NSW INDEPENDENT SCHOOLS

2010
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ....

Total marks – 120
Attempt Questions 1 - 10
All questions are of equal value.

Answer the questions on your own paper or writing booklet, if provided.
Start each question on a new page.

Marks

Question 1 (12 marks)

(a) Solve the equation $e^x = 4$. Give your answer correct to three significant figures. **2**

(b) Solve $\dfrac{3x+4}{x-1} = 2$. **2**

(c) Find the gradient of the tangent to the curve $y = \dfrac{2}{x}$ at the point $(-1,-2)$. **2**

(d) Solve $|3x-1| = 10$. **2**

(e) Sketch the graph of $2x+3y=9$, showing the intercepts on both axes. **2**

(f) Find the exact value of x such that $\sec x + 1 = 3$ where $0 \le x \le \dfrac{\pi}{2}$. **2**

Marks

Question 2 (12 marks) Start a new writing booklet.

(a) Shade the region in the plane defined by $y \le 0$ and $y \ge x^2 + 3x$ **2**

(b) Differentiate with respect to x:

(i) $\ln\sqrt{3x^2 - 1}$. **2**

(ii) $x\cos 2x$. **2**

(c) (i) Find $\int dt$. **1**

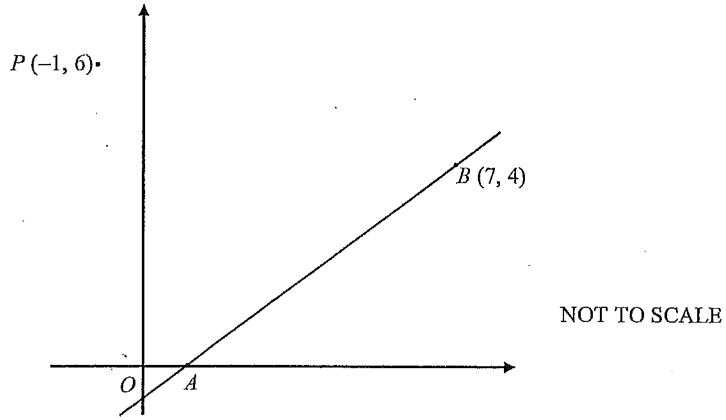
(ii) Find $\int \frac{4}{(2x-1)^3}\,dx$. **2**

(iii) Evaluate $\int_1^e 2x + \frac{1}{x}\,dx$. Leave your answer in exact form. **3**

Marks

Question 3 (12 marks) Start a new writing booklet.

(a)



The line AB has a gradient of $\frac{2}{3}$. The point B has coordinates (7, 4).

(i) Find the equation of AB in the form $ax + by + c = 0$. **2**

(ii) Find the shortest distance of the point P (–1, 6) from the line AB. **2**

(iii) Find the coordinates of A, the point where the line AB intersects with the x-axis. **1**

(iv) Find the distance AB. **1**

(v) Find the angle the line AB makes with the positive direction of the x-axis. **1**

(b) Evaluate $\sum_{m=2}^{6} 10 - m^2$. **2**

(c) Use the Trapezoidal Rule with 4 subintervals to find an approximation for $\int_1^3 f(x)\,dx$ given that **3**

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$f(x)$	11.2	17.8	9.3	4.1	11.6

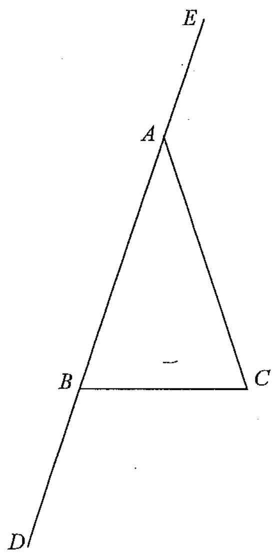
Marks

Question 4 (12 marks) Start a new writing booklet.

(a) Find the common ratio of a geometric series with a first term of 3 and a limiting sum of $\frac{9}{5}$. **1**

(b) Find the values of k for which the expression $x^2-(k-2)x+(k+13)$ is positive definite. **3**

(c)



NOT TO SCALE

ABC is an isosceles triangle in which $AB = AC$. E is a point in BA produced.
D is a point in AB produced such that $BD = BC$.

Copy or trace the diagram into your answer booklet showing all given information.

Show, giving reasons, that $\angle CAE = 4\angle BDC$. **2**

Question 4 continues.

Marks

Question 4 (continued)

(d) Juan started work at 20 and at the beginning of each month he invested \$150 into a superannuation fund. Interest was paid at 6% p.a. compounded monthly on the investment. Juan retired at 65 after having contributed to the fund for 45 years.

(i) How much did Juan contribute to the fund over the 45 years? **1**

(ii) How much did Juan's investment amount to after 45 years? **3**

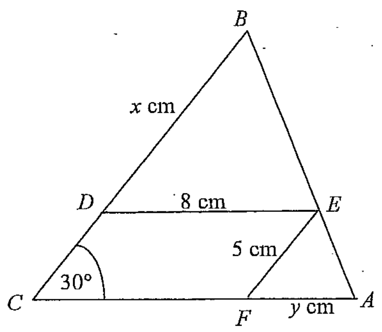
(iii) Juan plans to reinvest some of the money into an account which offers 8% p.a. compound interest compounded annually. **2**
He plans to have \$200 000 at the end of the 10 year investment period.
How much does Juan need to reinvest to achieve this amount.
(Give your answer to the nearest \$10).

End of Question 4

Marks

Question 5 (12 marks) Start a new writing booklet.

(a) In the diagram ABC is a triangle in which $\angle ACB = 30°$ and D, E and F lie on the lines BC, BA and AC respectively.
$CDEF$ is a parallelogram with $DE = 8$ cm and $EF = 5$ cm.



Let $BD = x$ cm and $AF = y$ cm,

(i) Show triangles BDE and EFA are similar. **2**

(ii) Show that $xy = 40$. **1**

(iii) Show that the area, A, of triangle ABC is given by **2**

$$A = 20 + 2x + \frac{50}{x}$$

(iv) Find the values of x and y which will minimise the area of triangle ABC. Justify your answer. **3**

Question 5 continues

Marks

Question 5 (continued)

(b) During July the probability that it rains on any day is $\frac{1}{3}$.
Find the probability that during a 7 day week

(i) it rains on the 1st and 5th day. **1**

(ii) it rains only on the first 3 days. **1**

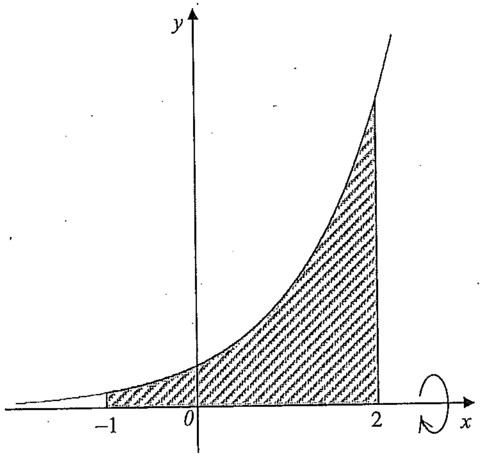
(iii) there is at least one rainy day. **2**

End of Question 5

Marks

Question 6 (12 marks) Start a new writing booklet.

(a) The diagram shows the region bounded by the curve $y = e^x$, the lines $x = -1$ and $x = 2$, and the x-axis. **3**



The region is rotated about the x-axis. Find the volume of the solid of revolution formed. Leave your answer in exact form.

(b) A particle moves in a straight line. At time t seconds, its distance x metres from a fixed point O on the line is given by

$$x = 1 - \cos 2t.$$

(i) Sketch the graph of x as a function of t for $0 \le t \le \pi$. **3**

(ii) Using your graph, or otherwise, find the times when the particle is at rest and the position of the particle at these times. **2**

(iii) Find the velocity of the particle when $t = \frac{\pi}{4}$. **2**

(iv) Over which time periods is the particle's velocity greater than 1 m/s? **2**

Marks

Question 7 (12 marks) Start a new writing booklet.

a) (i) Differentiate $\sqrt{(2x^2+1)^3}$ **2**

(ii) Hence evaluate $2\int_0^2 x\sqrt{2x^2+1}\,dx$ **2**

b) The acceleration of a particle moving in a straight line is given by $6 - 6t$ ms^{-2}
The particle starts from a point 2 metres to the right of the origin and moves towards the origin with an initial velocity of -3 ms^{-1}.

(i) Find an expression for the velocity of the particle in terms of time t. **2**

(ii) Find an expression for the displacement of the particle in terms of time t. **2**

(iii) Find when and where the particle comes to rest. **2**

(iv) Show the particle passes through the origin after 2 second of motion. **1**

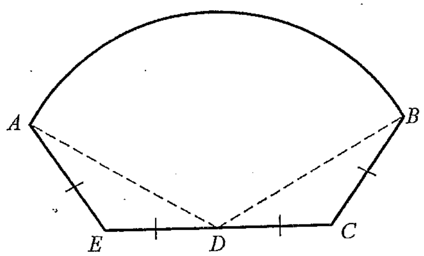
(v) Show the particle never changes direction. **1**

Marks

Question 8 (12 marks) Start a new writing booklet.

a) Find the equation of the parabola with vertex $(1,\ 2)$ and focus $(1,\ 4)$. **2**

b) The Trumpets are building an unusually shaped pool on their country property.



In the diagram, $ABCDE$, represents the shape of the surface of the pool.
The sector ABD has centre D and $\angle ADB = \frac{2\pi}{3}$.
The points C, D, E lie on a straight line.
The arc AB has a length of 6π metres. $AE = ED = DC = CB$.

(i) Show that $AD = 9$ metres. **1**

(ii) Find the length of the BC. **2**

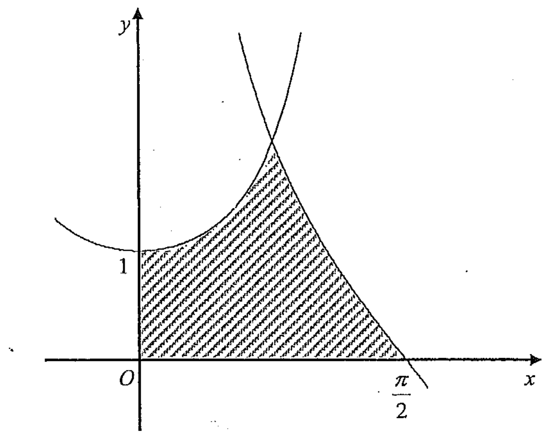
(iii) Find the area of the pool's surface. **2**

Question 8 continues

Marks

Question 8 ((continued)

c)



The diagram shows the region bounded by the curves $y = \sec^2 x$, $y = 2\cot x$ and the coordinate axes.

(i) Verify, by substitution, that the point $\left(\frac{\pi}{4},\ 2\right)$ lies on both $y = \sec^2 x$ and $y = 2\cot x$. **1**

(ii) Differentiate $\ln(\sin x)$ **1**

(iii) Hence, or otherwise, find the exact area of the shaded region. **3**

End of Question 8

Marks

Question 9 (12 marks) Start a new writing booklet.

(a) E. coli bacteria is growing on a piece of chicken at a rate proportional to the amount of bacteria present according to the formula

$$N = Ae^{kt}$$

Time t is in minutes. Initially there are 2000 bacteria present on the chicken. After 5 minutes there are 3500 bacteria on the chicken.

(i) Show that $k = 0.112$, correct to 3 decimal places. **2**

(ii) How many bacteria are on the chicken after 1 hour? **1**

(iii) How long will it take for the number of E. coli bacteria on the chicken to reach 10000? Give answer to the nearest minute. **2**

(b) Consider the function $y = (x^2+1)e^{-x}$.

The first derivative and second derivatives of this function are:

$\frac{dy}{dx} = -e^{-x}(x-1)^2$ and $\frac{d^2y}{dx^2} = e^{-x}(x^2-4x+3)$ (You don't need to show these)

(i) Find any stationary points and determine their nature. **2**

(ii) Find the points of inflexion. **2**

(iii) Sketch the function clearly showing the stationary point, any points of inflexion and any intercepts. **3**

Marks

Question 10 (12 marks) Start a new writing booklet.

a) For the function $y = 2\sin 3x + 4\cos 2x$ find p if $\frac{d^2y}{dx^2} + 4y = p\sin 3x$. **2**

b) At the beginning of 2010 the Watersheds borrowed \$ 1 500 000 to purchase a new race horse. The annual interest rate on their loan is 6% pa compounded monthly. The loan is to be repaid by equal annual repayments of \$ 243 161, the first repayment being made at the end of 2010.

(i) Show that the Watersheds still owe \$ 1 189 420 after they have made their 2nd repayment. (Answer correct to the nearest \$10.) **2**

(ii) Let A_n be the balance owing after the nth repayment. **3**

Show that $A_n = 1500000(1.005)^{12n} - 243161\left(\frac{1.005^{12n}-1}{1.005^{12}-1}\right)$

(iii) In which year will the Watersheds pay off their debt? **2**

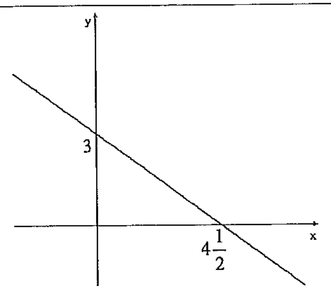
c) Pedro is playing in a tennis competition. He is required to play 8 matches and he must win all 8 to win the competition. **3**
He has a 10% chance of winning the competition.
In each match his probability of winning decreases by 5% of the preceding match's probability.

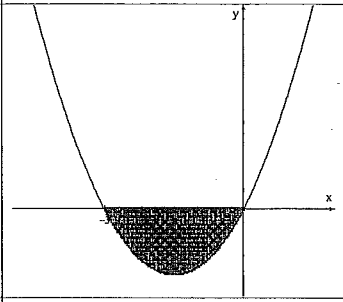
What is the probability that he wins the **first** match?

End of Paper

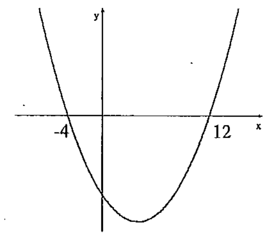
NSW INDEPENDENT TRIAL EXAMS – 2010

MATHEMATICS HSC TRIAL EXAMINATION
MARKING GUIDELINES

Question		Solution	Marks
1	a)	$e^x = 4$ $x = \ln 4$ $x = 1.386294361$ $= 1.39$	**1** ln 4 **1** for sig figures
	b)	$\frac{3x+4}{x-1} = 2$ $3x+4 = 2(x-1)$ $3x+4 = 2x-2$ $x = -6$	**1** **1** for correct answer
	c)	$y = \frac{2}{x}$ $= 2x^{-1}$ $\frac{dy}{dx} = -2x^{-2}$ $= -\frac{2}{x^2}$ At $x = -1$ $\frac{dy}{dx} = -\frac{2}{(-1)^2}$ $= -2$	**1** mark for correct derivative **1** mark for correct gradient
	d)	$\lvert 3x-1 \rvert = 10$ $3x-1 = 10$ $3x-1 = -10$ $3x = 11$ $3x = -9$ $x = \frac{11}{3}$ $x = -3$	**1** mark for each correct answer **(2 marks total)**
	e)		**1** mark for line and one correct intercept. **1** mark for 2nd intercept
	f)	$\sec x + 1 = 3$ $\sec x = 2$ $\cos x = \frac{1}{2}$ $x = \frac{\pi}{3}$	**1** mark **1** mark for correct solution

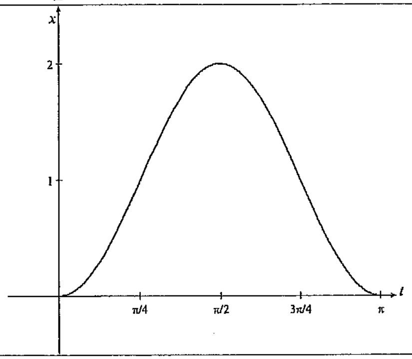
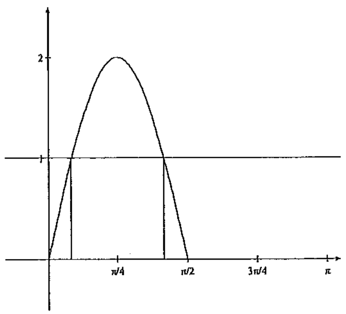
Question		Solution	Marks
2	a)		**1** mark for parabola and correct intercepts **1** mark for correct region shaded
	b)(i)	$\frac{d}{dx}\ln\sqrt{3x^2-1} = \frac{d}{dx}\ln\left(3x^2-1\right)^{\frac{1}{2}}$ $= \frac{d}{dx}\frac{1}{2}\ln\left(3x^2-1\right)$ $= \frac{1}{2}.\frac{6x}{3x^2-1}$ $= \frac{3x}{3x^2-1}$	**1** attempt a differentiation without applying log rules **1** for correct answer
	b)(ii)	$\frac{d}{dx}x\cos 2x = 1.\cos 2x + x.-2\sin 2x$ $= \cos 2x - 2x\sin 2x$	**1** applying product rule **1** for correct answer
	c)(i)	$\int dt = t + c$	**1** mark constant required
	c)(ii)	$\int \frac{4}{(2x-1)^3}dx = \int 4(2x-1)^{-3}dx$ $= \frac{4(2x-1)^{-2}}{-2\times 2}$ $= -\frac{1}{(2x-1)^2} + c$	**1** mark for correct rearrangement and attempted integration **1** mark for correct answer
	c)(iii)	$\int_1^e 2x + \frac{1}{x}\,dx = \left[x^2 + \ln x\right]_1^e$ $= \left[e^2 + \ln e\right] - \left[1 + \ln 1\right]$ $= e^2 + 1 - 1$ $= e^2$	**1** mark for correct integration **1** mark for correct substitution **1** mark correct answer

Question		Solution	Marks
3	a)(i)	$y-4=\frac{2}{3}(x-7)$ $3y-12=2x-14$ $2x-3y-2=0$	1 mark for correct method 1 mark for correct answer in correct form
	a)(ii)	$d=\left\|\frac{2(-1)-3(6)-2}{\sqrt{2^2+3^3}}\right\|$ $=\left\|\frac{-2-18-2}{\sqrt{4+9}}\right\|$ $=\left\|\frac{-22}{\sqrt{13}}\right\|$ $=\frac{22}{\sqrt{13}}$ units	1 mark correct substitution into correct formula 1 mark correct answer
	a)(iii)	At $y=0$, $2x-2=0$ $2x=2$ $x=1$ $A\ (1,\ 0)$	1 mark correct answer
	a)(iv)	$d=\sqrt{(7-1)^2+(4-0)^2}$ $=\sqrt{6^2+4^2}$ $=\sqrt{36+16}$ $=\sqrt{52}$ $=2\sqrt{13}$ units	1 mark $\sqrt{52}$
	a)(v)	$\tan\theta=\frac{2}{3}$ $\theta=\tan^{-1}\frac{2}{3}$ $=33°41'$ or $34°$	1 mark correct answer Degrees or minutes
	b)	$\sum_{m=2}^{6}10-m^2=6+1-6-15-26$ $=-40$	1 mark for series 1 mark for answer
	c)	$\int_1^3 f(x)\,dx=\frac{{}^1\!/_2}{2}(11.2+2(17.8+9.3+4.1)+11.6)$ $=\frac{1}{4}(85.2)$ $=21.3$	1 mark for correct h 1 mark correct sub into correct formula 1 mark correct answer

Question		Solution	Marks
4	a)	$S=\frac{a}{1-r}$ $\frac{9}{5}=\frac{3}{1-r}$ $9-9r=15$ $9r=-6$ $r=-\frac{2}{3}$	1 mark for correct answer
	b)	$x^2-(k-2)x+(k+13)$ Positive Definite V< 0 V$=(k-2)^2-4(k+13)$ $=k^2-4k+4-4k-52$ $=k^2-8k-48$ $=(k-12)(k+4)$ $(k-12)(k+4)<0$ $-4<k<12$ 	1 mark for V 1 mark for V< 0 1 mark correct answer
	c)	Let $\angle BDC=x$ VDBC is isosceles since given $BD=BC$ $\angle DCB=x$ (base $\angle$'s of isosceles VDBC =) $\angle ABC=2x$ (exterior $\angle$ of VDBC = sum opp. interior $\angle$'s) $\angle BCA=2x$ (base $\angle$'s of isosceles VABC =) $\angle EAC=4x$ (exterior $\angle$ of VABC = sum opp. interior $\angle$s) $\therefore \angle CAE=4\angle BDC$	1 mark for realizing VDBC isosceles & using fact 1 mark correct proof
	d)(i)	Investment = $150\times12\times45$ $=\$\ 81\ 000$	1 mark
	d)(ii)	$I=150(1.005)^{540}+150(1.005)^{539}+......+150(1.005)^1$ $=150(1.005)\left(\frac{1.005^{540}-1}{1.005-1}\right)$ $=\$415465.89$	1 mark correct interest rate 1 mark correct series 1 mark correct answer
	d)(iii)	$200000=A(1.08)^{10}$ $A=\frac{200000}{1.08^{10}}$ $A=92638.70$ $A=\$92640$	1 mark 1 mark correct answer

Question		Solution	Marks
5	a)(i)	Since *CDEF* is a parallelogram $DE \parallel CA$ $BC \parallel EF$ $\angle BDE = 30°$ (Matching $\angle BCF$, $DE \parallel CA$) $\angle EFA = 30°$ (Matching $\angle BCF$, $BC \parallel EF$) In $\triangle BDE$ and $\triangle EFA$ $\angle BDE = \angle EFA$ (both = 30°, proven above) $\angle BED = \angle EAF$ (matching $\angle$'s $DE \parallel CA$) $\therefore \triangle BDE \parallel\mid EFA$ (equiangular)	1 mark 1 mark
	a)(ii)	Corresponding sides in similar triangles are proportional. $\frac{x}{5} = \frac{8}{y}$ $xy = 40$ $y = \frac{40}{x}$	1 mark
	a)(iii)	Area $A = \frac{1}{2}ab\sin C$ $= \frac{1}{2}(x+5)(8+y)\sin 30°$ $= \frac{1}{2}(8x+40+xy+5y)\frac{1}{2}$ $= \frac{1}{4}\left(8x+40+x.\frac{40}{x}+5.\frac{40}{x}\right)$ $= \frac{1}{4}\left(8x+40+40+\frac{200}{x}\right)$ $= 2x+20+\frac{50}{x}$	**1** mark for correct expression for area involving x and y **1** correct substitution of $y = \frac{40}{x}$ and simplification
	a)(iv)	$A = 20+2x+\frac{50}{x}$ $= 20+2x+50x^{-1}$ $A' = 2-50x^{-2}$ S.P occur when $A' = 0$ $0 = 2-50x^{-2}$ $0 = 2-\frac{50}{x^2}$ $\frac{50}{x^2} = 2$ $2x^2 = 50$ $x^2 = 25$ $x = 5$ (-5 ignored as x is a length)	**1** mark for finding x

Question		Solution	Marks
5	a)(iv) cont.	$A'' = 100x^{-3}$ at $x = 5$ $A'' = \frac{100}{5^3} > 0$ $\therefore x = 5$ is a minimum $x = 5,\ y = 8$	**1** mark for test of Min **1** mark for finding y
	b)(i)	$P(E) = \frac{1}{3}\times\frac{1}{3}$ $= \frac{1}{9}$	**1** mark
	b)(ii)	$P(E) = \frac{1}{3}.\frac{1}{3}.\frac{1}{3}.\frac{2}{3}.\frac{2}{3}.\frac{2}{3}.\frac{2}{3}$ $= \frac{16}{2187}$	**1** mark
	b)(iii)	$P(\text{at least one rainy day}) = 1 - P(\text{all dry days})$ $= 1-\left(\frac{2}{3}\right)^7$ $= 1-\frac{128}{2187}$ $= \frac{2059}{2187}$	**1** mark for all dry days **1** mark for correct answer

Question		Solution	Marks
6	a)	$A=\pi\int_{-1}^{2}\left(e^{x}\right)^{2}dx$ $=\pi\int_{-1}^{2}e^{2x}dx$ $=\pi\left[\frac{e^{2x}}{2}\right]_{-1}^{2}$ $=\frac{\pi}{2}\left(e^{4}-e^{-2}\right)$ $units^{3}$	**1** mark for correct integral expression **1** mark for correct integration **1** mark correct answer
	b)(i)		**1** mark cos curve **1** mark period **1** mark range
6	b)(ii)	Times when particle is at rest: $t=0,\ \frac{\pi}{2},\ \pi$ seconds Position of particle at these times: $x=0,\ 2,\ 0$	**1** mark for times **1** mark for positions
	b)(iii)	$x=1-\cos 2t$ $v=2\sin 2t$ At $t=\frac{\pi}{4}$ $v=2\sin\left(2\frac{\pi}{4}\right)$ $v=2\text{ ms}^{-1}$	**1** mark for velocity equation **1** mark for velocity
	b)(iv)	$2\sin 2t>1$ $2\sin 2t=1$ $\sin 2t=\frac{1}{2}$ $2t=\frac{\pi}{6},\frac{5\pi}{6}$ $t=\frac{\pi}{12},\frac{5\pi}{12}$ $\frac{\pi}{12}<t<\frac{5\pi}{12}$ Graph may be used. 	**1** mark for $t=\frac{\pi}{12},\frac{5\pi}{12}$ **1** mark for correct solution

Question		Solution	Marks
7	a)(i)	$y=\sqrt{\left(2x^{2}+1\right)^{3}}$ $=\left(2x^{2}+1\right)^{\frac{3}{2}}$ $\frac{dy}{dx}=\frac{3}{2}\left(2x^{2}+1\right)^{\frac{1}{2}}.4x$ $=6x\sqrt{2x^{2}+1}$	**1** mark correct index form **1** mark correct solution
	a)(ii)	$2\int_{0}^{2}x\sqrt{2x^{2}+1}\ dx=\frac{1}{3}\int_{0}^{2}6x\sqrt{2x^{2}+1}\ dx$ $=\frac{1}{3}\left[\sqrt{\left(2x^{2}+1\right)^{3}}\right]_{0}^{2}$ $=\frac{1}{3}\left[\sqrt{\left(2(2)^{2}+1\right)^{3}}-\sqrt{\left(2(0)^{2}+1\right)^{3}}\right]$ $=\frac{1}{3}\left(\sqrt{9^{3}}-1\right)$ $=\frac{1}{3}(27-1)$ $=\frac{26}{3}$	**1** mark for correct integration **1** mark correct solution
7	b)(i)	$a=6-6t$ $v=6t-3t^{2}+c$ $v=-3$ when $t=0$ $-3=c$ $v=6t-3t^{2}-3$	**1** mark for integration **1** mark for c and expression
	b)(ii)	$v=6t-3t^{2}-3$ $x=3t^{2}-t^{3}-3t+k$ $x=2$ when $t=0$ $2=k$ $x=3t^{2}-t^{3}-3t+2$	**1** mark for integration **1** mark for k and expression
	b)(iii)	Particle comes to rest when $v=0$ $v=6t-3t^{2}-3$ $0=-3\left(t^{2}-2t+1\right)$ $0=(t-1)^{2}$ $t=1$ $x=3(1)^{2}-(1)^{3}-3(1)+2$ $=1$	**1** mark for t **1** mark for x
	b)(iv)	When $t=2$ $x=3(2)^{2}-(2)^{3}-3(2)+2$ $=12-8-6+2$ $=0$	**1** mark for correct sub and evaluation
	b)(v)	$v=-3(t-1)^{2}$ $v\le 0$ $\therefore$ particle never changes direction.	**1** mark perfect square & conclusion

Question		Solution	Marks
8	a)	Concave up parabola. Focal length 2. $(x-1)^2=8(y-2)$	**1** mark focal length **1** mark correct solution
	b)(i)	$l=\theta r$ $6\pi=\frac{2\pi}{3}r$ $18\pi=2\pi r$ $r=9$ $AD=9$	**1** mark correct substitution into arc/length formula
	b)(ii)	$\angle BDC=\frac{1}{2}\left(\pi-\frac{2\pi}{3}\right)$ $=\frac{\pi}{6}$ $\cos\frac{\pi}{6}=\frac{4.5}{BC}$ $BC=\frac{9}{2}\div\frac{\sqrt{3}}{2}$ $=\frac{9}{\sqrt{3}}$ $=3\sqrt{3}$ OR $\frac{BC}{\sin\frac{\pi}{6}}=\frac{9}{\sin\frac{2\pi}{3}}$ $BC=9\div\frac{\sqrt{3}}{2}\times\frac{1}{2}$ $=\frac{9}{2}\times\frac{2}{\sqrt{3}}$ $=3\sqrt{3}$	**1** mark for correct formula & substitution **1** mark correct solution
	b)(iii)	$A=2\times\frac{1}{2}\times9\times3\sqrt{3}\sin\frac{\pi}{6}+\frac{1}{2}\times\frac{2\pi}{3}\times9^2$ $=\frac{27\sqrt{3}}{2}+27\pi\ \text{m}^2$	**1** mark area of sector **1** mark area of triangles
	c)(i)	$y=\sec^2\left(\frac{\pi}{4}\right)$ $=\frac{1}{\cos^2\left(\frac{\pi}{4}\right)}$ $=\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$ $=2$ $y=2\cot\left(\frac{\pi}{4}\right)$ $=2\times\frac{1}{\tan\frac{\pi}{4}}$ $=2\times1$ $=2$	**1** mark for substitution into both equations
	c)(ii)	$y=\ln(\sin x)$ $\frac{dy}{dx}=\frac{\cos x}{\sin x}$ $=\cot x$	**1** mark for correct answer
	c)(iii)	$A=\int_0^{\frac{\pi}{4}}\sec^2 x\,dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}2\cot x\,dx$ $=\left[\tan x\right]_0^{\frac{\pi}{4}}+\left[2\ln(\sin x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $=\left[\left(\tan\frac{\pi}{4}\right)-\tan 0\right]+\left[2\ln\left(\sin\frac{\pi}{2}\right)-2\ln\left(\sin\frac{\pi}{4}\right)\right]$ $=1-0+\left(2\ln1-2\ln\frac{1}{\sqrt{2}}\right)$ $=1+0-2\ln(2)^{-\frac{1}{2}}$ $=1+\ln 2$	**1** mark **1** mark **1** mark correct answer

Question		Solution	Marks
9	a)(i)	$N=Ae^{kt}$ At $t=0,\ N=2000$ $2000=Ae^0$ $A=2000$ $N=2000e^{kt}$ At $t=5,\ N=3500$ $3500=2000e^{5k}$ $1.75=e^{5k}$ $\ln(1.75)=5k$ $k=\frac{\ln(1.75)}{5}$ $=0.112$	**1** mark correct formula and A value **1** mark correct sub and log
	a)(ii)	$t=60$ $N=2000e^{60\times0.112}$ $=1657635$ ($k=0.112$ used) OR $N=2000e^{60\times0.1119....}$ $=1650010$ (exact value of k used)	**1** mark for answer for either k value
	a)(iii)	$10000=2000e^{0.112t}$ $5=e^{0.112t}$ $\ln5=0.112t$ $t=\frac{\ln5}{0.112}$ $t=14$ minutes (either k same answer)	**1** mark correct substitution **1** mark correct answer
	b)(i)	Stationary points occur when $\frac{dy}{dx}=0$ $0=-e^{-x}(x-1)^2$ $x=1$ $y=2e^{-1}$ $(1,\ 2e^{-1})$ Test. $\frac{d^2y}{dx^2}=0$ $\begin{array}{c\|c\|c\|c} x & 0 & 1 & 2 \\ \hline \frac{dy}{dx} & -1 & 0 & -e^{-2} \end{array}$ Decreasing function at $x=1$. $\therefore$ $(1,\ 2e^{-1})$ is a horizontal P.O.I	**1** mark for point **1** mark for test and conclusion
	b)(ii)	Possible points of inflexion occur when $\frac{d^2y}{dx^2}=0$ $0=e^{-x}(x^2-4x+3)$ $0=(x-1)(x-3)$ $x=1$ or 3.	(marks over page)

Question		Solution	Marks
9	b)(ii)	$(1,\ 2e^{-1})$, $(3,\ 10e^{-3})$	$(1,\ 2e^{-1})$ should have been identified as a P.O.I in part (i) **1** mark for $(3,\ 10e^{-3})$ as a possible point of inflexion

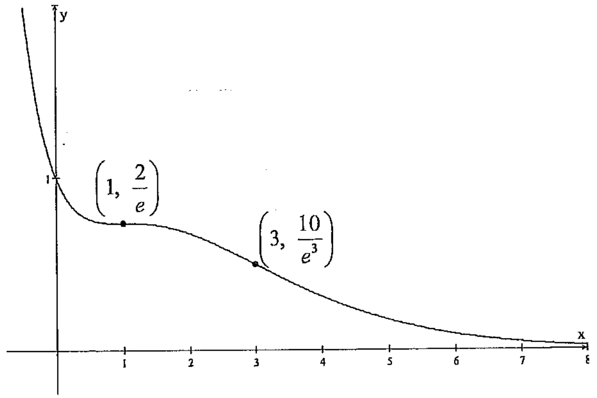
x	0	1	2	3	4
$\frac{d^2y}{dx^2}$	3>0	0	$-e^2<0$	0	$3e^{-4}>0$

Change in concavity, Change in concavity.

$\therefore (1,\ 2e^{-1})$ ia a horizontal Point of Inflexion

& $(3,\ 10e^{-3})$ is a point of Inflexion.

1 mark test & conclusion

Question		Solution	Marks
	b)(iii)		**1** mark for y-intercept and shape **1** mark for P.O.I's **1** mark for horizontal asymptote

Question		Solution	Marks
10	a)	$y=2\sin 3x+4\cos 2x$ $\frac{dy}{dx}=6\cos 3x-8\sin 2x$ $\frac{d^2y}{dx^2}=-18\sin 3x-16\cos 2x$ $-18\sin 3x-16\cos 2x+4(2\sin 3x+4\cos 2x)=p\sin 3x$ $-18\sin 3x-\cancel{16\cos 2x}+8\sin 3x+\cancel{16\cos 2x}=p\sin 3x$ $-10\sin 3x=p\sin 3x$ $p=-10$	**1** mark correct substitution **1** mark correct answer
	b)(i)	$r=0.005$ $A_1=1500000(1.005)^{12}-243161$ $=\$1349355.72$ $A_2=1349355.72(1.005)^{12}-243161$ $=\$1189420.03$ $=\$1189420$	**1** mark for A_1 **1** mark for A_2
	b)(ii)	$A_1=1500000(1.005)^{12}-243161$ $A_2=A_1(1.005)^{12}-243161$ $=\left(1500000(1.005)^{12}-243161\right)(1.005)^{12}-243161$ $=1500000(1.005)^{2\times12}-243161(1.005)^{12}-243161$ $=1500000(1.005)^{2\times12}-243161\left((1.005)^{12}+1\right)$ $A_3=A_2(1.005)^{12}-243161$ $=\left(1500000(1.005)^{2\times12}-243161(1.005)^{12}-243161\right)(1.005)^{12}-243161$ $=1500000(1.005)^{3\times12}-243161(1.005)^{2\times12}-243161(1.005)^{12}-243161$ $=1500000(1.005)^{3\times12}-243161\left((1.005)^{2\times12}+(1.005)^{12}+1\right)$ $A_n=1500000(1.005)^{n\times12}-243161\left(\frac{1\left((1.005^{12})^n-1\right)}{1.005^{12}-1}\right)$ $=150000(1.005)^{12n}-243161\left(\frac{1.005^{12n}-1}{1.005^{12}-1}\right)$	**1 mark** for expression for A_2 **1 mark** recognition of series & expression for A_3 **1 mark** sum of series
	b)(iii)	$0=1500000(1.005)^{12n}-243161\left(\frac{1.005^{12n}-1}{1.005^{12}-1}\right)$ $1500000(1.005)^{12n}(1.005^{12}-1)=243161(1.005^{12n}-1)$ $1500000(1.005)^{12n}(1.005^{12}-1)=243161(1.005^{12n})-243161$ $243161=243161(1.005^{12n})-1500000(1.005^{12n})(1.005^{12}-1)$ $243161=(1.005^{12n})(243161-1500000(1.005^{12}-1))$ $243161=(1.005^{12n})(150644.2822)$ $1.614140254=1.005^{12n}$ $\ln(1.614140254)=12n.\ln(1.005)$ $12n=\ln(150644.2822)/\ln(1.005)$ $n=8$ years	**1 mark** for $A_n=0$ and attempt a rearranging **1 mark** for solution

Question		Solution	Marks
	c)	Let x be the probability that he wins the first match $P(2nd)=0.95x$ $P(3rd)=(0.95)^2 x$ $P(4th)=(0.95)^3 x$ $P(8th)=(0.95)^7 x$	1 mark for developing series
		$P(\text{wins comp})=0.1$ $P(1).P(2)....P(8)=0.1$ $x.(0.95x).(0.95^2 x).....(0.95^7 x)=0.1$ $x^8.(0.95)^{1+2+...+7}=0.1$ $x^8.(0.95)^{28}=0.1$	1 mark for establishing equation
		$x^8=0.420473908$ $x=\sqrt[8]{0.420473908}$ $x=0.897361393$ $x=90\%$	1 mark for solution