Sydney Girls High School

Yearly Examination

2010

Mathematics

Year 9

Time Allowed: 70 minutes

Topics: Equations, Inequations & Formulae, Consumer Arithmetic, Coordinate Geometry, Factorising Algebraic Expressions, Statistics, Simultaneous Equations

Instructions:

- Attempt ALL 5 (five) questions.

- Not all questions are of equal value.

- Show all necessary working for each question.

- Marks will be deducted for careless or badly arranged work.

- Start each question on a new sheet.

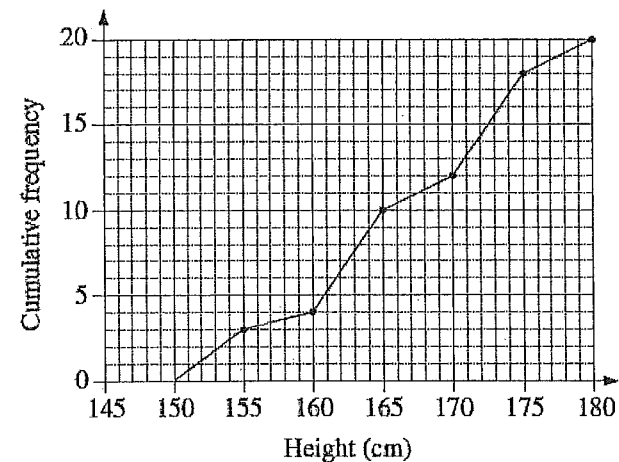- Write on one side of the paper ONLY.

- **DO NOT SPLIT PAGES INTO COLUMNS.**

Total Marks: 80

Name: _____   Teacher: _____

a. Solve $\dfrac{x}{7} = -4$.   1

b. State the gradient of the line $2x + y = -1$.   1

c. Factorise $9x + 6$.   1

d. Express $y = -\dfrac{3}{4}x + 2$ in general form.   1

e. For the scores: 2, 5, 9, 9, 7, 10. Find: 4

    i.   the mean;
    ii.   the median;
    iii.   the range;
    iv.   the mode.

f. The heights of 20 year 8 students are plotted on the cumulative frequency polygon below.

What is the median height of students in year 8?   1

g. Stephanie buys a car for $5742 including 10% GST. What is the value of the GST component?   1

h. Using the graph, what is the solution to the simultaneous equations? 1
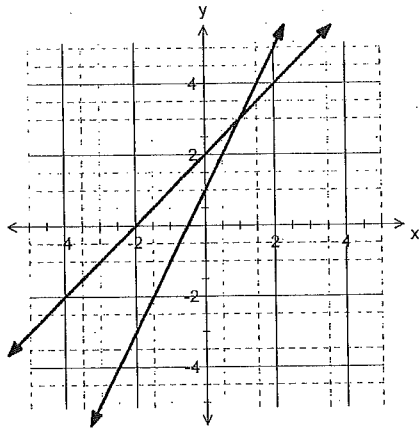
i. What is the equation of the straight line which passes through the origin and is parallel to the line $y = 3x + 2$? 1

j. Donald is paid $14.50 an hour to flip burgers at Mc Ronald's. Calculate his weekly income in a week where he is paid for 25 hours at normal time, 3 hours at time and a half and 2 hours at double time. 2

k. Simplify $\dfrac{x}{3} + \dfrac{3x-1}{2}$. 2

l. Eliza worked for 24 hours as shown on her payslip.

What was her hourly rate of pay? 1

a. Karen bought a new Mercedes Benz for $65000. Two years later she sold it for $42000. Calculate her loss as a percentage of the cost price of the car. 2

b. Solve the equation $4x - 2(x - 3) = 14$. 2

c. Milo is sold in either 350g or 500g containers. The 350g container sells for $7 and the 500g sells for $11. Which container is better value for money? Justify your response with suitable calculations. 2
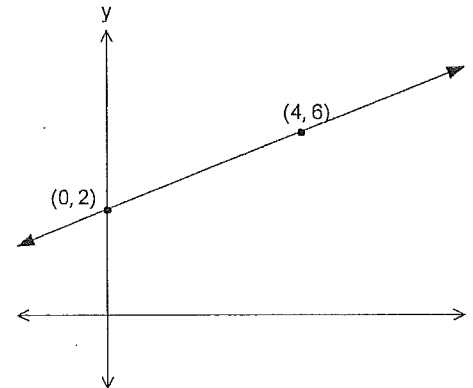
d. Solve $-3x + 5 \le -4$. 2

e. Graph the line $y = -2x + 3$, clearly showing where the graph cuts the x and y axes. 2

f. Find the equation of the line graphed below. 2

g. If $s = ut + \dfrac{1}{2}at^2$, find the value of a when $s = 44$, $u = 5$ and $t = 4$. 2

h. If $y = 3x + 5$, and x is increased by 3, what will be the corresponding increase in y? 2

QUESTION 3 (15 Marks)

a. Fully factorise the following:

    i. $4ac - 3bd + 2ad - 6bc$ 2

    ii. $9x^2 - 4y^2$ 2

    iii. $6x^2 + 17x + 5$ 2

b. Solve simultaneously: 3

$$5x - y = 19$$
$$2x + 5y = -14$$

c. Simplify the following:

    i. $\dfrac{x^2 - x}{2x - 2}$ 2

    ii. $\dfrac{3}{x+1} - \dfrac{3+x}{x+3}$ 2

    iii. $\dfrac{3(b-3)(b+3)}{a(b+4)(b-4)} \div \dfrac{b+3}{2(b+4)}$ 2

QUESTION 4 (17 Marks)

a. For the points $A(2,-1)$, $B(3,4)$ and $C(4,-7)$:

    i.   Find the distance AB in exact form.   2

    ii.   Find the midpoint of AC.   2

    iii.   Find the gradient of AC.   2

    iv.   Find the equation of the line which passes through B and is perpendicular to AC.   2

b. The table below shows personal income tax rates

Taxable income	Tax on this income
$0 - $6000	Nil.
$6001 - $20 000	17 cents for each $1 over $6000.
$20 001 - $50 000	$2380 plus 30 cents for each $1 over $20 000.
$50 001 - $60 000	$11 380 plus 42 cents for each $1 over $50 000.
$60 001 and over	$15 580 plus 47 cents for each $1 over $60 000.

In 2010, Alice had a gross income of $52300 and tax deductions that totalled $4250.

    i.   Calculate Alice's taxable income.   1

    ii.   Calculate the income tax payable on Alice's taxable income.   2

    iii.   During the year Alice paid $11500 in tax instalments. Find her refund or balance payable and calculate the amount.   1

c. The weekly income of 100 employees is listed below in a grouped frequency distribution table.

Class	Class Centre (c.c.)	Frequency (f)	f × c.c.	c.f.
$400 - $499		2		
$500 - $599		33		
$600 - $699		10		
$700 - $799		37		
$800 - $899		18		
		$\sum f =$	$\sum f \times c.c. =$	

i. On your answer sheet, copy and complete the frequency distribution table.   3

ii. What is the median class?   1

iii. Calculate an estimate for the mean of the data.   1

a. A group of 39 people went to see Whitney Houston in concert. There were both adults and children in the group. The total cost of the tickets was $939, with children paying $17 each and adults paying $29 each.

     i. Form two equations to represent the information given.   1

     ii. Solve the equations to find how many adults were in the group?   2

b. In order to pass her Maths course, Alexis must average at least 50% over 5 assessment tasks. After the first 4 assessment tasks, Alexis has a mean mark of 45%. All tasks have equal weight. What is the minimum mark, out of 100, that Alexis must score in the fifth assessment task to pass the course?   2

c. Sketch the region represented by the intersection of: 4

$$x - y > 2$$
$$y \geq -2x + 1$$

d. Make y the subject of $x^2 = \dfrac{y}{y-2}$.   3

e. Increasing average speed from 80km/h to 100km/h saves 10 minutes on a certain trip. How far is the trip?   3

Question One:

a. $x = -28$

b. $y = -2x - 1$ ∴ $m = -2$

c. $3(3x + 2)$

d. $3x + 4y - 8 = 0$

e. i) mean $= 7$
   ii) median $= 8$
   iii) range $= 8$
   iv) mode $= 9$

f. Median $= 165$ cm   or $\dfrac{165 + 170}{2} = 167.5$ cm

g. $5742 - $5220 = $522

h. $(1, 3)$

i. $y = 3x$

j. Pay $= 25 \times \$14.50 + 3 \times 1.5 \times 14.50 + 2 \times 2 \times 14.50$
      $= \$485.75$

k. $\dfrac{2x + 9x - 3}{6} = \dfrac{11x - 3}{6}$

l. $291.20 \div 26 = $11.20

Q2

a) Loss $= \$65000 - \$42000$

    $= \$230000$

    Loss as percentage of cost $= \dfrac{23000}{65000} \times 100$

    $= 35.38\%$

b) $4x - 2(x - 3) = 14$

    $4x - 2x + 6 = 14$

    $2x = 8$

    $x = 4$

c) Cost per $100g = \dfrac{7}{3.5}$

    Cost per $100g = \$2$ (for $350g$ at $\$7$)

    Cost per $100g = \dfrac{11}{5}$

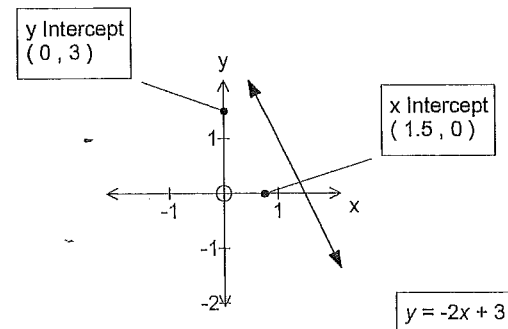   Cost per $100g = \$2.2$ (for $500g$ at $\$11$)

    The best value is 350 g for $\$7$

d) $-3x + 5 \le -4$

    $-3x \le -9$ divide by -3

    $x \ge 3$

e)

y Intercept (0 , 3)

x Intercept (1.5 , 0)

$y = -2x + 3$

f) $m = \dfrac{6 - 2}{4 - 0}$

    $m = \dfrac{4}{4}$

    $m = 1$

    $y - 2 = 1(x - 0)$

    ∴ $x - y + 2 = 0$

g) $s = ut + \dfrac{1}{2}at^2$

    $44 = 5 \times 4 + \dfrac{1}{2}a \times 4^2$

    $44 = 20 + \dfrac{1}{2}a \times 16$

    $44 = 20 + 8a$

    $24 = 8a$

    $a = 3$

h) $y = 3x + 5$

    $y = 3(x + 3) + 5$

    $y = 3x + 9 + 5$

    ∴ the increase is 9

Question 3

a.

i.
$$4ac + 2ad - 3bd - 6bc = 2a(2c+d) - 3b(d+2c)$$
$$= (2a-3b)(2c+d)$$

ii. $9x^2 - 4y^2 = (3x-2y)(3x+2y)$

iii.
$$6x^2 + 17x + 5 = 6x^2 + 2x + 15x + 5$$
$$= 2x(3x+1) + 5(3x+1)$$
$$= (2x+5)(3x+1)$$

b.
$$5x - y = 19 \rightarrow (1)$$
$$2x + 5y = -14 \rightarrow (2)$$
From (1) $y = 5x - 19$
Sub into (2) $2x + 5(5x-19) = -14$
$$2x + 25x - 95 = -14$$
$$27x = 81$$
$$x = 3$$
Sub $x = 3$ into (3) $y = 5\times3 - 19$
$$= -4$$
$$\therefore x = 3, \ y = -4$$

c.

i.
$$\frac{x^2-x}{2x-2} = \frac{x(x-1)}{2(x-1)}$$
$$= \frac{x}{2}$$

ii.
$$\frac{3}{x+1} - \frac{3+x}{x+3} = \frac{3}{x+1} - 1$$
$$= \frac{3}{x+1} - \frac{x+1}{x+1}$$
$$= \frac{3-x-1}{x+1}$$
$$= \frac{2-x}{x+1}$$

iii.
$$\frac{3(b-3)(b+3)}{a(b+4)(b-4)} \div \frac{b+3}{2(b+4)} = \frac{3(b-3)(b+3)}{a(b+4)(b-4)} \times \frac{2(b+4)}{b+3}$$
$$= \frac{6(b-3)}{a(b-4)}$$

Question 4 - Year 9 $\quad\boxed{17}$

a) $A\ \overset{x_1, \ y_1}{(2,-1)} \quad B\ (3,4) \quad C\ \overset{x_2\ y_2}{(4,-7)}$

i) distance $AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$$= \sqrt{(3-2)^2 + (4+1)^2}$$
$$= \sqrt{1 + 5^2}$$
$$= \sqrt{26} \quad ②$$

ii) Midpoint $AC \left(\dfrac{x_1+x_2}{2}, \ \dfrac{y_1+y_2}{2}\right)$
$$= \left(\frac{2+4}{2}, \ \frac{-1-7}{2}\right)$$
$$= (3, -4) \quad ②$$

iii) Gradient $AC = \dfrac{y_2-y_1}{x_2-x_1}$
$$= \frac{-7+1}{4-2}$$
$$= \frac{-6}{2}$$
$$m_1 = -3 \quad ②$$

iv) Perpendicular to AC
$$m_2 = \frac{1}{3}$$
$$B(3,4)$$
Equation of line
$$y - y_1 = m(x-x_1)$$
$$y - 4 = \frac{1}{3}(x-3)$$
$$3y - 12 = x - 3$$
$$x - 3y + 9 = 0 \quad ②$$
OR/ $y = \frac{1}{3}x + 3$

b) i) Taxable Income
$$= \text{gross income} - \text{tax deductions}$$
$$= 52\,300 - 4250$$
$$= \$48\,050 \quad ①$$

ii) Tax payable
$$= 2380 + \left(\frac{30}{100} \times 28\,050\right)$$
$$= 2380 + 8415$$
$$= \$10\,795 \quad ②$$

iii) Refund $= \$11\,500 - \$10\,795$
$$= \$705. \quad ①$$

c) i)

class	class centre cc ①	frequency f ①	f×cc ①	cf. ①
$ 400-499	449.5	2	899	2
$ 500-599	549.5	33	18133.5	35
$ 600-699	649.5	10	6495	45
$ 700-799	749.5	37	27731.5	
$ 800-899	849.5	18	15291	100
		$\Sigma f = 100$	$\Sigma f_x cc = $ 68550	

ii) \$700-799 $\quad ①$

iii) Mean $\bar{x} = \dfrac{\Sigma f \times cc}{\Sigma f}$
$$= \frac{68550}{100}$$
$$\bar{x} = 685.5 \quad ①$$

Question 5

a.

i. Let the number of children attending be x and the number of adults be y.

$$x + y = 39 \rightarrow (1)$$
$$17x + 29y = 939 \rightarrow (2)$$

ii.

$$x + y = 39 \rightarrow (1)$$
$$17x + 29y = 939 \rightarrow (2)$$
$$17x + 17y = 663 \rightarrow (3)\{(2)\times 1\}$$
$$12y = 276 \rightarrow (2) \div (3)$$
$$y = 23$$
$$x = 16$$

$\therefore$ There were 23 adults in the group.

b.

let x be the minimum number of marks Alexis must score in her 5th assessment task.
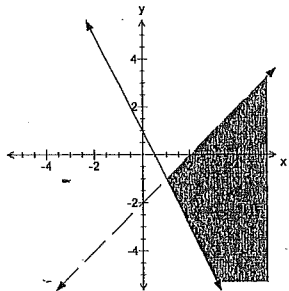
$$\frac{4 \times 45 + x}{5} = 50$$
$$\frac{180 + x}{5} = 50$$
$$180 + x = 250$$
$$x = 70$$

$\therefore$ She must score at least 70 to pass the course.

c.

d.

$$x^2 = \frac{y}{y-2}$$
$$x^2(y-2) = y$$
$$x^2 y - 2x^2 = y$$
$$x^2 y - y = 2x^2$$
$$y(x^2 - 1) = 2x^2$$
$$y = \frac{2x^2}{x^2 - 1}$$

e.

Let x hours be the time taken when travelling at $80km/h$

$\therefore$ the time taken when travelling at $100km/h$ is $\left(x - \frac{1}{6}\right)$ hours.

In both instances the distance travelled is the same therefore:

$$80x = 100\left(x - \frac{1}{6}\right)$$
$$80x = 100x - \frac{50}{3}$$
$$-20x = -\frac{50}{3}$$
$$x = \frac{5}{6}$$

$\therefore$ Therefore the time taken when travelling at 80km/h is $\frac{5}{6}$ of an hour.

$$D = 80 \times \frac{5}{6}$$
$$= 66\frac{2}{3} km$$