SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2012
YEAR 7 HALF-YEARLY EXAMINATION

Mathematics

Directions to Candidates:

- Answer all questions in the spaces provided in this question booklet.

- If additional working space is needed, use the spare pages at the end of the booklet. Show clearly which question you are continuing and draw a line through the faulty answer.

- Full marks may not be awarded for careless or badly arranged work.

- Answers should be given in simplest exact form unless otherwise stated.

- Use black or blue pen for written answers, but pencil for diagrams and graphs.

- Calculators may **NOT** be used.

Time allowed: 70 minutes
Examiner: Mr R. Boros

Your name: _____

Your Mathematics Class (Tick the box)	
7E	Mr McQuillan
7F	Mr Elliott
7M	Mr Fuller
7R	Mr Comben
7S	Mr Comben
7T	Ms Kilmore

| Markers' Use Only	
Question 1	/20
Question 2	/20
Question 3	/20
Question 4	/20
Question 5	/20
Question 6	/20
Total	/120

Question 1 (20 marks)

(a) $\sqrt[3]{64} =$ .... [1]

(b) List the first five prime numbers. { } [1]

(c) If **A** is the set of students who like maths, what symbol is used for the set of students who don't like maths? .... [1]

(d) Write down the set of multiples of 8 which are less than 48. { } [1]

(e) What number is represented by the Roman numerals XLIX ? .... [1]

(f) Write down the two symbols we can use to represent the empty set. .... [2]

(g)
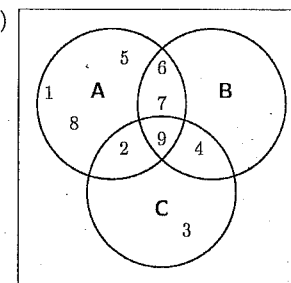
(i) List the elements in set **B**
B = { } [4]

(ii) List the set **A** ∩ **B**
{ }

(iii) List the set **A** ∩ **B** ∩ **C**
{ }

(iv) Find $n(\mathbf{A} \cup \mathbf{B} \cup \mathbf{C})$ ....

(h) In Ron's classroom there is the same number of desks in each row, and the rows are straight. His desk is third from the front and third from the back. It has one desk to its left and five to its right. How many desks are there in the room? [1]

....

....

(i) Find the average of $-3, 11, 19, 27$. ☐1

....

....

....

(j) Perform this addition:
$\begin{array}{r} 19208 \\ 7537 \\ 38690 + \\ \hline \end{array}$ ☐1

(k) Perform this multiplication: $25 \times 72 \times 1 \times 4$ ☐1

....

(l) Find three consecutive numbers which add up to 114. ☐2

....

....

(m) $4715 = 4 \times 10^3 + 7 \times 10^n + 1 \times 10^1 + 5$, find n. ☐1

....

(n) Evaluate: ☐2
(i) $\sqrt{2^2} + \sqrt{3^2} + \sqrt{4^2}$,

....

....

(ii) $\sqrt{2^2 + 3^2 + 4^2 - 2^2}$.

....

....

....

Question 2 (20 marks)

(a) Find the basic numeral for ☐3
(i) $18 \div 4 \times 7 - 9$

....

....

(ii) $8 - (-13 + 8 + 2) \times 2$

....

....

(iii) $5 \times 7 - 10 \times 7 + 16 \div 8$

....

....

(b) Write down the missing number from each pattern. ☐3
(i) $1, 1, 2, 3, 5, 8, \ldots$

(ii) $12, 13, 11, 12, 10, \ldots, 9, 10, 8$

(iii) $24, 12, 6, \ldots$

(c) Write 878 in Roman numerals. .... ☐1

(d) Express $3 \times 10^4 + 5 \times 10^2 + 9 + 3 \times 10^{-1}$ as a basic numeral. .... ☐1

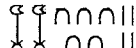(e) Given $(12)_6$ means $12 \times 11 \times 10 \times 9 \times 8 \times 7$ and ☐1
$(9)_2$ means 9×8,
find the *value* of $(8)_3$

....

(f) What is the remainder when 1518 is divided by 9? ☐1

....

....

....

(g) What is the value of this Egyptian numeral? .... $\boxed{1}$

....

(h) Measure this length to the nearest millimetre. $\boxed{1}$

(i) Draw a number line and indicate the position of the following numbers. $\boxed{2}$

$$\{2, -1, \tfrac{1}{2}, -1\tfrac{3}{4}\}$$

(j) What is the place value of the 4 in the number 234 000? .... $\boxed{1}$

(k) Write down 21 207 in words (spelling is important). $\boxed{1}$

....

....

(l) Consider 7329; what would the digit 9 in the number have to be changed to for the (new) number to be divisible by 9? $\boxed{2}$

....

....

....

(m) The average of four numbers is 48. If 8 is subtracted from each number, what is the average of the four new numbers? $\boxed{1}$

....

....

(n) A number is divided by 19 giving an answer of 25 with a remainder of 2. What is this number? $\boxed{1}$

....

....

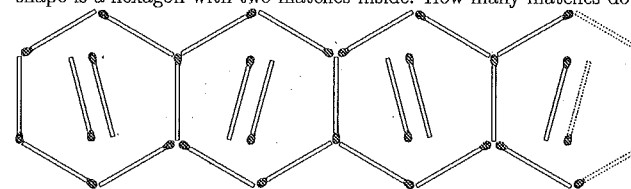...

Question 3 (20 marks)

(a) Each letter in the subtraction below represents a single digit. Find the value of each of the four pronumerals. $\boxed{4}$

$$\begin{array}{ccccc} & 6 & a & b & c \\ x & 3 & 5 & 9 & - \\ \hline 1 & 5 & 8 & 8 \end{array}$$

$x = \ldots,$ $a = \ldots,$ $b = \ldots,$ $c = \ldots$

(b) Doug has the exact number of matches to make a pattern of 63 shapes. Each shape is a hexagon with two matches inside. How many matches does he have? $\boxed{2}$
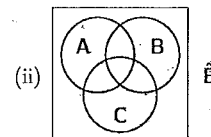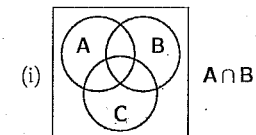
....

....

....

(c) Write down the set of single digit composite numbers. $\boxed{2}$

$$\{ \qquad\qquad\qquad\qquad \}$$

(d) By how much does the difference between one million and one hundred exceed their quotient? $\boxed{2}$

....

....

....

(e) Shade the sets given. $\boxed{2}$

(i) $A \cap B$

(ii) $\tilde{B}$

(f) List all the subsets of $\{p, q, r\}$ .... [2]

....

(g) Use the rule $b = 2a + 4$ to complete the following table: [2]

a	0	1	2	3
b				

(h) 101_2 written in expanded form is $1 \times 2^2 + 0 \times 2^1 + 1$. Write $100\,101_2$ in expanded form. [1]

....

(i)   Building a fence   The diagram shows a fence being built. [3]
                                  The vertical lines are posts and the
       —post   horizontal lines are rails.

       —rails

(i) Complete the table:

Number of posts (P)	Number of rails (R)
1	0
2	4
3	...
4	...

(ii) Write the rule using pronumerals P and R showing how the number of rails is found using the number of posts.

....

....

Question 4 (20 marks)

(a) At a maths competition, twenty questions were given. Five marks were given for a correct answer and two marks were deducted for an incorrect answer. Ron's score was 72. How many correct answers did he have? [3]

....

....

....

(b) Convert $101\,101_2$ to a base 10 number. [2]

....

....

(c) List the [2]

(i) trivial factors of 24 $\{\qquad\}$

(ii) non-trivial factors of 24 $\{\qquad\qquad\}$

(d) Find the HCF of 75 and 225. [1]

....

....

(e) Find the LCM of 36 and 44. [1]

....

....

(f) Evaluate [3]

(i) $|-9|$ ....

(ii) $|3 - 7|$ ....

(iii) $9 - |12|$ ....

(g) Given **A** = {2, 4, 6}, **B** = {5, 9, 11}, **C** = {1, 2, 3, 4, 5}, **D** = {a, b, c}, [7]
  and **E** = {5, 11, 9}, write true (T) or false (F) for these statements:
  (i) $5 \in \mathbf{B}$ (v) $\mathbf{A} \cup \mathbf{D} = \emptyset$

  (ii) $n(\mathbf{D}) = 3$ (vi) $6 \notin \mathbf{A}$

  (iii) $\mathbf{A} \longleftrightarrow \mathbf{B}$ (vii) $\mathbf{B} = \mathbf{E}$

  (iv) $\mathbf{A} \subset \mathbf{C}$

(h) If **S** = {A, B, d, e, F, g}, list the subset of capital letters **L** = {   } [1]

Question 5 (20 marks)

(a) Find $n(\mathbf{A} \cap \mathbf{B})$ given $n(\mathbf{A}) = 27$, $n(\mathbf{B}) = 11$, and $n(\mathbf{A} \cup \mathbf{B}) = 35$. [2]

....
....
....

(b) If the Universal set is {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, **A** = {2, 4, 6}, [2]
  B = {1, 2, 4, 7}, and **C** = {0, 2, 4, 8}, draw a full Venn diagram
  to illustrate these data.

(c) Arrange in descending order {0, −4, −2, 5, −6}. [1]

....

(d) Arrange the numbers 1, −1, 3, −3, −5 [5]
  on the pentagon so that the numbers
  on each side add up to −2.
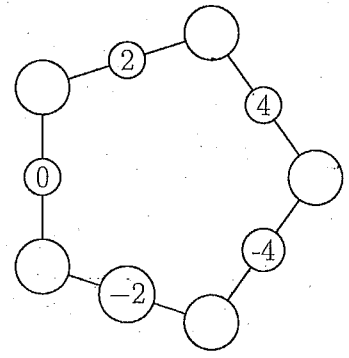
(e) Graph and label these points on the number plane:
$A(1, -3)$
$B(5, 1)$
$C(-1, 2)$
$D(3, 0)$.

$\boxed{4}$

(f) Evaluate

$\boxed{3}$

  (i) -5^2 ....
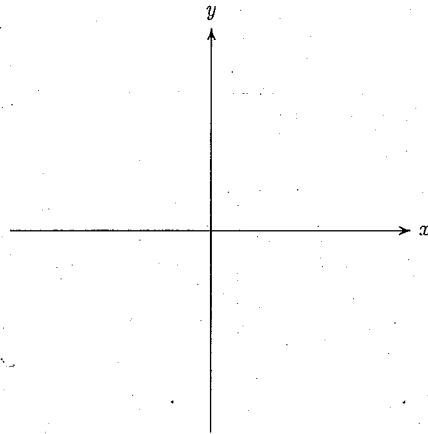
  (ii) $-7 - {}^-2 + 3$ ....

  (iii) $(-2)^3 \times 6$ ....

(g) Write down the smallest number in this set: $\{-48, -100, -150, 1\}$. .... $\boxed{1}$

(h) Which are true (T) and which are false (F)? $\boxed{2}$

  (i) $-3.5 \leqslant 3.5$ .....

  (ii) $-{}^-2 > 2$ .....

Question 6 (20 marks)

(a) Taronga Park Zoo in Sydney has 500 animals of which 400 can walk, 150 can swim, and 130 can both walk and swim. $\boxed{3}$

  (i) Create a Venn diagram to illustrate this information.

  (ii) How many can walk but not swim? ....

  (iii) How many can neither walk nor swim? ....

(b) $\boxed{2}$
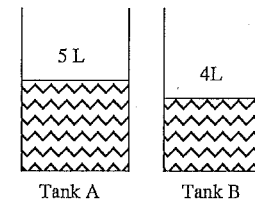
Tank A   Tank B

Here are two identical tanks which initially have 5 L of liquid in tank A and 4 L in tank B. How much liquid must be poured from A into B so that A has half as much in it as B?

....

....

(c) A book has 148 pages numbered from 1 to 148 inclusive. How many times is the digit 1 used (in total) in the numbering of the pages? $\boxed{3}$

....

....

....

(d) A mother hangs 12 nappies on a washing line using only her last 18 clothes pegs. Each nappy requires 2 pegs if hung separately, but they may be hung in groups of 2 or more. If she hangs as many as possible separately, how many must she hang together? $\boxed{3}$

....

....

....

(e) A rubber ball bounces half the height from which it falls. If the ball is thrown to a height of 24 metres, how high will it rise on the fifth bounce? $\boxed{3}$

....

....

....

(f) Three children sit along one bench in a railway carriage: | Darren | Ross | Brenda |. They decide to change their seating order. How many different *new* arrangements are possible? $\boxed{2}$

....

....

(g) $\boxed{4}$
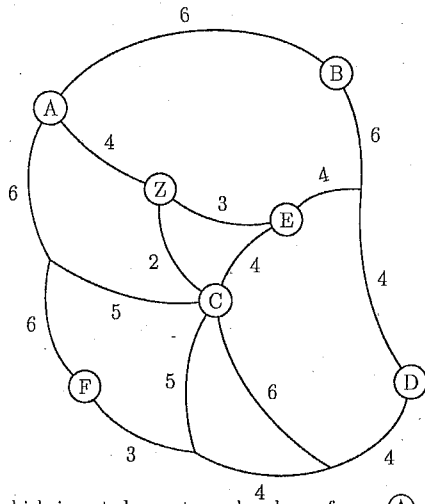
The map above, which is not drawn to scale, shows farms (A), (B), (C), (D), (E), and (F) from which the milk truck must collect milk each day and take to the depot (Z). The milk truck must start and finish at (Z) each day. Show the shortest route that the truck could take (on the above map) and give its total length.

....

....

....

End of Paper

SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2012
YEAR 7 HALF-YEARLY EXAMINATION

Mathematics Solutions

Question 1 (20 marks)

(a) $\sqrt[3]{64} = \ldots\ldots\ldots\ldots$ `1`

> Solution: 4.

(b) List the first five prime numbers.   {   }   `1`

> Solution: $\{2, 3, 5, 7, 11\}$

(c) If **A** is the set of students who like maths, what symbol is used for the set of students who don't like maths? $\ldots\ldots\ldots\ldots$ `1`

> Solution: $\overline{\mathbf{A}}$

(d) Write down the set of multiples of 8 which are less than 48.   `1`
{   }

> Solution: $\{8, 16, 24, 32, 40\}$

(e) What number is represented by the Roman numerals XLIX ? $\ldots\ldots\ldots\ldots$ `1`

> Solution: 49

(f) Write down the two symbols we can use to represent the empty set. $\ldots\ldots\ldots$ `2`

> Solution: $\emptyset, \{\}$

(g)

(i) List the elements in set **B**
   B = {   }

   Solution: $\mathbf{B} = \{4, 6, 7, 9\}$

(ii) List the set $\mathbf{A} \cap \mathbf{B}$
   {   }

   Solution: $\{6, 7, 9\}$

(iii) List the set $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$
   {   }

   Solution: $\{9\}$

(iv) Find $n(\mathbf{A} \cup \mathbf{B} \cup \mathbf{C})$ ....

   Solution: 9

[4]

(h) In Ron's classroom there is the same number of desks in each row, and the rows are straight. His desk is third from the front and third from the back. It has one desk to its left and five to its right. How many desks are there in the room? [1]

   Solution: $5 \times 7 = 35$

(i) Find the average of $-3, 11, 19, 27$. [1]

   Solution: $\dfrac{-3 + 11 + 19 + 27}{4} = \dfrac{54}{4}$,
   $= 13^{1}/_{2}.$

(j) Perform this addition: 19208
                              7537
                             38690+
   _____
   Solution: 65435

[1]

(k) Perform this multiplication: $25 \times 72 \times 1 \times 4$ [1]

   Solution: $25 \times 4 = 100$, $\therefore$ product is 7200.

(l) Find three consecutive numbers which add up to 114. [2]

   Solution: $114 \div 3 = 38$, $\therefore$ the numbers are 37, 38, 39.

(m) $4715 = 4 \times 10^3 + 7 \times 10^n + 1 \times 10^1 + 5$, find n. [1]

   Solution: $n = 2$.

(n) Evaluate:
   (i) $\sqrt{2^2} + \sqrt{3^2} + \sqrt{4^2}$,

   Solution: $\sqrt{2^2} + \sqrt{3^2} + \sqrt{4^2} = 2 + 3 + 4$,
   $= 9.$

   (ii) $\sqrt{2^2 + 3^2 + 4^2 - 2^2}$.

   Solution: $\sqrt{2^2 + 3^2 + 4^2 - 2^2} = \sqrt{4 + 9 + 16 - 4}$,
   $= \sqrt{25}$,
   $= 5.$

[2]

Question 2 (20 marks)

(a) Find the basic numeral for $\boxed{3}$

(i) $18 \div 4 \times 7 - 9$

> Solution: $4\frac{1}{2} \times 7 - 9 = 31\frac{1}{2} - 9,$
> $= 22\frac{1}{2}.$

(ii) $8 - (-13 + 8 + 2) \times 2$

> Solution: $8 - (-3) \times 2 = 8 + 6,$
> $= 14.$

(iii) $5 \times 7 - 10 \times 7 + 16 \div 8$

> Solution: $-35 + 2 = -33.$

(b) Write down the missing number from each pattern. $\boxed{3}$

(i) $1, 1, 2, 3, 5, 8, \ldots$

> Solution: 13

(ii) $12, 13, 11, 12, 10, \ldots, 9, 10, 8$

> Solution: 11

(iii) $24, 12, 6, \ldots$

> Solution: 3

(c) Write 878 in Roman numerals. .... $\boxed{1}$

> Solution: DCCCLXXVIII

(d) Express $3 \times 10^4 + 5 \times 10^2 + 9 + 3 \times 10^{-1}$ as a basic numeral. .... $\boxed{1}$

> Solution: 30 509.3

(e) Given $(12)_6$ means $12 \times 11 \times 10 \times 9 \times 8 \times 7$ and $\boxed{1}$
    $(9)_2$ means 9×8,
    find the *value* of $(8)_3$

> Solution: $8 \times 7 \times 6 = 336.$

(f) What is the remainder when 1518 is divided by 9? $\boxed{1}$

> Solution: $9\overline{)1518}$ *i.e.* remainder is 6.
>   168 r6

(g) What is the value of this Egyptian numeral? $\boxed{1}$
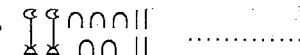
> Solution: 2054.

(h) Measure this length to the nearest millimetre. $\boxed{1}$

> Solution: 6.3 cm

(i) Draw a number line and indicate the position of the following numbers. $\boxed{2}$

$$\left\{ 2, -1, \tfrac{1}{2}, -1\tfrac{3}{4} \right\}$$
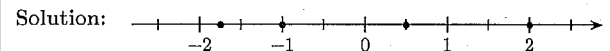
> Solution:
>

(j) What is the place value of the 4 in the number 234 000? .... $\boxed{1}$

> Solution: 4×1000

(k) Write down 21 207 in words (spelling is important). $\boxed{1}$

> Solution: Twenty one thousand, two hundred and seven.

(l) Consider 7329; what would the digit 9 in the number have to be changed to for the (new) number to be divisible by 9? [2]

Solution: $7 + 3 + 2 + 9 = 21.$
$\qquad 21 \div 9 = 2$ remainder $3.$
$\therefore$ We would have to change the 9 to a 6.

(m) The average of four numbers is 48. If 8 is subtracted from each number, what is the average of the four new numbers? [1]

Solution: $\dfrac{48 \times 4 - 4 \times 8}{4} = 40.$

(n) A number is divided by 19 giving an answer of 25 with a remainder of 2. What is this number? [1]

Solution: $2 + 19 \times 25 = 477.$

Question 3 (20 marks)

(a) Each letter in the subtraction below represents a single digit. Find the value of each of the four pronumerals. [4]

$$\begin{array}{ccccc} & 6 & a & b & c \\ x & 3 & 5 & 9 & - \\ \hline 1 & 5 & 8 & 8 \end{array}$$

$x = \ldots, \qquad a = \ldots, \qquad b = \ldots, \qquad c = \ldots$

Solution: $x = 5, \quad a = 9, \quad b = 4, \quad c = 7.$

(b) Doug has the exact number of matches to make a pattern of 63 shapes. Each shape is a hexagon with two matches inside. How many matches does he have? [2]
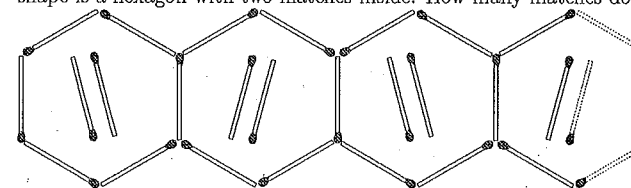
Solution: Each shape has 7 matches, plus we need one match to close the last shape. $\therefore 63 \times 7 + 1 = 442$, which is the number needed.

(c) Write down the set of single digit composite numbers. [2]

$\{ \qquad\qquad\qquad\qquad\qquad \}$

Solution: $\{4, 6, 8, 9\}$

(d) By how much does the difference between one million and one hundred exceed their quotient? [2]

Solution: $1\,000\,000 - 100 - \dfrac{1\,000\,000}{100} = 999\,900 - 10\,000,$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 989\,900.$

(e) Shade the sets given. $\boxed{2}$

(i)

$A \cap B$

Solution:

(ii)

$\tilde{B}$
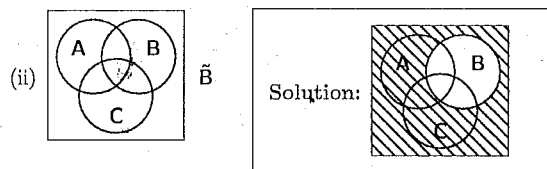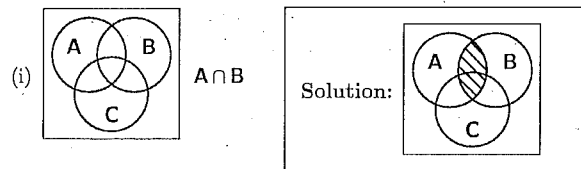
Solution:

(f) List all the subsets of $\{p, q, r\}$ .... $\boxed{2}$

Solution: $\emptyset$, $\{p\}$, $\{q\}$, $\{r\}$, $\{p, q\}$, $\{p, r\}$, $\{q, r\}$, $\{p, q, r\}$

(g) Use the rule $b = 2a + 4$ to complete the following table: $\boxed{2}$

a	0	1	2	3
b				

Solution:

a	0	1	2	3
b	4	6	8	10

(h) 101_2 written in expanded form is $1 \times 2^2 + 0 \times 2^1 + 1$. Write $100\,101_2$ in expanded form. $\boxed{1}$

Solution: $1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1$.

(i)

Building a fence
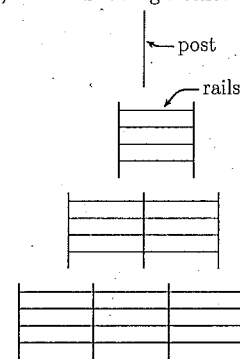
←— post

←— rails

The diagram shows a fence being built. The vertical lines are posts and the horizontal lines are rails. $\boxed{3}$

(i) Complete the table:

Number of posts (P)	Number of rails (R)
1	0
2	4
3	...
4	...

Solution:

Number of posts (P)	Number of rails (R)
1	0
2	4
3	8
4	12

(ii) Write the rule using pronumerals P and R showing how the number of rails is found using the number of posts.

Solution: $R = (P - 1) \times 4$

Question 4 (20 marks)

(a) At a maths competition, twenty questions were given. Five marks were given for a correct answer and two marks were deducted for an incorrect answer. Ron's score was 72. How many correct answers did he have? $\boxed{3}$

> Solution: 20 right, 0 wrong $= 100$ marks,
> 15 right, 5 wrong $= 65$ marks,
> 16 right, 4 wrong $= 72$ marks.
> Ron had 16 correct answers.

(b) Convert $101\,101_2$ to a base 10 number. $\boxed{2}$

> Solution: $1 + 0 + 2^2 + 2^3 + 0 + 2^5 = 1 + 4 + 8 + 32,$
> $= 45_{10}$

(c) List the $\boxed{2}$

(i) trivial factors of 24 $\{ \quad \}$

> Solution: $\{1,\ 24\}$

(ii) non-trivial factors of 24 $\{ \quad \}$

> Solution: $\{2,\ 12,\ 3,\ 8,\ 4,\ 6\}$

(d) Find the HCF of 75 and 225. $\boxed{1}$

> Solution: $75 = 3 \times 25,$
> $225 = 9 \times 25,$
> $\therefore$ HCF $= 3 \times 25,$
> $= 75.$

(e) Find the LCM of 36 and 44. $\boxed{1}$

> Solution: $36 = 9 \times 4,$
> $44 = 11 \times 4,$
> $\therefore$ LCM $= 4 \times 9 \times 11,$
> $= 396.$

(f) Evaluate $\boxed{3}$

(i) $|-9|$ ....

> Solution: 9

(ii) $|3 - 7|$ ....

> Solution: 4

(iii) $9 - |12|$ ....

> Solution: -3

(g) Given $\mathbf{A} = \{2, 4, 6\}$, $\mathbf{B} = \{5, 9, 11\}$, $\mathbf{C} = \{1, 2, 3, 4, 5\}$, $\mathbf{D} = \{a, b, c\}$, and $\mathbf{E} = \{5, 11, 9\}$, write true (T) or false (F) for these statements: $\boxed{7}$

(i) $5 \in \mathbf{B}$ (v) $\mathbf{A} \cup \mathbf{D} = \emptyset$

(ii) $n(\mathbf{D}) = 3$ (vi) $6 \notin \mathbf{A}$

(iii) $\mathbf{A} \longleftrightarrow \mathbf{B}$ (vii) $\mathbf{B} = \mathbf{E}$

(iv) $\mathbf{A} \subset \mathbf{C}$

> Solution:
> | | | | |
> |---|---|---|---|
> | (i) | $5 \in \mathbf{B}$ | T | (v) $\mathbf{A} \cup \mathbf{D} = \emptyset$ F |
> | (ii) | $n(\mathbf{D}) = 3$ | T | (vi) $6 \notin \mathbf{A}$ F |
> | (iii) | $\mathbf{A} \longleftrightarrow \mathbf{B}$ | T | (vii) $\mathbf{B} = \mathbf{E}$ T |
> | (iv) | $\mathbf{A} \subset \mathbf{C}$ | F | |

(h) If $\mathbf{S} = \{A, B, d, e, F, g\}$, list the subset of capital letters $\mathbf{L} = \{ \quad \}$ $\boxed{1}$

> Solution: $\{A,\ B,\ F\}$

Question 5 (20 marks)

(a) Find $n(\mathbf{A} \cap \mathbf{B})$ given $n(\mathbf{A}) = 27$, $n(\mathbf{B}) = 11$, and $n(\mathbf{A} \cup \mathbf{B}) = 35$. [2]

> Solution: $n(\mathbf{A} \cap \mathbf{B}) = 27 + 11 - 35,$
> $= 3.$

(b) If the Universal set is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $\mathbf{A} = \{2, 4, 6\}$, $\mathbf{B} = \{1, 2, 4, 7\}$, and $\mathbf{C} = \{0, 2, 4, 8\}$, draw a full Venn diagram to illustrate these data. [2]
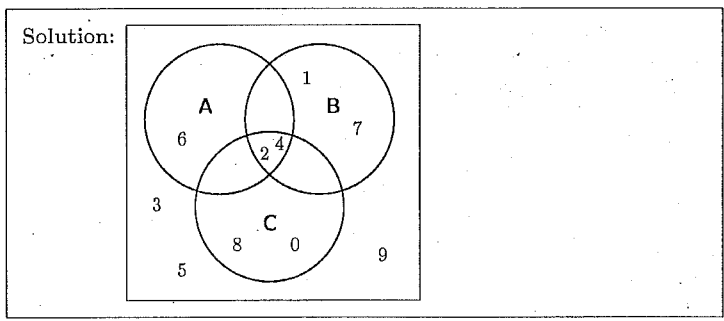
Solution:

(c) Arrange in descending order $\{0, -4, -2, 5, -6\}$. [1]

> Solution: $\{5, 0, -2, -4, -6\}$

(d) Arrange the numbers $1, -1, 3, -3, -5$ on the pentagon so that the numbers on each side add up to -2. [5]
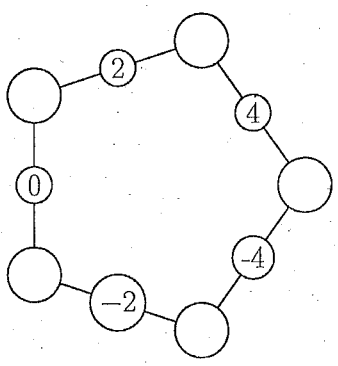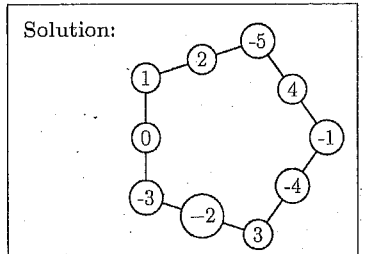
Solution:

(e) Graph and label these points on the number plane: [4]
$A(1, -3)$
$B(5, 1)$
$C(-1, 2)$
$D(3, 0)$.
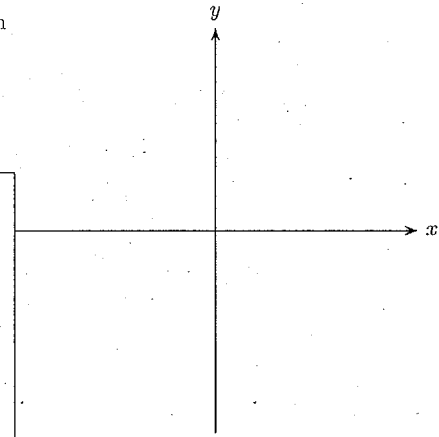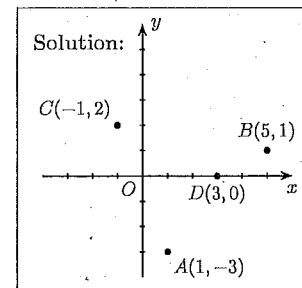
Solution:

(f) Evaluate [3]

(i) -5^2 .... Solution: -25

(ii) $-7 - {}^-2 + 3$ .... Solution: -2

(iii) $(-2)^3 \times 6$ .... Solution: -48

(g) Write down the smallest number in this set: $\{-48, -100, -150, 1\}$. .... [1]

> Solution: -150

(h) Which are true (T) and which are false (F)? [2]

(i) $-3.5 \leqslant 3.5$ ..... Solution: T

(ii) $-{}^-2 > 2$ ..... Solution: F

Question 6 (20 marks)

(a) Taronga Park Zoo in Sydney has 500 animals of which 400 can walk, 150 can swim, and 130 can both walk and swim.
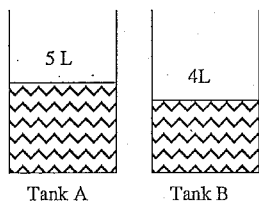
(i) Create a Venn diagram to illustrate this information.
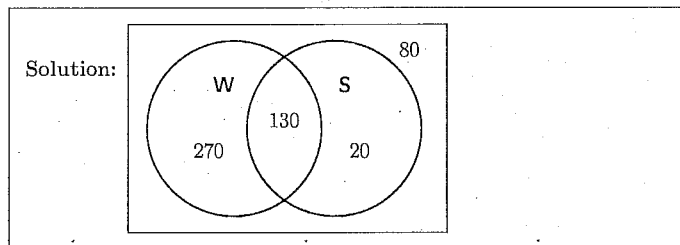
Solution:

(ii) How many can walk but not swim? .... | Solution: 270 |

(iii) How many can neither walk nor swim? .... | Solution: 80 |

(b)

Tank A   Tank B

Here are two identical tanks which initially have 5 L of liquid in tank A and 4 L in tank B. How much liquid must be poured from A into B so that A has half as much in it as B?

Solution: Total volume is 9 L.
B needs two parts, A needs one part.
One part = 3 L, $\implies$ A must lose 2 L.

(c) A book has 148 pages numbered from 1 to 148 inclusive. How many times is the digit 1 used (in total) in the numbering of the pages?

Solution: In the units column, 15 times (1, 11, 21, ... 131, 141).
In the tens column, 20 times (10, 11, 12, ... 19, 110, 111, 112, ... 119).
In the hundreds column, 49 times (100, 101, 102, ... 148).
Total times = 15 + 20 + 49 = 84.

(d) A mother hangs 12 nappies on a washing line using only her last 18 clothes pegs. Each nappy requires 2 pegs if hung separately, but they may be hung in groups of 2 or more. If she hangs as many as possible separately, how many must she hang together?

Solution: 12 together uses 13 pegs with 5 left over.
11 together and 1 separately uses 14 pegs with 4 left over.
10 together and 2 separately uses 15 pegs with 3 left over.
Following this pattern gives 7 together and 5 separately which uses all 18 pegs.

(e) A rubber ball bounces half the height from which it falls. If the ball is thrown to a height of 24 metres, how high will it rise on the fifth bounce?

Solution: Bounce 1,   12 m,
             bounce 2,   6 m,
             bounce 3,   3 m,
             bounce 4, $1\frac{1}{2}$ m,
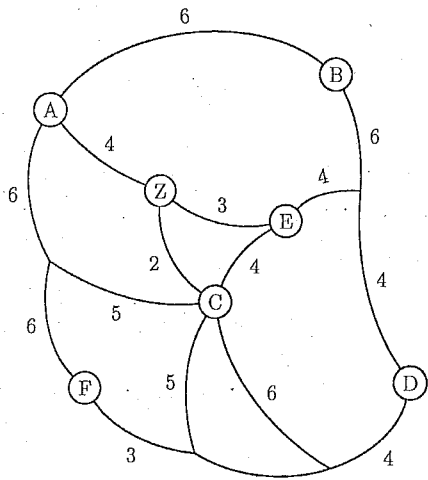   bounce 5, $\frac{3}{4}$ m.

(f) Three children sit along one bench in a railway carriage: | Darren || Ross || Brenda |.
They decide to change their seating order. How many different *new* arrangements are possible?

Solution: Any one of the three can sit in the left seat. This leaves one of two for the middle seat and only one for the last seat. Altogether there are $3 \times 2 \times 1 = 6$ possible arrangements, but this includes the original seating order. So there are only five *new* arrangements.

(g) $\boxed{4}$

The map above, which is not drawn to scale, shows farms Ⓐ, Ⓑ, Ⓒ, Ⓓ, Ⓔ, and Ⓕ from which the milk truck must collect milk each day and take to the depot Ⓩ. The milk truck must start and finish at Ⓩ each day. Show the shortest route that the truck could take (on the above map) and give its total length.

Solution:

Z—E = 3,
E—C = 4,
C—F = 5 + 3,
F—D = 3 + 4 + 4,
D—B = 4 + 6,
B—A = 6,
A—Z = 4,
total = 46.

End of Paper