of the cable from A to T.

(i) Explain why $\cos\theta = \frac{2}{x}$. **1**

(ii) Show that $s = 2\left[\pi - \cos^{-1}\left(\frac{2}{x}\right)\right] + \sqrt{x^2 - 4}$. **2**

(iii) Show that $\dfrac{ds}{dx} = \dfrac{\sqrt{x^2 - 4}}{x}$. **2**

(iv) The clip at T moves away from O along the line OR at a constant rate of 4 metres per second. **2**
Find the rate at which s changes when $x = 8$.

(b) A social club consists of n men and n women. A committee of 3 people is to be chosen.

(i) How many different committees consist of 2 men and 1 woman? **1**

(ii) Let p be the probability that a randomly chosen committee consists of 2 men and 1 woman. **2**

Show that $p = \dfrac{3n}{8n - 4}$.

(iii) Hence deduce that $\dfrac{3}{8} < p \le \dfrac{1}{2}$ **2**

End of paper

Question 1

(a) $P(x) = x^4 - 2x^3 + ax + b$

$(x-1)$ factor

$\therefore P(1) = 1 - 2 + a + b = 0$

$\therefore \boxed{a + b = 1}$ — ①

$(x+1)$ factor

$\therefore P(-1) = 1 + 2 - a + b = 0$

$\boxed{-a + b = -3}$ — ②

① + ② $2b = -2$

$\therefore b = -1,\ a = 2$

(b) $\dfrac{x+1}{x-1} \geq 2$

$\dfrac{(x+1)(x-1)^2}{(x-1)} \geq 2(x-1)^2, \quad x \neq 1$

$2(x-1)^2 \leq (x+1)(x-1)$

$2(x-1)^2 - (x+1)(x-1) \leq 0$

$(x-1)[2x - 2 - x - 1] \leq 0$

$(x-1)(x-3) \leq 0$

$\therefore 1 < x \leq 3$ since $x \neq 1$

(c) B I O L O G I S T

(i) $\dfrac{9!}{2!\,2!} = 90\,720$

(ii) [I I] B O L O G S T

$\dfrac{8!}{2!}$

$P(\text{I's together}) = \dfrac{8!}{2!} \Big/ \dfrac{9!}{2!\,2!}$

$= \dfrac{8! \times 2!}{9!} = \dfrac{2}{9}$

(d) $\tan 45° = \left| \dfrac{m-2}{1+2m} \right|$

$|1+2m| = |m-2|$

$1 + 2m = m - 2$ or $1 + 2m = -(m-2)$

$m = -3 \qquad 3m = 1$

$\therefore m = -3, \frac{1}{3}$

(e) $x(x+12) = 8^2$

$x^2 + 12x - 64 = 0$

$(x+16)(x-4) = 0$

$\therefore x = 4 \ [x > 0]$

Question 2

(a) $\displaystyle\int \frac{dx}{1+4x^2} = \frac{1}{4}\int \frac{dx}{\frac{1}{4} + x^2}$

$= \frac{1}{4} \cdot \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{x}{\frac{1}{2}}\right) + c$

$= \frac{1}{2} \tan^{-1} 2x + c$

(b) let roots $\alpha, \beta, \alpha - \beta$

sum of roots $\alpha + \beta + \alpha - \beta = -\frac{-4}{4}$

$2\alpha = 1$

$\alpha = \frac{1}{2}$

product of roots $\alpha\beta(\alpha - \beta) = \frac{-15}{4}$

$\frac{1}{2}\beta\left(\frac{1}{2} - \beta\right) = \frac{-15}{4}$

$\beta(1 - 2\beta) = -15$

$2\beta^2 - \beta - 15 = 0$

$(2\beta + 5)(\beta - 3) = 0$

$\beta = \frac{-5}{2}, 3$

$\therefore$ roots $\frac{1}{2}, \frac{-5}{2}, \frac{1}{2} - \frac{-5}{2} \rightarrow \frac{1}{2}, \frac{-5}{2}, 3$

$\frac{1}{2}, +3, \frac{1}{2} - 3 \rightarrow \frac{1}{2}, 3, \frac{-5}{2}$

Question 2 (ctd)

(c) (i) $S(0,1)\quad P(2p, p^2)\quad k:l = 1:2$

$R\left(\dfrac{2p + 2\times 0}{2+1}, \dfrac{p^2 + 2}{2+1}\right)$

$\therefore R\left(\dfrac{2p}{3}, \dfrac{p^2+2}{3}\right)$

(ii) $\therefore x = \frac{2p}{3} \quad \therefore p = \frac{3x}{2}$

$y = \dfrac{p^2 + 2}{3}$

$\therefore 3y = \left(\frac{3x}{2}\right)^2 + 2$

$3y - 2 = \dfrac{9x^2}{4}$

$9x^2 = 4(3y - 2)$

$9x^2 = 12y - 8$

(d) $\sin\theta = \dfrac{2t}{1+t^2} \quad \cos\theta = \dfrac{1-t^2}{1+t^2}$

$2\sin\theta + 4\cos\theta = 3$

$2\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right) = 3$

$4t + 4 - 4t^2 = 3 + 3t^2$

$7t^2 - 4t - 1 = 0$

$t = \dfrac{4 \pm \sqrt{16+28}}{14} = \dfrac{4 \pm \sqrt{44}}{14}$

$= 0.759\ldots,\ -0.188\ldots$

$\therefore \frac{\theta}{2} = 0.64\ldots \quad 2.955\ldots$

$\theta = 1.30, \quad 5.91$

Question 3

(a) (i) exterior angle of cyclic quad ($\angle CDE$)

= opposite interior angle ($\angle ABC$)

(ii) $\angle ABC = \angle BAC$ (equal angles isosceles ΔACB)

$\angle BDC = \angle BAC$ (angles in same segment)

$\therefore \angle BDC = \angle ABC$

but $\angle CDE = \angle ABC$ from (i)

$\therefore \angle BDC = \angle CDE$

$\therefore DC$ bisects $\angle BDE$

(b) $\displaystyle\lim_{x\to 0} \frac{\sin 2x}{3x} = \frac{2}{3}\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right) = \frac{2}{3}$

$= \frac{2}{3} \times 1 = \frac{2}{3}$

(c) $\displaystyle I = \int_0^{\ln 2} e^{2x}\sqrt{e^x - 1}\, dx$

let $u = e^x - 1$

$x = 0 \quad u = e^0 - 1 = 0$

$x = \ln 2 \quad u = e^{\ln 2} - 1 = 2 - 1 = 1$

$\dfrac{du}{dx} = e^x$

$\therefore du = e^x\, dx$

$\displaystyle\therefore I = \int_0^{\ln 2} e^x \sqrt{e^x - 1}\ .\ e^x\, dx$

$\displaystyle = \int_0^1 (u+1)\sqrt{u}\ .\ du$

$\displaystyle = \int_0^1 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$

$= \left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$

$= \frac{2}{5} + \frac{2}{3}$

$= \frac{16}{15}$

Question 5

(a) test $n=1$

$LHS = 2. \quad RHS = 2^1 + 1^2 - 1 = 2$

$\therefore$ true for $n=1$

assume true for $n=k$

i.e. $2+5+9+\dots+2^{k-1}+2k-1 = 2^k + k^2 - 1$

to show true for $n=k+1$

i.e. $2+5+\dots+\left[2^{k+1}+2(k+1)-1\right] = 2^{k+1}+(k+1)^2-1$

Now $LHS = S_k + T_{k+1}$

$= 2^k + k^2 - 1 \quad + 2^k + 2(k+1) - 1$

$= 2.2^k + k^2 - 1 + 2k + 2 - 1$

$= 2^{k+1} + k^2 + 2k$

$= 2^{k+1} + (k^2+2k+1) - 1$

$= 2^{k+1} + (k+1)^2 - 1$

$\therefore$ True for $n=k+1$

$\therefore$ Since true for $n=1$ and true for $n=k+1$ when true for $n=k$, true for all $n \geq 1$ by induction

(b) (i) $y = \dfrac{e^x}{e^x+4}$

$\dfrac{dy}{dx} = \dfrac{e^x(e^x+4) - e^x(e^x)}{(e^x+4)^2}$

$= \dfrac{4e^x}{(e^x+4)^2}$

$\neq 0$ as $4e^x > 0$

$\therefore$ No stationary points

(ii) $y=f(x)$ continuous and increasing.

$\therefore$ always inverse

(iii) $f(x)$ always positive $\therefore y > 0$

denominator > numerator $\therefore y < 1$

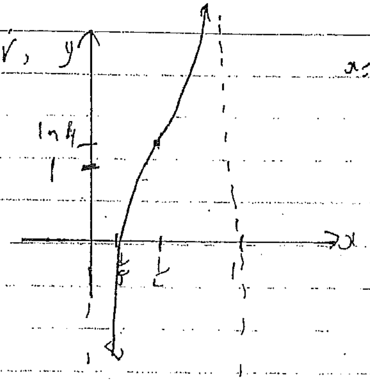
$\therefore 0 < y < 1$

OR $\lim\limits_{x \to \infty} \dfrac{e^x}{e^x+4} = \dfrac{1}{1+\frac{4}{e^x}} = 1$

$\lim\limits_{x \to -\infty} \dfrac{e^x}{e^x+4} = \dfrac{e^{-\infty}}{e^{-\infty}+4} = \dfrac{0}{4} = 0$

$\therefore 0 < y < 1$

(iv)



$x=0 \quad y = \frac{1}{1+4} = \frac{1}{5}$

$\therefore$ x int $= \frac{1}{5}$

(v) $y = \dfrac{e^x}{e^x+4}$

Inverse $x = \dfrac{e^y}{e^y+4}$

$xe^y + 4x = e^y$

$e^y(1-x) = 4x$

$e^y = \dfrac{4x}{1-x}$

$y = \ln\left(\dfrac{4x}{1-x}\right)$

Question 3

(a) (i) $1+\cos 2x = 1$

$\cos 2x = 0$

$2x = \pm\frac{\pi}{2}$

$x = \pm\frac{\pi}{4}$

(ii) $V = \pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1+\cos 2x)^2\,dx - \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\,dx$

$= 2\pi\int_0^{\frac{\pi}{4}} (1+2\cos 2x + \cos^2 2x - 1)\,dx$

$= 2\pi\int_0^{\frac{\pi}{4}} 2\cos 2x + \frac{1}{2}(1+\cos 4x)\,dx$

$= 2\pi\left[\sin 2x + \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right)\right]_0^{\frac{\pi}{4}}$

$= 2\pi\left[\sin\frac{\pi}{2} + \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{4}\sin\pi\right)\right]$

$= 2\pi\left[1 + \frac{\pi}{8} + 1\right]$

$= 2\pi\left[\frac{\pi}{8} + 1\right]$

$= \left(\frac{\pi^2}{4} + 2\pi\right)$ units^3

Question 4

(a) $\left(x - \frac{3}{x^2}\right)^{16}$

$T_k = \binom{16}{k}(x)^{16-k}\left(\frac{-3}{x^2}\right)^k$

$= \binom{16}{k} x^{16-k} (-3)^k x^{-2k}$

$= \binom{16}{k}(-3)^k x^{16-3k}$

coeff x: $16-3k=1$ $\therefore$

$\therefore 3k = 15$

$k=5$

$\therefore$ coefficient $x = \binom{16}{5}(-3)^5$

$= -1\,061\,424$

(b) (i) $N = A + Be^{-0.4t}$

$t=0 \quad N=500 \quad \therefore 500 = A+B$

$t \to \infty \quad N \to 100$ but $N \to A$

$\therefore A = 100$

$\therefore B = 400$

(ii) $110 = 100 + 400e^{-0.4t}$

$\frac{1}{40} = e^{-0.4t}$

$-0.4t = \ln\frac{1}{40}$

$t = \dfrac{\ln 40}{0.4}$

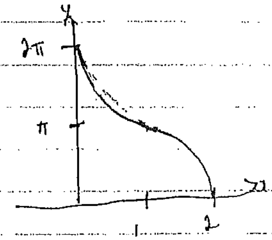
$= 9.22$ yrs

$\doteqdot 111$ months.

(c) (i) $-1 \leq x-1 \leq 1$

$0 \leq x \leq 2$

(ii)



(iii) $y = 2\cos^{-1}(x-1)$

$x = \cos\left(\frac{y}{2}\right)+1$

$A = \int_0^{2\pi}\left(\cos\left(\frac{y}{2}\right)+1\right)dy$

$= \left[2\sin\left(\frac{y}{2}\right) + y\right]_0^{2\pi}$

$= (2\sin\pi + 2\pi) - (2\sin 0 + 0) = 2\pi$ units^2

Question 6

(a) (i) $2\cos 5\theta = 1$

$\cos 5\theta = \frac{1}{2}$

$\therefore 5\theta = 2n\pi \pm \frac{\pi}{3}$

$\theta = \frac{2n\pi}{5} \pm \frac{\pi}{15}$

$= (6n \pm 1)\frac{\pi}{15}$

(ii) $(6n+1)\frac{\pi}{15}$

$n=0 \Rightarrow \frac{\pi}{15}$ $\quad n=1 \Rightarrow \frac{7\pi}{15}$

$(6n-1)\frac{\pi}{15}$

$n=1 \Rightarrow \frac{5\pi}{15} = \frac{\pi}{3}$

$\frac{\pi}{15}, \frac{7\pi}{15}, \frac{\pi}{3}$

(b) (i) $\frac{h}{BD} = \tan 45$ $\quad \frac{h}{DC} = \tan 60 = \sqrt{3}$

$\therefore BD = h$ $\quad DC = \frac{h}{\sqrt{3}}$

$\therefore h^2 = x^2 + \frac{h^2}{3} - 2 \times x \cdot \frac{h}{\sqrt{3}} \cdot \cos 30$

$h^2 = x^2 + \frac{h^2}{3} - \frac{2xh}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}$

$h^2 = x^2 + \frac{h^2}{3} - xh$

$3h^2 = 3x^2 + h^2 - 3xh$

$2h^2 + 3xh - 3x^2 = 0$

(ii) $2\left(\frac{h}{x}\right)^2 + 3\left(\frac{h}{x}\right) - 3 = 0$

$\frac{h}{x} = \frac{-3 \pm \sqrt{9 - 4 \times 2 \times -3}}{4}$

$= \frac{-3 \pm \sqrt{33}}{4}$

$\therefore \frac{h}{x} = \frac{\sqrt{33} - 3}{4}$ since both > 0

(c) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n$

(i) $\frac{(1+x)^{n+1}}{n+1} = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1} + c$

let $x=0$ $\quad \therefore c = \frac{1}{n+1}$

$\therefore \frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1} = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots \frac{1}{n+1}\binom{n}{n}x^{n+1}$

let $x=1$

$\frac{2^{n+1}}{n+1} - \frac{1}{n+1} = \binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \dots \frac{1}{n+1}\binom{n}{n}$

$\frac{2^{n+1}}{n+1} - \frac{1}{n+1} = \frac{2^{n+1} - 1}{n+1}$

$\therefore \binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \dots \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1} - 1}{n+1}$

(ii)

$\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \dots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1} - 1}{n+1}$ — ①

let $x=1$ in original

$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$ — ②

② − ①

$\frac{1}{2}\binom{n}{1} + \frac{2}{3}\binom{n}{2} + \dots \left(1 - \frac{1}{n+1}\right)\binom{n}{n} = 2^n - \frac{2^{n+1} - 1}{n+1}$

$\frac{1}{2}\binom{n}{1} + \frac{2}{3}\binom{n}{2} + \dots \frac{n}{n+1}\binom{n}{n} = \frac{2^n \times n + 2^n - 2^{n+1} + 1}{n+1}$

$RHS = \frac{n \cdot 2^n + 2^n - 2 \cdot 2^n + 1}{n+1}$

$= \frac{2^n(n-1) + 1}{n+1}$

Question 7

(a) (i) $\angle OBT = 90°$ (tangent ⊥ radius)

$\therefore \cos\theta = \frac{2}{x}$ (right triangle)

(ii) $l = r\theta$

$\angle AOB = \pi - \cos^{-1}\left(\frac{2}{x}\right)$

$\therefore l = 2\left[\pi - \cos^{-1}\left(\frac{2}{x}\right)\right]$

$BT^2 = x^2 - 2^2$ (pythagoras)

$\therefore BT = \sqrt{x^2 - 4}$

$\therefore S = l + BT$

$= 2\left[\pi - \cos^{-1}\left(\frac{2}{x}\right)\right] + \sqrt{x^2 + 4}$

(iii) $S = 2\pi - 2\cos^{-1}\left(\frac{2}{x}\right) + \sqrt{x^2 - 4}$

$\frac{ds}{dx} = 0 - 2 \cdot \frac{-1}{\sqrt{1 - \left(\frac{2}{x}\right)^2}} \cdot -2x^{-2} + \frac{1}{2} \cdot 2x\,(x^2 - 4)^{-\frac{1}{2}}$

$= \frac{-4}{x^2\sqrt{1 - \frac{4}{x^2}}} + \frac{x}{\sqrt{x^2 - 4}}$

$= \frac{-4}{x\sqrt{x^2 - 4}} + \frac{x}{\sqrt{x^2 - 4}}$

$= \frac{-4 + x^2}{x\sqrt{x^2 - 4}}$

$\frac{ds}{dx} = \frac{\sqrt{x^2 - 4}}{x}$

(iv) $\frac{ds}{dt} = \frac{ds}{dx} \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{4 \times \sqrt{8^2 - 4}}{8}$

$= \frac{\sqrt{60}}{2}$

$= \sqrt{15}$

(b) (i) 2 men 1 woman $= \binom{n}{2} \cdot \binom{n}{1}$

$= \frac{n(n-1)}{2} \times n$

$= \frac{n^2(n-1)}{2}$

(ii) $n(S) = \binom{2n}{3} = \frac{2n(2n-1)(2n-2)}{3!}$

$= \frac{4n(2n-1)(n-1)}{6}$

$P(\text{2 men 1 woman}) = \frac{n^2(n-1)}{2} \Big/ \frac{4n(2n-1)(n-1)}{6}$

$= \frac{n^2(n-1)}{2} \cdot \frac{6}{4n(2n-1)(n-1)}$

$= \frac{3n}{4(2n-1)} = \frac{3n}{8n-4}$

(iii) Least value for $n = 2$

$P = \therefore P(\text{2 men 1 woman}) = \frac{6}{16-4} = \frac{1}{2}$

as $n \to \infty$ $\quad \frac{3n}{8n-4} \to \frac{3}{8}$ $\quad \therefore \frac{3}{8} < p \leq \frac{1}{2}$

$\therefore$